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CRACK GROWTH INITIATION IN CONCRETE-LIKE MATERIALS IN THE PRESENCE OF CREEP

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ABSTRACT

A numerical procedure that employs the finite elements method and the state variables approach is proposed to analyze the critical condition of a crack in an ageing viscoelastic body under sustained load. A far field solution proposed by Schapery is used. An example shows how a crack can becomes critical after some finite time that depends on the characteristics of the concrete.

1. INTRODUCTION

Several authors have applied Fracture Mechanics to the analysis of concrete (see for example Wittmann [1], Cedolin [13]). In this paper we introduce the effect of creep deformation into the analysis. Now, when linear viscoelasticity theory is applied to the analysis of a traction boundary value problem for a cracked body, the correspondence principle [2,8] indicates that stresses remain constant in time. Thus, when a K-type stress intensity criterion for crack growth is applied, no deferred effect appears to be possible. On the other hand, experimental results [3,4,5] indicate that cracks in viscoelastic materials do grow under constant loads beneath the elastic fracture level.

The reason of this apparent paradox lies in the existence of a process zone or failure zone around the crack tip, where the material shows a highly nonlinear behaviour. Several authors have studied this problem [5], particularly for viscoelastic polymers. In this paper we shall follow the approach of Schapery [8], that allows an approximate treatment without the need of a precise modeling of the nonlinear behaviour at the crack tip, using instead a far field solution by means of a J-integral type approach.

In Section 2 we recall some linear viscoelasticity fundamental concepts. In Section 3 the criteria of crack growth initiation is presented. In Section 4 the formulation used in the implementation of a finite elements viscoelastic program is reviewed. Two benchmark tests are presented in Section 5. A numerical algorithm for fracture parameter evaluation is described in Section 6. In Section 7 the viscoelastic analysis program is applied to the determination of crack growth initiation conditions.

2. LINEAR VISCOELASTICITY FUNDAMENTALS

The constitutive relation for a linear viscoelastic material may be written in general

$$\varepsilon_{ij} = E_R \int_0^t D_{ijkl}(t,\tau)\, d\sigma_{kl} \tag{1}$$

where σ_{ij} and ε_{ij} are the stress and strain tensors, D_{ijkl} is the generalized creep compliance function and t,τ is the time.

The correspondence principle relates stresses and deformations in the viscoelastic body to the stresses and deformations in a elastic reference body with the same geometry and loading. Thus, we can write a general constituve equation

$$\varepsilon_{ij} = \int_0^t D(t,\tau)\, d\varepsilon^e_{ij} \tag{2}$$

where E_R is a free constant termed the reference modulus and ε^e_{ij} is the strains in the reference body, so that $\sigma_{ij} = E_R\ \varepsilon_{ij}$. For the operational use of the correspondence principle, the symbolic notation

$$\{Ddf\} = E_R \int_0^t D(t,\tau)\, df \tag{3}$$

is useful. So,incremental displacements Δu_i in the viscoelastic body may be found from the corresponding displacements Δu^R_i in the reference body using $\Delta u_i = \{Dd\Delta u^R_i\}$

It is assumed [8] that a potential Φ exists such that σ_{ij} may be written as $\sigma_{ij} = \partial\Phi/\partial\varepsilon^e_{ij}$. Φ is called pseudo strain energy density. In the linear isotropic case it coincides with the usual strain energy density. It must be evaluated with the elastic part of the stresses of the viscoelastic body, that are the same as the reference body stresses for the constant Poisson modulus case.

3. CRACK GROWTH INITIATION CRITERIA

Schapery considers the situation in Fig. 1 where the damaged zone is indicated. The line integrals

$$J_v = \int_{c_1} \left(\Phi\, dx_2 - T_i \frac{\partial u^R}{\partial x_i} ds \right) \quad ; \quad J_f = \int_{c_2} \left(\Phi\, dx_2 - T_i \frac{\partial u^R}{\partial x_i} ds \right) \tag{4}$$

are defined for the viscoelastic (undamaged) and failure zones respectively. x_2 , T_i ,ds, C_1 and C_2 are indicated in Fig.1. Both integration paths must be followed counterclockwise. The property of invariance of the line integral provides, taking into account the sense of integration along paths C_1 and C_2, the relation $J_v = J_f$.

The path C_1 for J_v is arbitrary except that it starts and ends up on the (unloaded) crack faces. The relation $J_f = J_v$ is very important because it provides a mean of determining indirectly the fracture driving force on the damage zone without more precise hypotheses on the behaviour of the material there (far field solution).

