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ENERGY RELEASE RATE FOR CRACKS IN NON-HOMOGENEOUS MEDIA

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ABSTRACT

This paper gives two extensions of the G-θ method to calculate the energy release rate in case of spatially varying material properties.

The first formulation is based upon the material parameter gradients. A practical application of this development is the case of a cracked component subjected to a thermal shock, with temperature dependent material properties. An additional term must be added to guarantee a path-independant G value.

The second formulation is applicable in case of a bimetallic interface. The additional term corresponds to a line integral along the interface, for taking into account the stress and energy discontinuities.

1 INTRODUCTION

In this paper, we give two new formulations that allow to compute the energy release rate in a media with spatially varying elastic properties.

In this special case, it is shown that the classical G or J formulae are not appropriate and that auxiliary terms should be added to the classical expressions to keep their independance with the contour of integration. The new integrals have been implanted in the finite element computer code CASTEM 2000, and the efficiency of the additional terms is presented on some examples.

2 CRACKED BODY WITH CONTINUOUSLY VARYING MATERIAL PARAMETERS

2.1 Theoretical expression for G

For a non-homogeneous cracked body Ω, under general loading conditions, in the elastic range, the following analytical expression for the energy release rate can be derived [1] :

$$G = G_{hom} + G_{add} \tag{1}$$

with $G_{hom} = \int_\Omega Tr(\sigma \nabla U \nabla \Theta)\,d\Omega - \int_\Omega w\,div\Theta\,d\Omega + \int_\Omega Tr(\sigma)\alpha \nabla T \Theta\,d\Omega - \int_\Omega F \nabla U \Theta\,d\Omega$

and $G_{add} = \int_\Omega Tr(\sigma) \nabla\alpha \Theta T\,d\Omega - \frac{1}{2} \int_\Omega Tr(\nabla R \Theta \varepsilon \varepsilon)\,d\Omega$

Where G_{hom} represents the energy release rate when the material is considered as homogeneous, and G_{add} an additional contribution when the material is non-homogeneous. In the expressions above ε is the total strain tensor, σ the stress tensor, U the displacement vector, w the strain energy density, F the body forces and T the temperatures (variation in comparison with the reference state). The displacement vector Θ maps the body containing the crack into a body where the crack length is infinitesimally increased.

2.2 Application : center-cracked plate with varying material parameters

Let us considerer the problem of a center-cracked plate, with non-homogeneous elastic material parameters, and under plane-stress condition. The loading is a linear thermal gradient, $\Delta T = 330°C$, between the crack faces and the plate boundary (figure 1). Young's modulus E and thermal expansion coefficient α are assumed to vary linearly in the x-direction, the extreme values on left and right boundaries being fixed to :

$$E_1 = 1{,}2\ 10^5 \text{ MPa} \qquad E_2 = 3{,}8\ 10^5 \text{ MPa}$$

$$\alpha_1 = 6\ 10^{-6}\,°C^{-1} \qquad \alpha_2 = 25\ 10^{-6}\,°C^{-1}$$

The calculation was performed using CASTEM 2000 computer code, on one quarter of the plate modelled with 370 constant strain triangular elements (figure 2). Five contours around the crack tip were defined to analyse the stability of the computed G-values (figure 3).

Results are plotted on figure 4, and we may concluded that :

- the additional term G_{add} guarantees the stability of the computed G values with more and more extended Θ displacement fields,
- the additional term G_{add} is not negligible.

3 CRACKED BODY WITH A BIMETALLIC INTERFACE

3.1 Theoretical expression for G

Refering to the formulation of Delale and al. [2], the cracked body with a bimetallic interface is divided into three parts, and the integration over the whole body Ω is the sum of the integrations over the three parts (figure 5) :

$$\int_{\Omega} d\Omega = \int_{\Omega_1} d\Omega + \lim_{\xi \to 0} \int_{\Omega^*} d\Omega + \int_{\Omega_2} d\Omega$$

Volume integrations over Ω_1 and Ω_2 correspond to the formula (1), since material parameters in the two media are assumed continuous.

