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DETERMINING THE J-R CURVE FOR A PIPE CONTAINING A THROUGH-WALL CIRCUMFERENTIAL CRACK

E. Smith

Manchester University-UMIST Materials Science Centre, Grosvenor Street, Manchester M1 7HS, UK

ABSTRACT

A theoretical analysis for a pipe subjected to bending deformation, and containing a through-wall circumferential crack, shows that the commonly used eta factor formulation based on limit load considerations, underestimates the deformation J integral and thus when used gives unduly low J_D and J_M crack growth resistance curves. The conclusion is reached by making a comparison between the results for the limit load situation and the small-scale yielding situation.

1. INTRODUCTION

When assessing the integrity of a pipe containing a through-wall circumferential crack, which is subjected to bending deformation, it is important to have a reasonably accurate description of the material's crack growth resistance, more especially the J integral - crack extension behaviour associated with a through-wall circumferential crack. Recognizing that the deformation J integral J_D can be separated into elastic (J_E) and plastic (J_{DP}) components, and since J_E is readily obtained via use of the stress intensity factor, determination of J_D really depends on calculating J_{DP}. A commonly used method is based on relating J_{DP}, for a non-growing crack, to the plastic work integral via an eta proportionality factor; this eta factor is a function of the geometrical parameters of the cracked cross-section, and is determined from limit-load considerations. It is then possible to relate J_{DP} for a growing crack to the moment, plastic rotation and crack extension, the relation providing a means of obtaining J_D and also modified J integral (J_M) crack growth resistance curves for the material from moment, rotation and crack extension measurements using a single piping specimen. Against the background of recent developments in the J integral-eta factor area, the present paper shows that this procedure can underestimate J_{DP} particularly when the deformation is not extensive and with a small crack.

2. THE ETA FACTOR METHOD FOR DETERMINING J_{DP}

Figure 1 shows the cracked cross-section of a pipe. The pipe of radius $R \gg t$ (thickness) contains a circumferential through-wall crack of contained angle 2θ, and is subjected to bending deformation by an applied moment M. Having separated the deformation J integral J_D (for the non-growing crack situation) into its elastic (J_E) and plastic (J_{DP}) components, J_{DP} is given in terms of the plastic rotation ϕ_p by the standard expression for bending deformation[1]:

$$J_{DP} = -\int_0^{\phi_p} \left(\frac{\partial M}{\partial A}\right)_{\phi_p} d\phi_p \qquad (1)$$

where $A = 2Rt\theta$ is the crack area. Assuming limit load conditions $M = M_L$ with the stress distribution across the cracked section being a tensile stress σ_o (independent of crack size) above the neutral axis and an equivalent compressive stress - σ_o below the neutral axis, and assuming no resultant axial force across the cracked section, moment equilibrium requires that

$$M_L = 4\sigma_o R^2 t\left[\cos\frac{\theta}{2} - \frac{1}{2}\sin\theta\right] \qquad (2)$$

Consequently relations (1) and (2), taken together, show that for extensive deformation at limit load conditions then

$$J_{DP} = -\int_0^{\phi_p} \left(\frac{\partial M_L}{\partial A}\right)_{\phi_p} d\phi_p = \sigma_o R\left[\sin\frac{\theta}{2} + \cos\theta\right] \qquad (3)$$

Furthermore, again for extensive deformation at limit load conditions,

$$\int_0^{\phi_p} M d\phi_p = \int_0^{\phi_p} M_L d\phi_p = 4\sigma_o R^2 t\left[\cos\frac{\theta}{2} - \frac{1}{2}\sin\theta\right]\phi_p \qquad (4)$$

using relation (2), whereupon relations (3) and (4) give J_{DP} in the form

$$J_{DP} = \eta \int_0^{\phi_p} M d\phi_p \qquad (5)$$

with

$$\eta = -\frac{1}{2Rt}\frac{f'(\theta)}{f(\theta)} \qquad (6)$$

where

$$f(\theta) = \left[\cos\frac{\theta}{2} - \frac{1}{2}\sin\theta\right] \qquad (7)$$

