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A MODIFIED THERMAL $\hat{J}_{or}$ INTEGRAL FOR ORTHOTROPIC COMPOSITE CRACKED STRUCTURES

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ABSTRACT

A modified thermal $\hat{J}_{or}$ integral is proposed and further study on the applicability of $\hat{J}_{or}$ integral is established using detailed numerical experiment. In particular, when consider the isotropic material, modified thermal $\hat{J}_{or}$ integral reduced to J as expected. On the basis of on a well-developed hybrid displacement finite element model for orthotropic fracture problem, the modified thermal $\hat{J}_{or}$ integral is computed and averaged by four different integral paths chosen and path independence is noted. In addition, the stress intensity factor for orthotropic fracture parameter K_I can be accurately inferred from the $\hat{J}_{or}$ integral. Thus the $\hat{J}_{or}$ integral could be served as a sound fracture parameter for orthotropic thermal fracture problem.

1 INTRODUCTION

The increasing utilization of some high-modules fiber-reinforced composite material in aerospace structural application desires to require fully understand the fracture characteristics and damage tolerance of laminated composites for proper structural design. High performance composites often have very strong fibers in a much weaker matrix so that crack formation and growth is constrained by fiber direction resulting in mixed mode crack deformation and growth. The fracture analysis method that has been well developed to isotropic materials [Wilson & Yu,1979]& [Aoki & Sakata, 1982] but has not been universally successful when transferred directly to orthotropic materials. This paper is concerned with the 2-dimensional thermal fracture problem of a orthotropic materials structure.

2. MODIFIED THERMAL $\hat{J}_{or}$ INTEGRAL

The path independent J-integral for isothermal elastic homogeneous material is defined as [Rice, 1968]

$$J = \int_{\Gamma} W dx_2 - T_i \frac{\partial u_i}{\partial x_1} ds$$

Where W is the elastic strain energy density function. T_i is a traction vector along the arbitrary integral contour Γ that begins and ends at the crack faces. . u is a displacement vector. However, the path independence of J is not maintained when thermal loadings are present. The modified $\bar{J}_k$ integral for a thermal elastic crack is proposed [Chen & Chen,1981]

$$\bar{J}_k = \int_{\Gamma} W_t dx_2 - T_i \frac{\partial u_i}{\partial x_1} ds + \iint_A \beta \varepsilon_{ii} \frac{\partial \theta_T}{\partial x_1} dA$$

where W_t is the isothermal strain energy. $W_t = W - \beta \theta_T \varepsilon_{ii}$

Valaire el.[1990] have been derived a path independence J-integral for mode I and mode II in orthotropic composite material and obtained the relationship

$$\Delta > 0 \quad J_{or} = \frac{K_I^2 a_{22} (\beta_1 + \beta_2)}{2\beta_1 \beta_2}$$

$$\Delta < 0 \quad J_{or} = \frac{K_I^2 a_{22} \beta_1}{\alpha_1^2 + \beta_1^2}$$

The present work is concerned with the 2-dimensional thermal fracture problem of a orthotropic materials structure. A modified thermal $\hat{J}_{or}$ integral is proposed as:

$$\bar{J}_{or} = \int_{\Gamma} W_t dx_2 - T_i \frac{\partial u_i}{\partial x_1} ds + \iint_A (\beta_{11}\varepsilon_{11} + \beta_{22}\varepsilon_{22} + \beta_{33}\varepsilon_{33}) \frac{\partial \theta_T}{\partial x_1} dA$$

Substitutive $E_1 = E_2$; $\beta_{11} = \beta_{22} = \beta_{33}$ into above equation , then we can obtain the thermal elastic J integral [Chen & Chen, 1981].

$$\bar{J}_k = \int_{\Gamma} W_t dx_2 - T_i \frac{\partial u_i}{\partial x_1} ds + \iint_A \beta \varepsilon_{ii} \frac{\partial \theta_T}{\partial x_1} dA$$

3 FINITE ELEMENT MODEL

Due to high efficiency and accuracy, the assumed hybrid displacement finite element model[Chen &Chen,1981; Chen &Wu,1980] with relaxed continuity requirement for displacements at the inter element boundary has been demonstrated to an attractive method in dealing with isothermal cracked structures. On the basis of the 2-D theory of thermoelasticity for orthotropic structure, an analogous to the functional of the hybrid displacement model for isothermal stress crack problems can be generalized in this work as follows:

