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LOCAL INSTABILITY ANALYSIS OF THIN WALLED STRUCTURAL MEMBERS USING PLATE FINITE ELEMENTS

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Abstract

An analysis of the instability of axially compressed thin walled columns is performed, using a plate finite element, obtained through an incremental lagrangian formulation, which is based on the Mindlin-Reissner plate theory. The secondary bifurcation phenomena, which are characterized by the local failure of the flanges, in standard thin walled columns with L and U cross sections, are studied.

1. INTRODUCTION

The great effort devoted during the last decades in using numerical methods applied to structural analysis have consolidated their reliability for solving a significant number of problems in engineering. Particularly, the finite element method (FEM) has experienced major developments, compared to other numeric techniques used and, considering the solution of elastic stability problems of structures, the FEM has been utilized with great success (Gallagher, 1987).

Nevertheless, the correct solutions of certain non-linear problems in beams and plates, even with the help of the FEM, are still being highly investigated in order to assure the appropriate simulation of these phenomena. One of these cases is the instability analysis of thin walled structures, such as columns with thin webs and flanges. To solve these type of problems, the classical approach for the elastic stability problems of columns is often used. However, with the use of beam-column finite elements, some local effects, such as the local buckling in axially compressed columns, which characterizes a secondary bifurcation case, are not allowed to be simulated. This phenomenon is characterized by the failure of a part of the column, such as the flange and, therefore, differs very much from the classical Euler type of buckling, especially considering the critical loads and the correspondent mode shapes (Rhodes and Walker, 1980).

This work presents a study of the instability of axially compressed columns, made of thin webs and flanges, using a plate finite element. This plate finite element is obtained through an incremental updated lagrangian formulation of the non-linear plate bending problem, based on the Mindlin-Reissner plate theory. To solve properly this type of problem, the coupling between bending and stretching has to be taken into account. The use of the Mindlin-Reissner plate theory has the advantages of requiring only C^o-continuity for the displacements interpolation functions to be used, and the uncoupling between the rotational and translational degrees of freedom of the element (Pica et alli., 1980).

The updated lagrangian formulation leads to a set of finite element equations, which is solved at each increment of the load, thus producing the equilibrium paths of the thin walled columns studied. To avoid the problems of bad conditioning of this set of equations, when the equilibrium path passes by a bifurcation point or a limit load point, small imperfections are introduced into the flanges of these thin walled columns, in order to investigate the occurrence of secondary bifurcations, which are characterized by the local instability of the flanges (Brush and Almroth, 1975). From this incremental formulation, one can also obtain the linearized eigenvalue problem for an axially loaded thin walled column, which gives the Euler critical loads and the correspondent mode shapes (Ramm and Stegmüller, 1982).

In this work, an application of this methodology is presented, which constitutes a study about local instabilities in standard thin walled columns with L and U cross sections, using a Lagrange-type plate finite element of 9 nodes, obtained from the above referred incremental formulation.

2. THE PLATE FINITE ELEMENT

An incremental lagrangian formulation, often used in the study of non-linear problems in structural stability (Pica et alli., 1980), is taken as the basis upon which the updated lagrangian version used here is developed, and which results in the finite element equations for the problem of buckling and post-buckling of flat plates. The incremental principle of virtual work for an elastic body, such as a plate, can be written, in its updated lagrangian version, as (Washizu, 1982)

$$\int_\Omega \left(\Delta S_{ij}\delta\varepsilon_{ij} + \sigma_{ij}\Delta u_{k,i}\delta u_{k,j}\right)d\Omega = \int_\Omega \Delta b_i \delta u_i d\Omega + \int_{\Gamma_F} \Delta t_i^F \delta u_i d\Gamma + R \quad , \tag{1a}$$

where

$$R = -\int_\Omega \sigma_{ij}\delta\varepsilon_{ij} d\Omega + \int_\Omega b_i \delta u_i d\Omega + \int_{\Gamma_F} t_i^F \delta u_i \quad , \tag{1b}$$