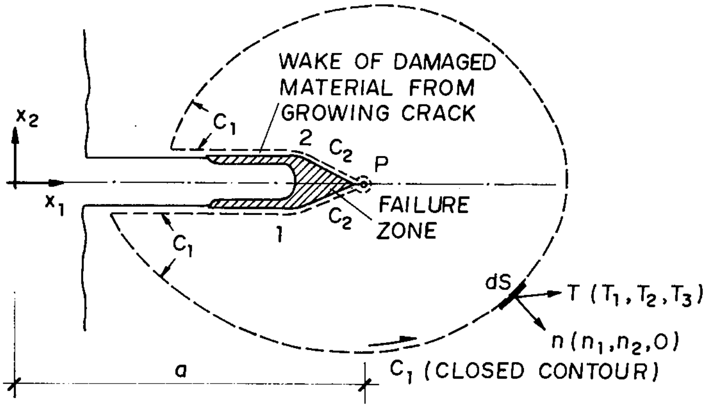

Figure 1 - Process zone and line integrals in the crack tip. *(from* [8]*)*

The failure stress distribution is assumed as that of a time-independent rigid-plastic body. Thus, neglecting the contribution to (4) of singular stresses we have $J_f = \tau_i\ \Delta u^R_{i\alpha}$ where $\Delta u^R_{i\alpha}$ is the crack opening displacement in the apparent crack tip at the moment of rupture. The work input on the crack tip is analogously written $W_f = \tau_i\ \Delta u_{i\alpha}$ and represents the mechanical work available for producing failure. Thus, from this and (3) we obtain $W_f = \tau_i\ \{Dd\Delta u^R_{i\alpha}\} = \{DdJ_f\}$. This result, together with the relation $J_v = J_f$ yields

$$W_f = E_R \int_0^{t_i} D(t,\tau)\ \frac{\partial J_v}{\partial \tau}\ d\tau \qquad (5)$$

An equation for predicting t_i is obtained introducing the work $2\Gamma_i(t)$ required to break an element at $x = a$: $2\Gamma_i(t) = W_f(t)$.

For the opening mode of crack tip deformation in a locally isotropic linear elastic material [5] we have $J_v = (1 - \nu^2)\ K_I^2\ /\ E_R$ so that from (5) we have

$$2\Gamma_i(t) = W_f(t) = J_v\ D(t_i,t)\ E^R \qquad (6)$$

Equation (5) represents the value of J_v transformed to the viscoelastic plane via the viscoelastic operator; it is valid when the Poisson modulus is constant in time (i.e. when the viscoelastic behaviour is characterized by a single function $D(t,\tau)$), as usually assumed for concrete. On the other hand, in many technically important situations, one must consider different rheological laws for deviatoric and volumetric deformations, that results in a varying Poisson modulus. In such a case, $J_v(t)$ may be determined using the well known expression [5]: $J_v(t) = -\,\partial\Phi(t)/\partial a$ where $\Phi(t) = \Phi^s(t) + \Phi^o(t)$ is the internal energy divided into deviatoric and hydrostatic parts. Those parcels are determined using

$$\Phi^o(t) = K^R \int K(t,\tau)\, d\Phi^o_e \quad ; \qquad \Phi^s(t) = G^R \int G(t,\tau)\, d\Phi^s_e \tag{7}$$

where the subindex *e* indicates (linear) elastic component and the superindexes *s* and *o* indicate deviatoric and hydrostatic components, respectively. $K(t,\tau)$ and $G(t,\tau)$ are the creep compliance functions for hydrostatic and deviatoric parts, respectively.

The values in (7) are determined using

$$\Phi^s_e(t) = \int_V \left(\int_0^{e_{ij}} s_{ij}\, de_{ij} \right) dV \quad ; \qquad \Phi^o_e(t) = \frac{1}{3} \int_V \left(\int_0^{\varepsilon_o} \sigma^o\, d\varepsilon^o \right) dV \tag{8}$$

where s_{ij} , e_{ij} , σ^o , ε^o are the deviatoric and hydrostatic stresses and strains, respectively, defined as

$$\begin{aligned} \sigma^o &= \sigma_{ii} \quad ; \quad s_{ij} = \sigma_{ij} - \frac{1}{3}\sigma^o \delta_{ij} \\ \varepsilon^o &= \varepsilon_{ii} \quad ; \quad e_{ij} = \varepsilon_{ij} - \frac{1}{3}\varepsilon^o \delta_{ij} \end{aligned} \tag{9}$$

We remark that the elastic stresses and strains used in (8) and (9) must be determined for each time of interest through the viscoelastic analysis; in that way, they include the eventual effects of redistributions due to nonhomogeneities, that can be inherent to the structure or be developed along the process. The later case may correspond to thermal effects or damage effects.

4. FINITE ELEMENT FORMULATION FOR VISCOELASTIC ANALYSIS

Because of space limitations, only the formulation for the deviatoric part is presented, indicated by a superindex *s*. Similar expressions for the hydrostatic terms may be analogously obtained. Rewriting (1) in rate form using matrix notation, it results, for an isotropic material

$$\dot{\tilde{e}}^s(t) = \frac{\dot{\tilde{s}}(t)}{2G_o(t)} + \int_\tau^t \dot{G}(t,\tau)\, \dot{\tilde{s}}(\tau)\, d\tau \tag{10}$$

where $\dot{G}(t,\tau) = \partial G(t,\tau)/\partial t$ is the derivative of the creep compliance function for the deviatoric part.

It is possible to approximate the derivatives of the creep compliance function through Dirichlet-Prony exponential series as

$$\dot{G}(t,\tau) = \sum_{k=1}^{n} \frac{1}{\eta_k^s(\tau)} e^{-(t-\tau)/\theta_k^s} \tag{11}$$

where e = 2.71828218 is not to be mistaken with the strain e(t).