Relation 5 for J_{DP} has been derived assuming extensive deformation at limit-load conditions. The same expression for J_{DP} can be derived[2] by recognizing that the stress distribution across the cracked section can always, not merely at limit load conditions, be expressed in terms of a tensile stress $+\sigma_*$ acting above the neutral axis, and a compressive stress-σ_* acting below the neutral axis. Dimensional considerations then require that ϕ_p is given by the functional form[3]

$$\phi_p = H_*\left(\frac{\sigma_*}{\sigma_y},\theta,n\right) \tag{8}$$

where σ_y is the tensile yield stress and n is the work hardening exponent. The bending moment $M = 4\sigma_* R^2 tf(\theta)$ and it follows from relation (8), by elimination of σ_*, that ϕ_p is given by the functional form

$$\phi_p = H_*\left(\frac{M}{4\sigma_y R^2 tf(\theta)},\theta,n\right) \tag{9}$$

As emphasised by the present author[3], Zahoor and Kanninen[2] did not recognize the generality of expression (9), but instead used the simplified relation

$$\phi_p = H_*\left(\frac{M}{4\sigma_y R^2 tf(\theta)},n\right) \tag{10}$$

when it is then possible to use relations (1) and (10) to give expression (5) for J_{DP} with η again being given by relation (6). Thus the J_{DP} formulation for a non-growing crack as given by relations (5)-(7) can be obtained by either assuming extensive deformation at limit load conditions, or alternatively by using dimensional considerations and assuming separation of variables, i.e. by using relation (10) instead of relation (9).

When it is possible to express J_{DP} for a non-growing crack via the single eta factor formulation (5), it is then possible[2] to express J_{DP} for a crack growing from θ_o to θ via the relation

$$J_{DP} = \int_0^{\phi_p} \eta M d\phi_p + \int_{\theta_o}^{\theta} \gamma J_{DP} d\theta \tag{11}$$

Furthermore the plastic component J_{MP} of the modified J integral J_M[4,5] is given by the relation

$$J_{MP} = \int_0^{\phi_p} \eta M d\phi_p \tag{12}$$

Relations (11) and (12) enable respectively the J_D and J_M crack growth resistance behaviours to be determined from moment, rotation and crack extension measurements using a single specimen; this is the great virtue in being able to express J_{DP} for a non-growing crack via a single eta factor relation of the form (5).

Now it has been emphasized in the preceding considerations that such a formulation is valid for extensive deformation at limit load conditions. However it has been used by many researchers quite generally, with no assurance that such conditions are indeed operative. The question therefore arises as to whether relation (5) can be used, generally, as a reasonable working approximation for J_{DP}. For small-scale yielding, $\phi_p = g(\theta)M^3$, where $g(\theta)$ is a function of the crack length θ, and then relation (1) shows that a single eta factor formulation is appropriate. However the eta factor η_s relevant to this small-scale yielding state is different to the eta factor η_L that is given by relations (6) and (7). Nevertheless if it can be shown that η_s is approximately equal to η_L then it is not unreasonable to assume that the single eta factor J_{DP} formulation based on relations (5)-(7) can be used generally and not merely for extensive deformation at limit load conditions. This issue has already been addressed by the author(6) for the compact tension specimen geometry, and will be addressed in the next section for a circumferential through-wall crack in a pipe.

3. DETERMINATION OF THE SMALL-SCALE YIELDING η_s AND COMPARISON WITH THE EXTENSIVE DEFORMATION η_L

For a point loading situation with load P, and with Δ_p being the plastic component of the load-point displacement, $\Delta_p \propto P^3$ for small-scale yielding and then J_{DP} for a non-growing crack can always be expressed in the form

$$J_{DP} = \eta_s \int_0^{\Delta_p} P d\Delta_p \qquad (13)$$

Furthermore, the small-scale yielding analysis(7), which is applicable to all configurations, gives η_s in the form

$$B\eta_s = \frac{1}{3G^4}\frac{d}{da}(G^4) \qquad (14)$$

where B is the thickness of the specimen and δa is an element of crack extension at a crack tip; the stress intensity K_I is expressed in the form $K_I = PG$, G being a function of the geometrical parameters of the configuration. It therefore follows, by analogy, that with any general bending situation with moment M and plastic rotation ϕ_p, where $\phi_p \propto M^3$, then J_{DP} for a non-growing crack can always be expressed at small-scale yielding in the form

$$J_{DP} = \eta_s \int_0^{\phi_p} M d\phi_p \qquad (15)$$

with

$$B\eta_s = \frac{1}{3G^4}\frac{d}{da}(G^4) \qquad (16)$$

where B is again the specimen thickness, δa an element of crack extension at a crack tip, and the stress intensity K_I is expressed in the form $K_I = MG$, G being a function of the geometrical parameters of the configuration. For a circumferential through-wall crack with contained angle 2θ, the function G is given by the approximate expression(8)