and where the displacement field, that represents the motion of a point outside the reference surface of the plate, is given by

$$u_\alpha(x,y,z) \cong u_\alpha^o(x,y) + z\theta_\alpha(x,y) \quad , \quad \alpha = 1,2 \tag{2a}$$

$$u_z(x,y,z) \cong w(x,y) \quad , \tag{2b}$$

u_α^o being the axial displacements, and w being the transversal displacement of a point on the reference surface, and θ_α representing the rotations of the normals to the reference surface, prior to the deformation process of the plate. The measure of deformation used is the Green strain tensor, written in its incremental form (Washizu, 1982). The incremental principle of virtual work, given by (1), is submitted to a discretization procedure in which the whole plate is assumed to be divided into E elements, and inside of each plate element the incremental and virtual displacements are interpolated in terms of the incremental and virtual nodal displacements, in the following manner:

$$\Delta u^{\circ}_{\alpha}(x,y) = N_I(x,y)\Delta u^{\circ}_{\alpha I} \tag{3a}$$

$$\Delta\theta_{\alpha}(x,y) = N_I(x,y)\Delta\theta_{\alpha I} \tag{3b}$$

$$\Delta w(x,y) = N_I(x,y)\Delta w_I \quad , \tag{3c}$$

where I varies from 1 to 9, and $N_I(x,y)$ are the interpolation functions for a Lagrange-type isoparametric finite element of 9 nodes, which are given by (Pica et alli., 1980)

$$N_I = \frac{1}{4}\left(1+\xi\xi_I\right)\left(1+\eta\eta_I\right)\left(\xi\xi_I+\eta\eta_I-1\right) \qquad I = 1,3,5 \text{ and } 7 \tag{4a}$$

$$N_I = \frac{1}{2}\left(1-\xi^2\right)\left(1+\eta\eta_I\right) \qquad I = 2 \text{ and } 6 \tag{4b}$$

$$N_I = \frac{1}{2}\left(1+\xi\xi_I\right)\left(1-\eta^2\right) \qquad I = 4 \text{ and } 8 \tag{4c}$$

$$N_I = \left(1-\xi^2\right)\left(1-\eta^2\right) \qquad I = 9 \; . \tag{4d}$$

The discretization procedure leads to a set of equations, given by

$$\left[K^{\Delta u}_{\alpha I\beta J} + K^{\sigma}_{\alpha I\beta J}\right]\left\{\Delta U_{\beta J}\right\} = \left\{\Delta F_{\alpha I}\right\} \quad , \tag{5}$$

which are the incremental finite element equations for a problem of buckling of a plate, or of whatever structure, submitted to buckling efforts, and which is being modelled by plate finite elements. Notice in equation (5), that the terms $K^{\Delta u}_{\alpha I\beta J}$ are the components of incremental stiffness matrix, similar to the stiffness matrix obtained in a linear analysis, and that the terms $K^{\sigma}_{\alpha I\beta J}$ are the components of the geometric, or initial stresses, stiffness matrix. In this work, the finite element obtained is being used to study the phenomenon of local instability in thin walled columns, whose flanges and webs are discretized by using plate finite elements and to which small imperfections are added. With this procedure, one can study the behaviour of imperfect columns but, mainly, search for the possible secondary bifurcation cases that exist for certain ratios between the width and the thickness of the flanges.