The constitutive equation for an orthotropic material is given as :

$$\varepsilon_{11} = \frac{\sigma_{11}}{E_1} - \frac{\sigma_{22}\nu_{21}}{E_2} - \frac{\sigma_{33}\nu_{31}}{E_3} + \alpha_{11}\theta_T$$

$$\varepsilon_{22} = \frac{\sigma_{12}}{E_2} - \frac{\sigma_{11}\nu_{12}}{E_1} - \frac{\sigma_{33}\nu_{32}}{E_3} + \alpha_{22}\theta_T$$

$$\varepsilon_{33} = \frac{\sigma_{33}}{E_3} - \frac{\sigma_{11}\nu_{13}}{E_1} - \frac{\sigma_{22}\nu_{23}}{E_2} + \alpha_{33}\theta_T$$

For the plane stress case, we can simplified as

$$\begin{Bmatrix} \sigma_{11} \\ \sigma_{22} \end{Bmatrix} = \begin{bmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{bmatrix} \begin{Bmatrix} \varepsilon_{11} \\ \varepsilon_{22} \end{Bmatrix} - \begin{Bmatrix} \beta_{11} \\ \beta_{22} \end{Bmatrix} \theta_T$$

where

$$c_{11} = \frac{E_1^2}{E_1 - E_2\nu_{12}^2} \quad c_{12} = \frac{E_1 E_2 \nu_{12}}{E_1 - E_2\nu_{12}^2} \quad c_{22} = \frac{E_1 E_2}{E_1 - E_2\nu_{12}^2}$$

$$\beta_{11} = c_{11}\left(\alpha_{11} + \frac{E_2}{E_1}\nu_{12}\alpha_{22}\right) \quad \beta_{22} = c_{22}\left(\alpha_{22} + \nu_{12}\alpha_{11}\right)$$

Thermoelastic total potential functional can be stated as:

$$\pi_H(u_i, v_i, T_{Li}) = \sum_{m=1}^{M_p}\left[\int_{A_m}\left(\frac{1}{2}E_{ijkl}\varepsilon_{ij}\varepsilon_{kl} - \beta_{ij}\varepsilon_{ij}\theta_T\right)dA - \int_{A_m}\left(\overline{F}_i u_i\right)dA\right.$$
$$\left. - \int_{S_\sigma}\left(\overline{T}_i v_i\right)ds + \int_{\partial A_m} T_{Li}\left(v_i - u_i\right)ds\right]$$

$$\pi_c(u_i) = \sum_{m=M_p+1}^{M}\left[\int_{A_m}\left(\frac{1}{2}E_{ijkl}\varepsilon_{ij}\varepsilon_{kl} - \beta_{ij}\varepsilon_{ij}\theta_T\right)dA - \int_{A_m}\left(\overline{F}_i u_i\right)dA - \int_{S_\sigma}\left(\overline{T}_i u_i\right)ds\right]$$

where M_p is the number of singular elements. M is the total number of meshes. $u_i; v_i; T_{Li}$ are element interior displacements, inter element boundary displacement and inter element boundary traction, respectively. W_t is the isothermal strain energy. E_{ijkl} are Young's modulus matrices , ν is the Poison's ratio. ∂A_m is the entire boundary of the domain of mth element A_m. S_{σ_m} is a portion of ∂A_m in which traction is specified. $\overline{F}_i; \overline{T}_i$ are the prescribed body forces and traction, respectively. θ_T is the distribution of temperature difference. β_{ij} is the thermal elastic tensor. For the want of space , the detailed mathematical procedure of matrix form is not repeated here.

4. RESULTS AND DISCUSSION

The accuracy of the present analysis is first investigated by applying it to the case of a central isothermal cracked plate solved by Yum & Hong [1991]. The finite element mesh of the specimen used is shown in Fig.1 along with the four paths over which line integrals $\bar{J}_{or}$ are direct calculated. Glass/epoxy orthotropic material' properties: Young's modulus E_1 is 48.27×10^9Mpa; E_2 is 17.74×10^9 Mpa. Poison's ratio ν_{12} is 0.29. Shear modulus is 6.9×10^9. Thermal expansion coefficient α_1 is 3.5×10^{-6}; α_2 is 11.4×10^{-6}. As shown in Table 1, the accurate path independence of $\bar{J}_{or}$ integral is excellence for all the paths. The computed normalized stress intensity factors are displayed in Table 2. and in Fig. 2 under isothermal uniform tension loading.

When the orthotropic material plate is subject to a uniform temperature change $\Delta\theta_T = -100^0 F$ and constant gradient temperature load $\theta_T = \theta x$., the computed normalized thermal stress intensity factor $K_I \Big/ \left[\frac{E_2\theta_T}{1-(E_2/E_1)\nu_{12}^2}(\alpha_2+\nu_{12}\alpha_1)\sqrt{a} \right]$ and $\bar{J}_{or}$ integral values are shown in Table 3 and Table 4 , respectively. They exhibit a good agreement between indirect method by crack face displacement [Yum & Hong, 1991] and direct method by modified $\bar{J}_{or}$ integral in the present work.