For the integration over Ω^*, with ξ tending to zero, it has been demonstrated that this term corresponds to a line integral along the interface. In a bidimensionnal case, with a straight interface along the y-axis, the G_{line} term is expressed by :

$$G_{line} = \int_0^{y_o} (w_a - w_b)\, \Theta_x dy + \int_0^{y_o} \left[\sigma_b\, n \left.\frac{\partial U}{\partial x}\right|_b - \sigma_a\, n \left.\frac{\partial U}{\partial x}\right|_a \right] \Theta_x dy \qquad (2)$$

In the above expression, w is the elastic energy density, U the displacement vector, and n the normal to the discontinuity. a and b subscripts design left and right side of the discontinuity.

3.2 Application : crack perpendicular to a bimetallic interface

Erdogan and Lu ([3], [4]) give analytical solutions of the stress intensity factor, for cracks perpendicular to and on the interface of bonded elastic layers. Results are given in diagrams for fixed material combinaison or layer dimensions. More specifically, we analysed the case of a pressurized edge crack, with a tip located near the interface (figure 6).

Due to the symetry condition, one half of the plate was modelled with quadratic elements (figure 7).

Computed G value agrees very well with that readed in Erdogan's diagram :

$$K/p\sqrt{\pi a} : \begin{cases} 2{,}16 \text{ (calculated value)} \\ 2{,}17 \text{ (Erdogan's diagram)} \end{cases} \quad \text{with } K = \sqrt{\frac{E_2 G}{1-\upsilon_2^2}}$$

Furthermore, the stability of the G values for Θ displacement fields crossing the interface is verified (figure 8).

3.3 Special case : crack tip touching the interface

With the two new formulations of the G energy release rate presented above, the following problems are solved :

- if the elastic material properties vary spatially, the G_{add} term must be added,
- if the crack tip is located near the interface, the G_{line} term must be added.

But two particular cases have to be distinguished :

- for a crack parallel or lying along a bimetallic interface, no corrector term has to be added to the classical formula G_{hom}, since a virtual crack advance does not lead to cross the discontinuity,
- for a crack tip touching the interface, a demonstration is done below to explain why the energy release rate is indefined in this special case.

When the crack tip touchs the interface, the stress singularity is no more a function of $r^{-1/2}$.

$$\text{Stresses} \quad \sigma = A\, r^{-\alpha} \quad \text{with } \alpha \neq 1/2$$

Let us return to the definition of the energy release rate, and write the energy variation to return from a crack length a + da to a crack length a :

- Displacement $\quad U = B(da - r)^{1-\alpha}$.

- Energy variation $\quad -dw = \int_0^{da} A\, r^{-\alpha}\, B(da - r)^{1-\alpha}\, dr$

$$\text{and } -dw = G da$$

Using a change of variable, r = da t, we obtain :

$$G = AB(da)^{1-2\alpha} \int_0^1 t^{\alpha}\,(1 - t)^{1-\alpha}\, dt$$

If α is lower than 1/2, G is extremely small as $(da)^{1-2\alpha}$. On the contrary, if α is greater than 1/2, G is extremely large as $1/(da)^{2\alpha-1}$.

4 CONCLUSION

The interest of the two new formulations of the G-θ method, in case of spatially varying material properties, has been proved on some examples.

The first formulation is relative to continuously varying properties. A practical application is the case of a thermal shock problem with temperature dependent material characteristics.

The second formulation concerns a bimaterial discontinuity, and can be used to analyse a crack in a cladded component. But in the particular case of a crack with one tip located exactly at the interface, it is pointed out in the paper, that G is undefined. However the stress intensity factor remains defined, and analytical solutions are given in references [4].

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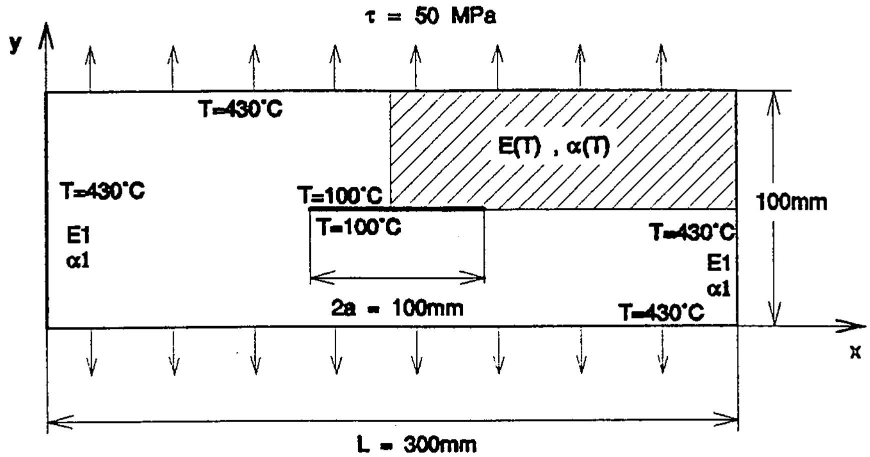