Additionally, to compute the classical Euler buckling loads, and the correspondent mode shapes, of these thin walled columns, one can obtain from the equation (5), the following linear eigenvalue problem for the linearized problem of buckling of a plate (Ramm and Stegmüller, 1982):

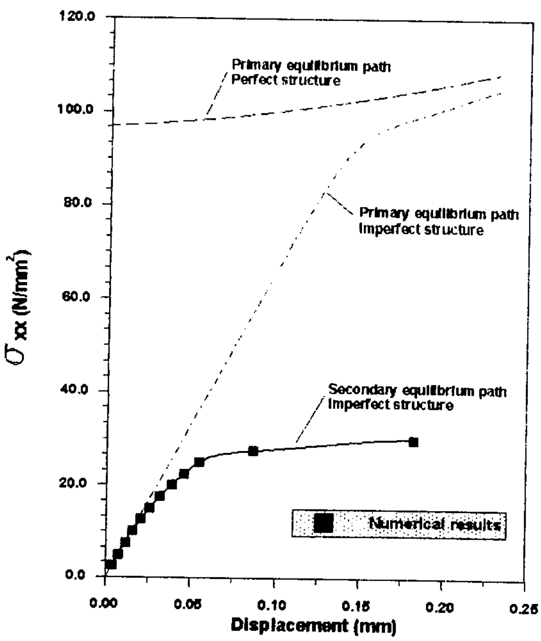
$$\left[K^{\Delta u} + \lambda K^{\sigma}\right] X = 0 \quad , \qquad (6)$$

where λ is the factor that has to multiply the value of stress or load at any given configuration, in order that the critical load can be achieved.

3. NUMERICAL RESULTS

The plate finite element above described was used to numerically study the local instabilities in thin walled columns. In this work, aluminium thin walled columns, with E=74.5 GPa and ν=0.30, were considered.

The first standard thin walled column investigated is the one with the L cross section, whose flanges have width b and thickness t. Various ratios between the thickness and the width of the flanges were tested for the same column length of 559.0 mm (Boresi and Sidebottom, 1985). The results of these analyses are shown in Figure 1 where, along with the loads that characterize the secondary bifurcations for certain ratios t/b, the classical Euler buckling curve is also presented. One can conclude that below the value t/b = 0.032 , a structural designer should be aware of the inadequacy of the classical Euler approach to evaluate the critical loads of these structures since, below that value, the local instability of the flanges are the biggest possible reason for the failure of the column. The behaviour of a L-shaped cross section column, with a given ratio t/b, below the value above discussed, can be illustrated by the load(stress) X displacement diagram shown in



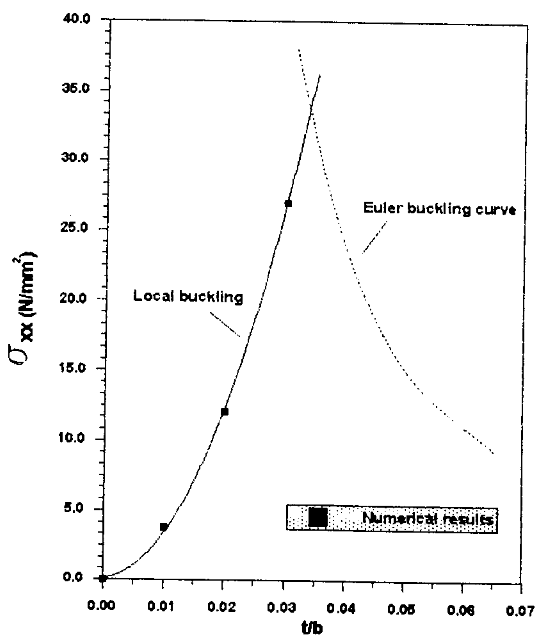


Figure 1: Diagram load(stress) x ratio width/thickness for flanges of L-shaped cross section columns.

Figure 2: Diagram load(stress) x displacement of a point on the flange, for a L-shaped cross section column of t/b = 0.020.

Figure 2, where is very clear the occurrence of a secondary bifurcation at a level of load way below the value of the classical Euler critical load. The numerical values presented should be compared to reliable experimental data, which were not available to the authors while this study was being performed.