$$G = \frac{\sqrt{\pi R \theta}}{\pi R^2 t} F(\theta) \qquad (17)$$

with $F(\theta)$ being given by the expression

$$F(\theta) = 1 + 8\left(\frac{\theta}{\pi}\right)^{5/2} \tag{18}$$

which is valid for $0 < \theta < 100°$. It immediately follows from relations (16), (17) and (18) with $B = t$ and noting that $\delta a = R\delta\theta$, that η_s is given by the expression

$$\eta_s = \frac{2}{3\pi Rtx}\left[\frac{1 + 48x^{5/2}}{1 + 8x^{5/2}}\right] \tag{19}$$

with $x = \theta/\pi$. Table I gives η_s for different values of the contained crack angle, together with the value of the eta factor, i.e. η_L, appropriate to extensive deformation at limit load conditions, as given by relations (6) and (7), i.e.

$$\eta_L = \frac{\left[\sin\frac{\theta}{2} + \cos\theta\right]}{4Rt\left[\cos\frac{\theta}{2} - \frac{1}{2}\sin\theta\right]} \tag{20}$$

TABLE I

Values of η_s and η_L for different size crack

2θ	$\eta_s Rt$	$\eta_L Rt$
60°	1.80	0.39
120°	1.72	0.58
180°	1.67	0.85

4. DISCUSSION

The results in Table I clearly show that $\eta_s > \eta_L$ for all the crack angles considered and that the difference between η_s and η_L is particularly marked with small crack sizes. The implication of these results is that the eta factor formulation based on limit load considerations, i.e. relation (5) coupled with relations (6) and (7), underestimates J_{DP} and consequently leads to an underestimation of J_D (and J_M) crack growth resistance behaviours. There is limited support for this conclusion from a finite element study[9] where limited results suggest that the eta factor formulation underestimates the J_D crack growth resistance curve. There is clearly scope for a systematic finite element study focussed on a comparison of J_D as obtained by finite element methods and J_D as obtained via the single eta factor formulation, with the crack size and deformation level being important parameters whose effects should be studied. The results from such a study should provide a clear indication of the range of parameters over which the single eta factor J_{DP} formulation is viable.

It is worth at this stage mentioning, for sake of comparison, the results obtained for the compact tension specimen[6]. With this geometry it was shown that when $b/W < 0.5$ (b = ligament width and W = specimen width), η_s is approximately equal to η_L. However, for large b/W, η_s is greater than η_L and then the single eta factor J_{DP} formulation based on η_L

underestimates J_{DP}, as is the case with a circumferentially cracked pipe, as predicted via the present paper's analysis.

5. CONCLUSION

- It has been shown that the commonly used eta factor formulation, based on limit load considerations, underestimates the plastic component J_{DP} of the deformation J integral, and therefore when used, gives unduly low J_D and J_M crack growth resistance curves.

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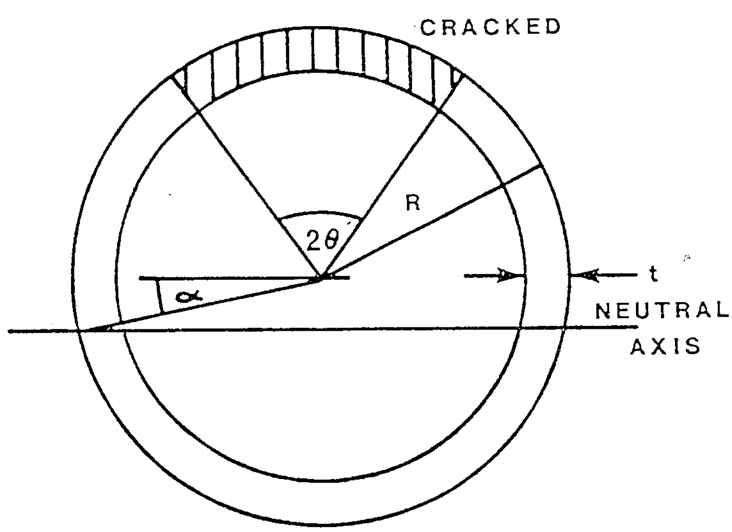

FIGURE 1 The cracked section of a pipe.