Fig. 1 : Center-cracked plate with a non-homogeneous material

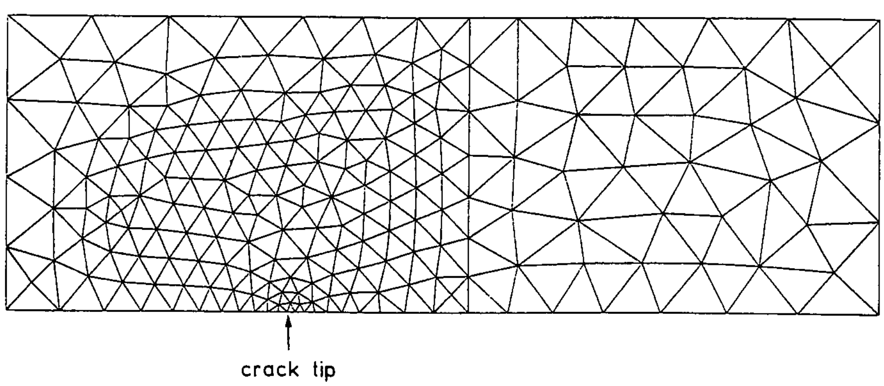

Fig. 2 : FE model for CCP specimen

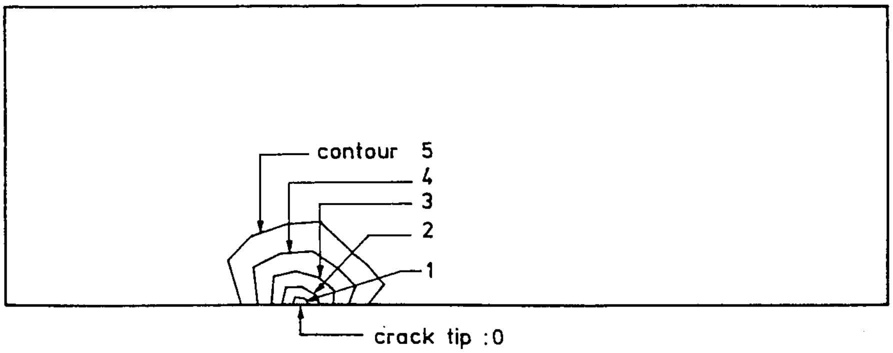

Fig. 3 : Contours limiting the θ-displacement field

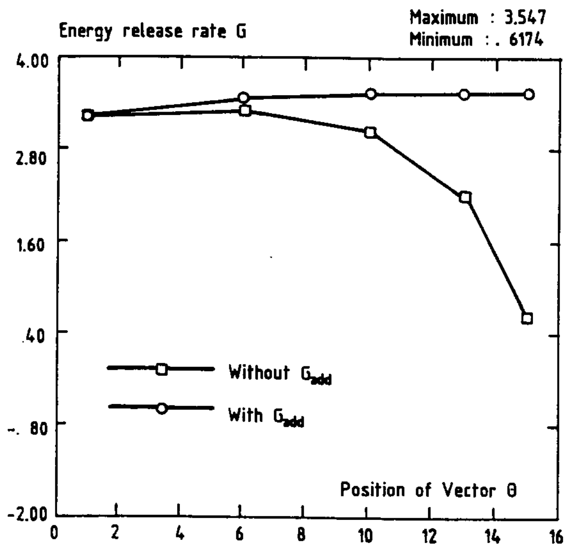

Fig. 4 : G function of the θ displacement field

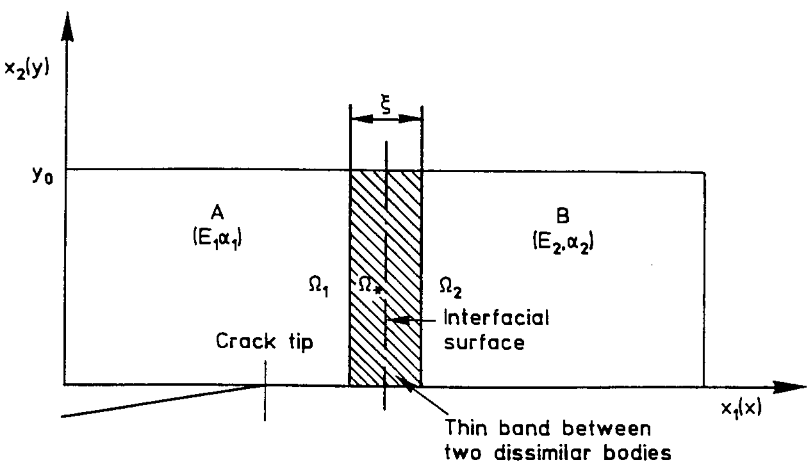

