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PLASTIC BUCKLING ANALYSIS OF AXIALLY STIFFENED SHELLS

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INTRODUCTION

The frequent use of stiffened cylindrical shells as components of offshore and aeronautical structures is at the origin of most analysis on the buckling behavior of this kind of structures.

In the case of axisymetric shells with a high number of regularly spaced stiffeners, a multi-layer approach is used. In the elastic field this approach is equivalent of the smeared technic but in the plastic field the originality is that a similar plastic criteria is applied separately to the shell and the stiffeners. The tangent stiffness of the structure is obtained by the addition of the tangent stress-strain matrix of differents layers.

A numerical analysis is presented at the end of this article with a comparison with experimental results.

1 - ELASTIC STRESS-STRAIN MATRIX

The general approach is to make appropriate kinematical assumption which relate the displacements of the stiffener to the displacements of the shell. The stiffness matrix is derived for the combined structure. In our model, the stiffened shell is considered as a multi-layer shell. It is the same approach than the smeared technic which distribute the rigidity of the stiffeners on all the surface of the shell with the difference that we separate the global rigidity in layers. One layer for the shell and one layer for the each family of stiffeners (axial or circumferencial).

1.1 - SHELL MATRIX :

Relation between resultant stresses and strains of the shell is :

$$\left\{\begin{matrix}\{N\}\\ \{M\}\end{matrix}\right\} = \begin{bmatrix} h.[G] & [0] \\ [0] & \frac{h^3}{12}[G] \end{bmatrix} \left\{\begin{matrix}\{\varepsilon\}\\ \{\chi\}\end{matrix}\right\} \quad \text{with} \quad [G] = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix}$$

1.2 - EQUIVALENT MATRIX FOR STIFFENERS :

All displacement fields of the stiffeners are expressed in terms of the displacement of the meddle surface of the shell with deformation due to bending and torsion varying linearly. The stiffener is treated as a beam so that the cross section do not distort. The general beam stresses taken into account are the axial extension N^r, the out of plate bending M^r, the warping T^r and the St VENANT torsion M_t^r (fig 1). All interaction between the shell and the stiffener is neglected.

$$\tau_{def} = \begin{Bmatrix} \varepsilon_0^r \\ 0 \\ 2\tau_0^r \\ \chi_0^r \\ 0 \\ 2\theta_0^r \end{Bmatrix}^T \begin{bmatrix} \frac{E.A}{b} & 0 & 0 & g\frac{E.A}{b} & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 \\ & & \frac{G.A}{b} & 0 & 0 & 2.g\frac{G.A}{3.b} \\ & & & g^2\frac{E.A}{b}+\frac{E.I}{b} & 0 & 0 \\ & (sym.) & & & 0 & 0 \\ & & & & & \frac{G.J}{4.b}+g^2\frac{G.A}{2.b} \end{bmatrix} \begin{Bmatrix} \varepsilon_s \\ \varepsilon_\theta \\ \varepsilon_{s\theta} \\ \chi_s \\ \chi_\theta \\ \chi_{s\theta} \end{Bmatrix}$$

We have now the elastic stress-strain matrix of the equivalent eccentric layer. In an elastic analysis the geometry of the section of the stiffener is described its characteristics A,I and J. All kind of section are treated. When the elastic limit is overload a plastic criteria is applied. This criteria is written only for rectangular sections. So for example to represent a T section in the plastic field, two rectangles are needed and the structure is designed by one layer for the shell and two layers for the stiffeners (fig 2). The plastic criteria is applied in each layer. Thus we can have local plastification of the shell or the stiffeners. It is the main difference of our analysis in comparison with the classical smeared technic.

2 - PLASTIC ANALYSIS

Our model consider isotropic hardening materials. Two functions γ and β of an equivalent plastique strain and of the traction curve of the material are introduiced in ILYUSHIN model.

$$f = \frac{\bar{N}}{No^2} + \frac{s}{\sqrt{3}} \cdot \frac{\overline{NM}}{\gamma . No . Mo} + \frac{\bar{M}}{\gamma^2 . Mo^2} = \beta^2$$

Flow rule is applied with the omission of γ variation ($d\gamma=0$).

3 - PLASTIC BUCKLING

After the incremental non linear analysis including plasticity and large displacements, the stability of the equilibrium can be performed. We have then to evaluate the local stress-strain matrix on the actual configuration.
Three various theories are used with the flow potential :

3.1 - FLOW THEORY:

The application of the flow rule on the approximate yield criterion gives (2.12):

$$\begin{Bmatrix} \{dN\} \\ \{dM\} \end{Bmatrix} = \begin{bmatrix} D_T \end{bmatrix} \begin{Bmatrix} \{d\varepsilon\} \\ \{d\chi\} \end{Bmatrix}$$

$\begin{bmatrix} D_T \end{bmatrix}$ is the local matrix function of the stress and the equivalent plastic deformation ζ.

3.2 - TANGENT MODULUS THEORY:

The second theory and the most simple consist on replacing the Young modulus of the elastic matrix by the tangent young modulous when the section is plastic.

