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THE EFFECT OF AXIAL FORCES ON THE CONSERVATISM OF THE NET-SECTION STRESS CRITERION FOR THE FAILURE OF CRACKED STAINLESS STEEL PIPING

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ABSTRACT

The integrity of cracked stainless steel piping is often examined from the basis that failure conforms to a net-section stress criterion, with the stresses at the cracked section being calculated via a purely elastic analysis assuming that the piping system is uncracked. However, a piping system is usually built-in at its ends into a larger component which restricts the amount of elastic follow-up,and also the onset of crack extension requires some plastic deformation; consequently, the net-section stress approach, when used in this manner, can lead to overly conservative failure predictions. Earlier work has quantified the extent of this conservatism for the case where the effect of an axial force normal to the cracked section was ignored. The present paper extends the earlier considerations by examining the effect of an axial force on the degree of conservatism.

1. INTRODUCTION

The integrity of cracked piping systems, and in particular stainless steel piping systems, is often examined via the net-section stress methodology(1), which can be viewed as a special case, appropriate for ductile materials, of the CEGB R6 methodology(2). The net-section stress methodology uses as input a knowledge of the anticipated loadings and the critical net-section stress, which is usually taken to be the average of the yield and ultimate tensile stresses of the material under consideration. In applying this methodology, the standard practice is to calculate the stresses in the region of the cracked section via a purely elastic analysis based on the piping system being uncracked. However, a piping system is usually built-in at its ends into a larger component which restricts the amount of elastic follow-up and also the onset of crack extension requires some plastic deformation; consequently, use of the net-section stress approach in this manner gives conservative predictions for the onset of crack extension. Earlier work(3) has quantified the extent of this conservatism for the case where the effect of an axial force normal to the cracked section was neglected. The present paper extends the earlier work by quantifying how the degree of conservatism is affected by an axial force, induced by for example internal pressure.

2. THEORETICAL ANALYSIS

With regard to the present paper's objective, it is sufficient to consider the behaviour of a piping system containing a through-wall circumferential crack. Thus Figure 1 shows the cross-section of a circular cylindrical pipe, thickness t and radius $R \gg t$, which contains a through-wall crack with contained angle 2θ. Provided that plastic deformation is confined to the cracked section, the behaviour of this section can be decoupled from the remainder of the system, which is assumed to deform in accord with the laws of linear elasticity. If it is assumed that the discontinuity at the cracked section is entirely in the form of a rotation, simple bending theory shows that the actual moment M and rotational discontinuity ϕ at the cracked section are related to the moment M_u, calculated on the basis that the piping system is uncracked, by an expression of the form

$$M + \frac{EI\phi}{L_*} = M_u \tag{1}$$

where E is Young's modulus and $I = \pi R^3 t$ is the second moment of area of the piping at the cracked section. The moment M_u is the bending moment due to all sources that give non self-equilibrating static forces at the cracked section, i.e. deadweight, thermal expansion, applied loadings or displacements, and the system pressure. Forces that are self- equilibrating, i.e. arising for example from local thermal gradients or welding processes are not considered in the present discussion. The effect of seismically induced (dynamic) forces has been considered separately[(4)], and it has been shown that their effects can be assessed in the same way as non self-equilibrating static forces. The parameter L_* in relation (1) is a length parameter, and is a measure of the degree of elastic follow-up or flexibility of the piping system, with a high L_* value implying significant elastic follow-up. It is a function of the geometrical parameters of the system, including the crack location, and can be obtained via a very simple procedure[(5)], which involves a separation of the complete piping system into two elastic parts at the cracked section with the system ends remaining fixed, the application of equal and opposite moments M_A to the cut faces, and the correlation of M_A with the rotational discontinuity ϕ_A generated at this section, while maintaining compatibility of displacement, i.e.

$$L_* = \frac{EI\phi_A}{M_A} \tag{2}$$

For a straight pipe segment, built-in at its ends, subjected to equal and opposite rotations at these ends, and with a cracked section at the mid-length position, $L_* = L$, the pipe length[(6)].

To facilitate the analysis, and following the pattern adopted in the author's earlier work[(3)], it is assumed that: (a) the material does not work-harden, (b) the rotation at the cracked cross-section is all plastic, (c) the material is sufficiently ductile that general yield precedes the onset of crack extension. The cracked section is therefore subjected to the plastic limit moment M_p, and following Tada, Paris and Gamble[(6)], the stress distribution across the section can then be described by a tensile stress σ_o acting within the region above the neutral axis (see Figure 1), with a compressive stress of similar magnitude being operative below the neutral axis. In the earlier analysis[(3)], the effect of any axial force acting perpendicular to the cracked cross-section was ignored, and consequently, the location of the neutral axis, defined by the angle α, was readily shown by balancing forces, to be given by $\alpha = \theta/2$. If however, there is a tensile force P acting perpendicular to the cracked section, then balancing forces gives the relation