Other standard thin walled column studied is the one with the U cross section, whose flanges have width b and thickness t. Once again, various thickness/width ratios of the flanges were tested, and the results are presented in Figure 3 where, as well as the classical Euler buckling curve, the loads that characterized the secondary bifurcations are also shown. Following similar behaviour observed in the study of the L-shapped cross section column, for values below the ratio t/b = 0.030, if the use of this type of column cannot be avoided, one should be aware that, in order to evaluate its capacity of taking loads, he cannot use the classical Euler approach, which is clearly inadequate for this range of t/b ratio. The behaviour of a U-shapped cross section column, with a ratio t/b below the value above presented, is illustrated in the Figure 4, where the occurrence of a secondary bifurcation at a load level much smaller than the classical Euler critical load is well demonstrated. Once again, reliable experimental data, that could be used for checking the performance of this numerical simulation, were not available.

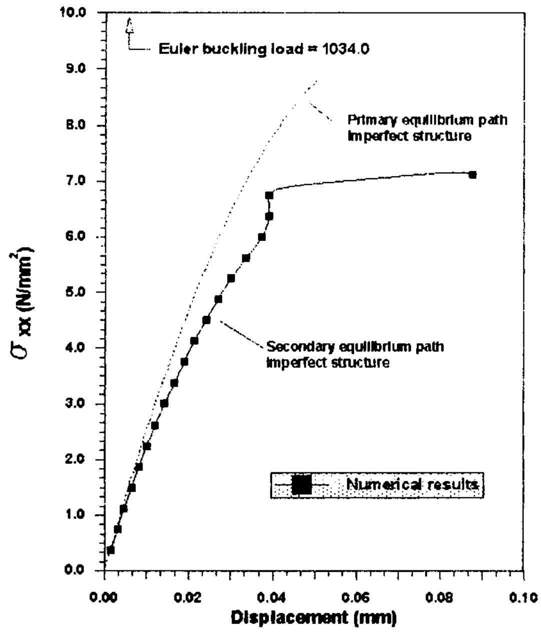


Figure 3: Diagram load(stress) X ratio width/thickness for flanges of U-shapped cross section columns.

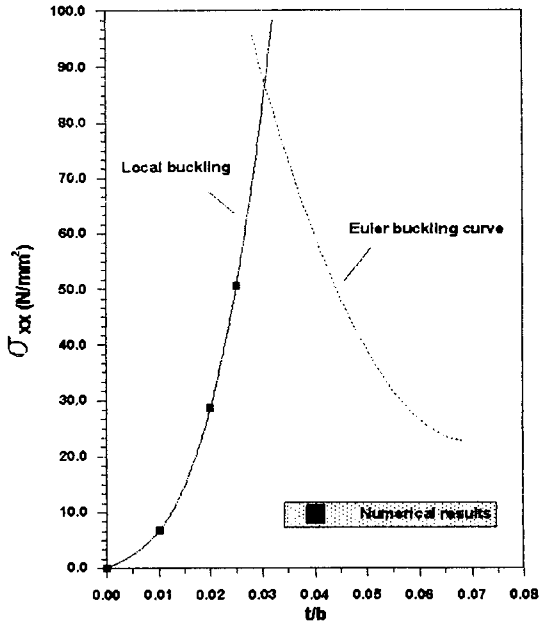


Figure 4: Diagram load(stress) X displacement of a point on a flange, for a U-shapped cross section cross section column of t/b = 0.010.

4. CONCLUSIONS

A methodology for solving nonlinear structural stability problems of thin walled columns, using plate finite elements to discretize its webs and flanges, was proposed. The main objective was the study of the local instabilities, which, for certain values of the ratio between the thickness and the width of the flanges, may cause the failure of the column at load levels much smaller than the ones predicted by the classical Euler approach. The results presented have shown that this methodology is capable of modelling such undesirable situations. Nevertheless, additional studies are needed to obtain a more precise evaluation of this secondary bifurcation loads, and to provide a comparative study with experimental results. The inclusion of the residual stresses at the formulation is a key matter to be addressed in future works.

5. ACKNOWLEDGEMENTS

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