5. CONCLUSION

A modified thermal $\hat{J}_{or}$ integral is proposed and further study on the applicability of $\hat{J}_{or}$ integral is established using detailed numerical experiment. The modified thermal $\hat{J}_{or}$ integral is computed and averaged by four different integral paths chosen and path independence is noted. In addition, the stress intensity factor for orthotropic fracture parameter K_I can be accurately inferred from the $\hat{J}_{or}$ integral. Thus the $\hat{J}_{or}$ integral could be served as a sound fracture parameter for orthotropic thermal fracture problem.

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TABLES AND FIGURES

Table 1. Path independent of $\bar{J}_{or}$ integral under uniform loading.

	$\bar{J}_{or}$ integral $(\times 10^6)$				
a/w	Path 1	Path 2	Path 3	Path 4	average.
0.3	301.9	302.1	300.0	304.0	302.0
0.4	450.4	452.4	452.1	454.2	452.3
0.5	642.6	641.4	642.1	644.2	642.6
0.6	936.0	931.7	932.1	933.3	933.3

Table 2. $\bar{J}_{or}$ integral and $K_I/\sigma\sqrt{a}$ value under uniform loading..

	$\bar{J}_{or}$	normalized $K_I/\sigma\sqrt{a}$	
a/w	values($\times 10^6$)	Yum & Hong	$\bar{J}_{or}$ integral
0.3	302.0	1.10	1.08
0.4	452.3	1.17	1.15

0.5	642.6	1.24	1.22
0.6	933.3	1.37	1.35

Table 3. $\bar{J}_{or}$ integral and $K_I/\sigma\sqrt{a}$ value under uniform temperature loading.

Material No	$\bar{J}_{or}$ integral ($\times 10^5$)	$K_I \Big/ \left[\dfrac{E_2\theta_T}{1-(E_2/E_1)\nu_{12}^2} (\alpha_2+\nu_{12}\alpha_1)\sqrt{a} \right]$ indrect method	direct method
1	66.86	1.01	1.03
2	4.12	1.42	1.53
3	3.65	1.37	1.46

Table 4. $\bar{J}_{or}$ integral and $K_I/\sigma\sqrt{a}$ value under gradient temperature loading.

Material No	$\bar{J}_{or}$ integral ($\times 10^4$)	$K_I \Big/ \left[\dfrac{E_2\theta_T}{1-(E_2/E_1)\nu_{12}^2} (\alpha_2+\nu_{12}\alpha_1)\sqrt{a} \right]$ indirect method	direct method
1	67.05	1.60	1.62
2	4.52	2.34	2.40
3	4.23	2.33	2.38

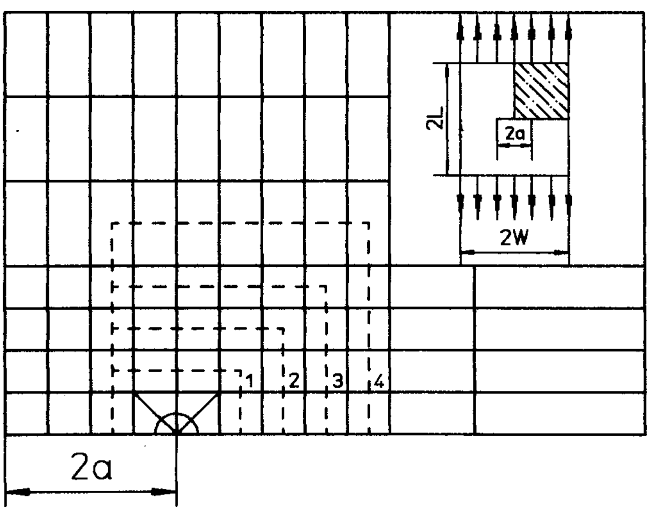

Fig.1 Finite element meshes & integral paths.

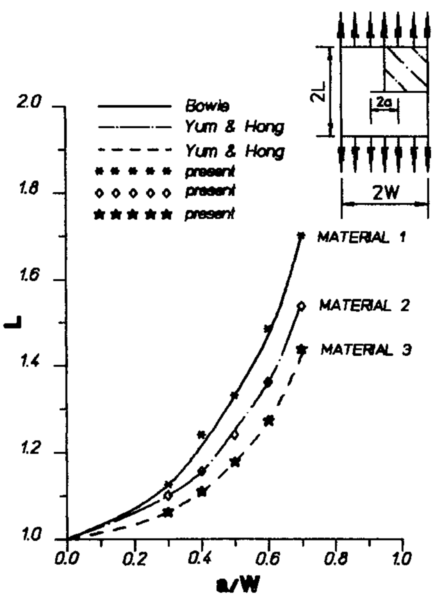

Fig.2 Stress intensity factor curve