$$\frac{P}{2\sigma_o Rt} = 2\alpha - \theta \tag{3}$$

while moment equilibrium provides the relation

$$\frac{M_p}{4\sigma_o R^2 t} = \cos\alpha - \frac{1}{2}\sin\theta \tag{4}$$

The plastic rotation ϕ_c about the neutral axis at the onset of crack extension is related to the critical crack tip opening displacement δ_{IC} by the expression

$$\phi_c = \frac{\delta_{Ic}}{R(\sin\alpha + \cos\theta)} \tag{5}$$

Now, with the net-section stress approach as it is usually employed, crack extension is presumed to occur when the moment M_{UNC} at the cracked section, calculated on the basis of the piping system being uncracked, equates with the plastic limit moment M_p. However, the onset of crack extension actually occurs when the moment M_{IN} at the cracked section, calculated on the same basis, equals $M_p + (EI\phi_c/L_*)$. Thus a measure of conservatism is the parameter $f = M_{IN}/M_{UNC}$, the physical meaning of f being that the cracked section can withstand a moment f times the moment predicted via the net-section stress approach without fracture initiating. Consequently

$$f = 1 + \frac{EI\phi_c}{M_p L_*} = 1 + \frac{\pi E\delta_{IC}}{4\sigma_o(\sin\alpha + \cos\theta)(\cos\alpha - \frac{1}{2}\sin\theta)L_*} \tag{6}$$

using relations (4) and (5), with α being given by relation (3). With P being the axial force, the axial tensile stress within the pipe is $\sigma_* = P/2\pi Rt$. Table I shows f (as given by relation (6)) for a crack with $\theta = 30°$, for values of σ_*/σ_o up to 0.3 and for the values $(\pi E\delta_{IC}/4\sigma_o L_*) = 1$, 0.5, 0.1 and 0.05. To put these latter values in perspective, with typical values for ductile stainless steel or ductile weld material: $\sigma_o = 50 \times 10^3$ psi (350 MPa), $E = 30 \times 10^6$ psi (210×10^3 MPa), $\delta_{IC} = 0.1$ in (0.25 cm), the values 1, 0.5, 0.1 and 0.05 correspond to $L_* \sim 4$, 8, 40 and 80 ft, i.e. 120, 240, 1200 and 2400 cm. Table II shows the corresponding results for a crack with $\theta = 60°$. The results in Table I show that, for the case $\theta = 30°$, the factor of conservatism f is essentially unaffected by an axial tensile force of magnitude up to $\sigma_*/\sigma_o = 0.3$. However, for the case $\theta = 60°$, the factor of conservatism is higher when there is an axial tensile force of magnitude up to $\sigma_*/\sigma_o = 0.3$.

It should be emphasised that the result that the factor of conservatism can be higher, i.e. for the case $\theta = 60°$, when there is an axial force does not imply that failure is necessarily more difficult in this situation. This point is readily seen by considering the case where a straight pipe of length L containing a crack at the mid-length position is subjected to equal and opposite rotations $\pm\psi/2$ at the pipe ends. In this case $M_u = EI\psi/L$, $L_* = L$ and with $M = M_p$ as given by relation (4), the criterion for the onset of crack extension is given by relations (1) and (5) as

$$\psi = \frac{\delta_{IC}}{R(\sin\alpha + \cos\theta)} + \frac{4L\sigma_o}{\pi RE}\left[\cos\alpha - \frac{1}{2}\sin\theta\right] \quad (7)$$

with α being given by relation (3); it is immediately seen that the applied rotation ψ needed for crack extension decreases as the applied tensile force increases, i.e. as the angle α increases.

3. DISCUSSION

The present paper's theoretical analysis has shown that an axial tensile force, induced by for example internal pressure, can have a significant effect on the safety margin when the net-section stress methodology is used to predict the extension of a circumferential crack, if the stresses at the cracked section are calculated on the basis that the piping is uncracked. The safety margin is quite marked in the absence of an axial tensile force, particularly with a stiff piping system, i.e. low L_* value, and can be even greater when there is an axial tensile force (see the results in Table II).

There is also a potential additional conservatism which stems from the fact that unstable failure need not necessarily be associated with the onset of crack extension[3]. For sake of completeness, it is worth considering how an axial tensile force affects the instability criterion; it is sufficient to focus on the criterion for crack growth to be unstable at the onset of crack extension, with the applied loadings fixed. Thus, with M_u fixed, and M being equal to the plastic limit moment M_p, relation (1) shows that the condition for crack growth to be unstable at the onset of crack extension is

$$\frac{EI}{L_*} + \left[\frac{dM_p}{d\phi}\right]_I < 0 \quad (8)$$

with $dM_p/d\phi$ being measured at the onset of crack extension. Following Tada, Paris and Gamble[6], relation (8) can be expressed in the tearing modulus format[7]. Thus the magnitude of the J integral for a given crack area $A = 2Rt\theta$ is

$$J = -\phi\frac{\partial M_p}{\partial\theta} = \sigma_o R\phi\left[\sin\alpha + \cos\theta\right] \quad (9)$$

using relation (4), whereupon relations (4), (8) and (9) give the instability condition as

$$L_* . \frac{2}{\pi}[\sin\alpha + \cos\theta]^2 > \frac{E}{\sigma_o^2}\frac{dJ}{d\theta} - \frac{EJ}{\sigma_o^2 R}\left[\frac{\frac{1}{2}\cos\alpha - \sin\theta}{\sin\alpha + \cos\theta}\right] \quad (10)$$