The tangent matrix is then obtained by : $\left[D_T \right]_g = \frac{E_T}{E} \left[D \right]_g$

4 - PLASTIC BUCKLING OF A CYLINDER UNDER COMBINED AXIAL COMPRESSION AND INTERNAL PRESSURE

A stiffened cylinder clamped at its two extremities and submitted to an axial compression and to a simultaneous internal pressure is tested against buckling. The experimental test was made by LIMAM on an electro deposit copper cylinder with 60 T stiffeners. The cylinder has a nominal radius of 135 mm, a length of 270 mm, a thickness of 0.2 mm in the current part and 1.2 mm at its extremities. The internal pressure is 0.18 Mpa (fig 5).

The numerical model is designed in the Inca code with three layers: One for the shell and one for each part of the stiffener. large deflection elasto-plastic calculation is used with two nodes shell elements. The plastic buckling is evaluated with the three presented methods.

(Fig 6) shows the load-displacement response in experimental and numerical cases and different plastic buckling values. The initial load F=-10.21 kN is the axial traction due to internal pressure. (Fig 7) shows the buckling mode obtained with the flow theory and the value of the equivalent stress $f = \begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix}^T \left[F \right] \begin{Bmatrix} \{N\} \\ \{M\} \end{Bmatrix}$ through the height of the cylinder for the axial load F=18.40 kN. We see that the buckle appears in a part where stiffeners are plastified.

In this example the flow theory leads to very good results because the internal pressure eclipse the initial imperfections of this very thin shell. The tested cylinder is the perfect example of weakly stiffened shell. We have 60 stiffeners which represents only 4% of the total structure. When the number of stiffeners decrease, buckling predictions with the presented smeared technic are too high because the model don't take account local displacements.

CONCLUSION

A finite element method has been presented for the plastic buckling of axially stiffened shells. The axisymetric calculation and the modified ILYUSHIN criterion which assume a sudden plastification through the depth is considerably economical and has been shown to give very good results.

The equivalent layer method associated with the Comu element of INCA code can evaluate the buckling of structures submitted to non axisymetric load or with initial imperfections. It was presented in this article axial stiffening, but it can be applied in the same way for circumferential stiffeners or a combinaison of axial and circumferencial stiffeners.

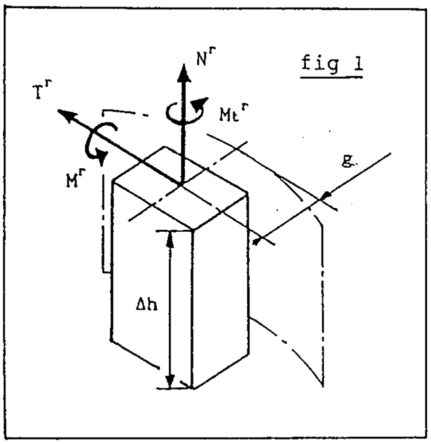


fig 1

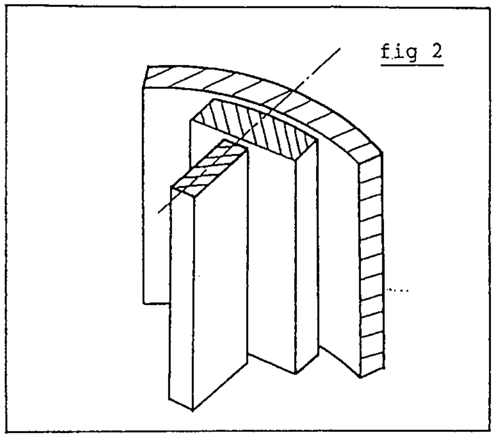
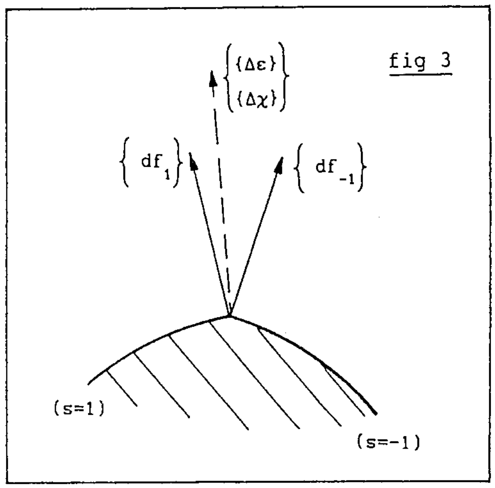


fig 2



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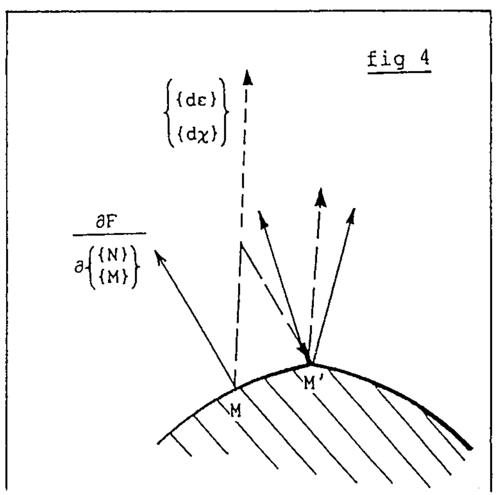