Fig. 5 : Cracked body with a bimetallic interface

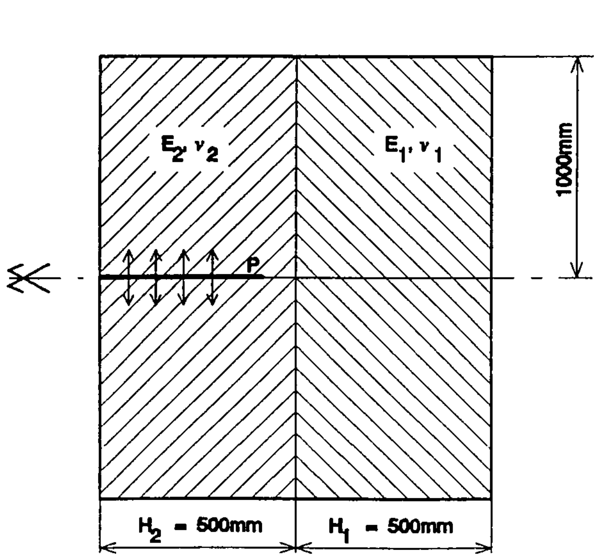

Fig. 6 : Pressurized edge crack perpendicular to a bimetallic interface

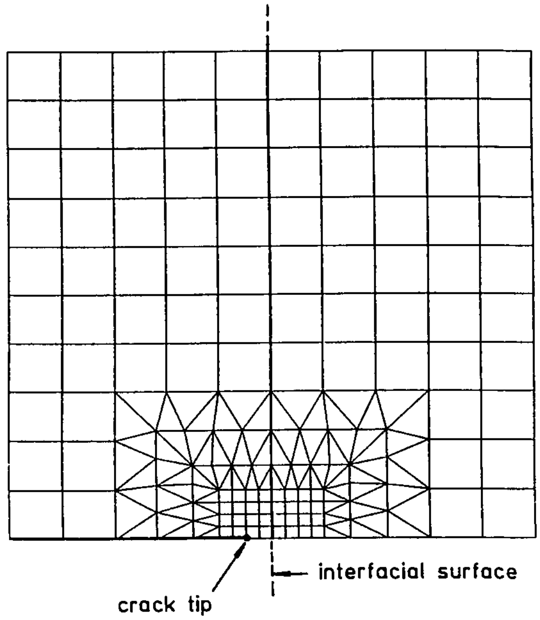

Fig. 7 : Mesh for the pressurized edge crack

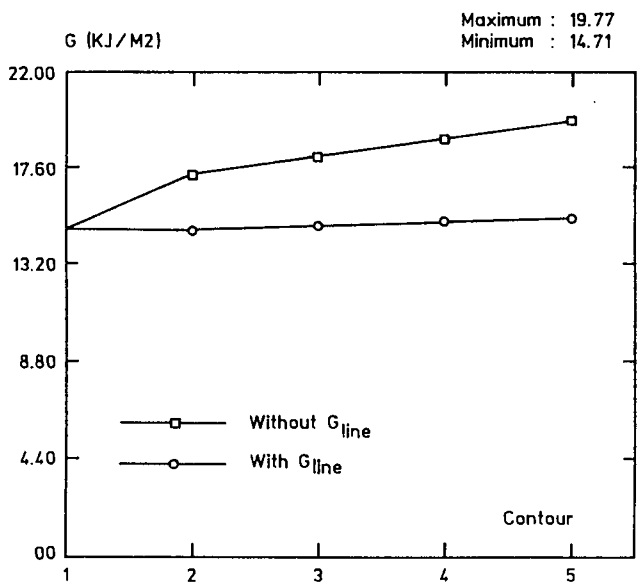

Fig. 8 : G-values stability analysis