With the tearing modulus format, the material tearing modulus T_{MAT} is related to the slope of the J-crack growth resistance curve:

$$T_{MAT} = \frac{E}{\sigma_o^2}\frac{dJ_{MAT}}{da} \quad (11)$$

where δa is an increment of crack length. Noting that $\delta a = R\delta\theta$, the condition (i.e. (10)) for crack growth to be unstable at the onset of crack extension, i.e. when $J = J_{IC}$, can be written in the simplified form

$$T_{APP} = \frac{L_*}{R}.\frac{2}{\pi}[\sin\alpha + \cos\theta]^2 > T_{MAT} \quad (12)$$

this simplified form being obtained[6] by recognizing that for a material, like Type 304 stainless steel, with a high crack growth resistance, $EJ_{IC}/\sigma_o{}^2R \ll T_{MAT}$; the second term on the right-hand side of the inequality (10) can then be neglected. Noting that α is given by relation (3), it is immediately observed that instability is favoured by the presence of an axial tensile force.

In summary therefore, one observes that an axial tensile force can lead to an increase in the factor of conservatism when the extension of a circumferential crack is predicted using the net-section stress methodology with the stresses at the cracked section being calculated on the basis that the piping system is uncracked. On the other hand, the likelihood of a crack being unstable at the onset of crack extension is increased by the presence of an axial tensile force. Consequently, both these competing effects should be taken into account when using the net-section stress approach if there is a significant axial tensile force.

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REFERENCES

1. American Society of Mechanical Engineers, Boiler and Pressure Vessel Code, Section XI, IWB-3640, (1983).
2. Milne, I., Ainsworth, R.A., Dowling, A.R. and Stewart, A.T., 1988, Int. Jnl. Pres. Ves. and Piping, 32, 3.
3. Smith, E., "The conservatism of the net-section stress criterion for the failure of cracked stainless steel piping", paper accepted for publication in Int.Jnl.Press.Vess and Piping.
4. Smith, E., 1991, ASME PVP Meeting, San Diego, USA, Seismic Engineering, p. 215 PVP-Vol 220, June (1991).,
5. Cotter, K.H., Chang, H.Y. and Zahoor, A., 1982, EPRI Research Project T118-9, NP-2261 Final Report.
6. Tada, H., Paris, P.C. and Gamble, R.M., 1980, ASTM STP 700, 296.
7. Paris, P.C., Tada, H., Zahoor. A. and Ernst, H., 1979, ASTM STP 668, 5.

Table I. The factor of conservatism f for the case $\theta = 30°$.

$\frac{\sigma_*}{\sigma_o}$	$\pi E\delta_{IC}/4\sigma_o L_*$			
	1	0.5	0.1	0.05
0	2.25	1.62	1.12	1.06
0.1	2.18	1.59	1.12	1.06
0.2	2.20	1.60	1.12	1.06
0.3	2.32	1.66	1.13	1.07

Table II. The factor of conservatism f for the case $\theta = 60°$.

$\frac{\sigma_*}{\sigma_o}$	$\pi E\delta_{IC}/4\sigma_o L_*$			
	1	0.5	0.1	0.05
0	3.31	2.15	1.23	1.11
0.1	3.57	2.28	1.26	1.13
0.2	4.41	2.70	1.34	1.17
0.3	7.69	4.34	1.67	1.33

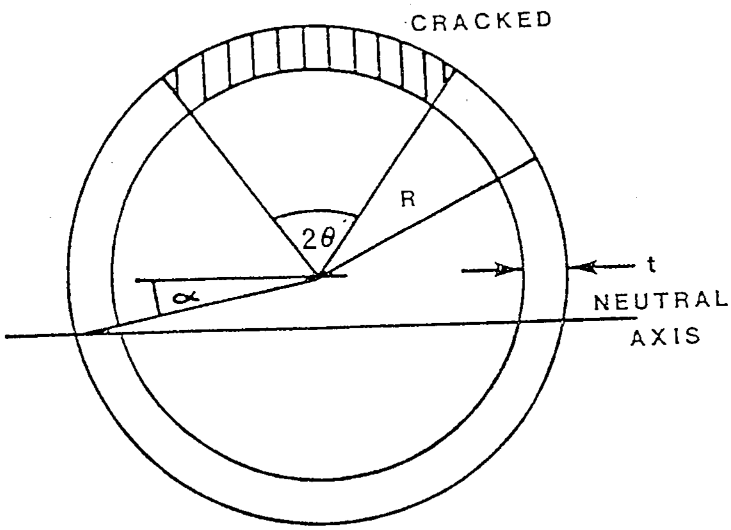

FIGURE 1 The geometry of the cracked cross-section of a pipe; the cross-section contains a through-wall crack with contained angle 2θ. The pipe radius is R and the pipe thickness is $t \ll R$.