fig 4

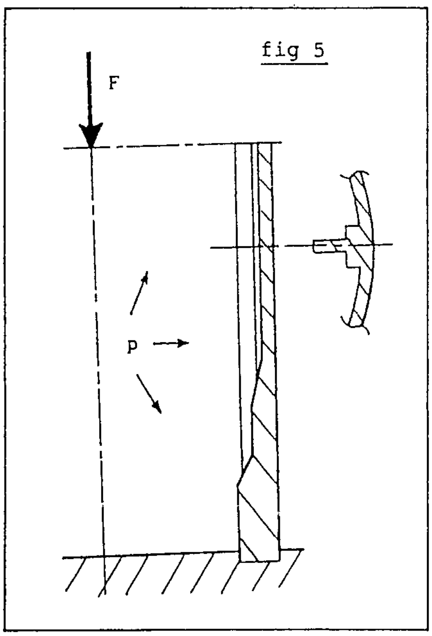
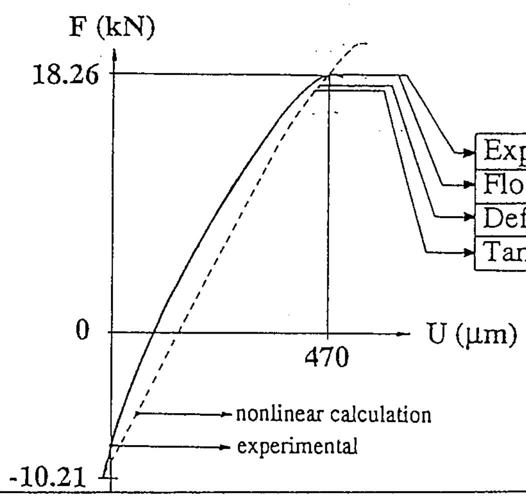


fig 5

Plastic buckling of an axially stiffened shell

Load-displacement response

fig 6



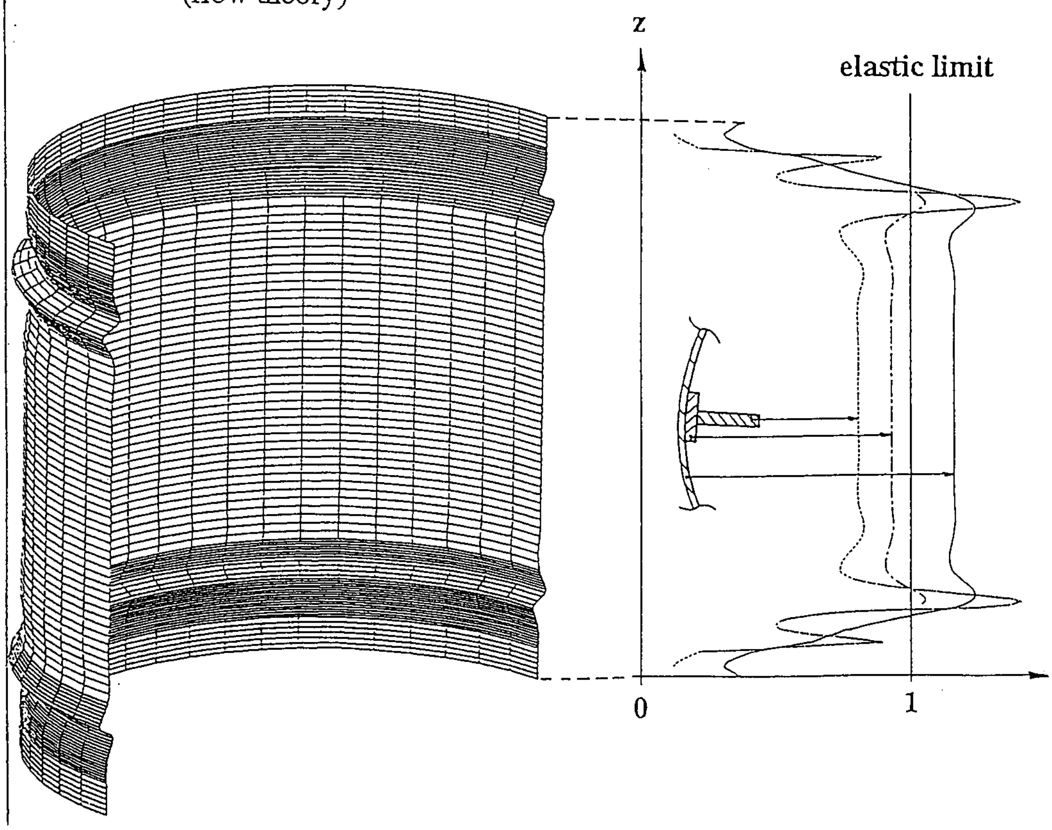
Buckling results

	F (kN)	U (µm)
Experimental	18.26	470
Flow theory	18.40	471
Deformation theory	17.56	450
Tangent modulous theory	17.27	442

Buckling mode (flow theory)

fig 7

Equivalent stress



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