This representation is complete, and any compliance functions (i.e. creep curve) can be approximated as closely as desired [2,12].

State variables are defined as

$$q_k^s(t) = \int_{\tau_o}^{t} \frac{1}{\eta_k^s(\tau)} e^{-(t-\tau)/\theta_k^s} \dot{\underset{\sim}{s}}(\tau)\, d\tau \tag{12}$$

The summation of these state variables provides the deviatoric deferred strains.

It can be shown [6] that, if $\dot{\underset{\sim}{s}}(\tau) / \eta_k^s(\tau)$ is considered as linear functions of τ in the interval {t, t+Δt} used in a time integration process, the state variables can be obtained in the incremental form described below:

$$\underset{\sim}{q}_k^s(t+\Delta t) = \underset{\sim}{q}_k^s(t)\, e^{-\Delta t/\theta_k^s} + \frac{\dot{\underset{\sim}{s}}(t)}{2G_k^*(t)} \left(1-e^{-\Delta t/\theta_k^s}\right) + \Delta\left(\frac{\dot{\underset{\sim}{s}}(t)}{2G_k^*(t)}\right) \left(\frac{\Delta t-\theta_k^s(1-e^{-\Delta t/\theta_k^s})}{\Delta\bar{t}}\right) \tag{13}$$

where $G_k^*(t) = \eta_k^s(t)/\theta_k^s$, $\Delta\bar{t}$ is the time interval of the previous step and $\Delta(\dot{\underset{\sim}{s}}(t)/2G_k^*(t))$ is the variation of $\dot{\underset{\sim}{s}}(t)/2G_k^*(t)$ along that interval.

Then, the strains are given by

$$\dot{\varepsilon}^o(t) = \frac{\dot{\sigma}^o(t)}{3K_o(t)} + \sum_{j=1}^{n} q_j^o(t) \quad : \quad \dot{\underset{\sim}{e}}^s(t) = \frac{\dot{\underset{\sim}{s}}(t)}{2G_o(t)} + \sum_{k=1}^{m} \underset{\sim}{q}_k^s(t) \tag{14}$$

where $K_o(t)$ is the elastic bulk modulus and $G_o(t)$ the elastic shear modulus at age t, $\varepsilon^o(t)$ is the hydrostatic strain, $\sigma^o(t)$is the hydrostatic stress and $q_j^o(t)$ is the state variable corresponding to hydrostatic deformations.

Total strains are obtained from the hydrostatic and deviatoric components and can be separated into elastic and deferred parcels (the last one formed by the summation of the state variables)

$$\dot{\underset{\sim}{\varepsilon}}(t) = \frac{\dot{\varepsilon}^o(t)}{3} \underset{\sim}{I} + \dot{\underset{\sim}{e}}^s(t) \quad \text{or} \quad \dot{\underset{\sim}{\varepsilon}}(t) = \dot{\underset{\sim}{\varepsilon}}^e(t) + \dot{\underset{\sim}{\varepsilon}}^v(t) \tag{15}$$

where $\underset{\sim}{I}$ is the unit matrix.

The formulation above corresponds to a generalized Kelvin model with spring modulus E(t) and dashpot viscosity $\eta(t)$, with the condition

$$\theta = \frac{\eta(t)}{E(t) + \dot{\eta}(t)} = \text{constant} \tag{16}$$

The influence of the temperature over the material characteristics may be evaluated using the same state variables concept. Thus, we can consider that both the retardation time θ and the viscosity η are functions of age and temperature T. In the case of concrete, the temperature history affects the ageing process. Thus, it is convenient to use the maturity m as an additional state variable, writing

$$\dot{m} + \frac{m}{\theta_m} = a\, T \tag{17}$$

where θ_m and *a* are constants. Then, we may use *m* as an additional parameter [9].

In the finite elements formulation, viscoelastic ($\underset{\sim}{\varepsilon}^v$) and thermal ($\underset{\sim}{\varepsilon}^T$) strains can be treated in a similar way as initial strains. Using the finite element formulation in incremental form , the general stress-strain relation is given by

$$\dot{\underset{\sim}{\sigma}}(t) = \underset{\sim}{E} \left(\underset{\sim}{B}\, \dot{\underset{\sim}{u}}(t) - \dot{\underset{\sim}{\varepsilon}}^T(t) - \dot{\underset{\sim}{\varepsilon}}^v(t) \right) \tag{18}$$

where $\underset{\sim}{E}$ is the elastic matrix and $\underset{\sim}{B}$ is the strain-nodal displacement matrix. The thermal strain $\underset{\sim}{\varepsilon}^T$ is determined as

$$\underset{\sim}{\varepsilon}^T(t) = \alpha\, (T(t) - T(0))\, \underset{\sim}{I} \tag{19}$$

where α is the linear thermal dilatation coefficient.

The same state variables formulation may be extended also to include some kind of nonlinear behaviour directly as in [10] or by using damage theory, as explained in [7]. More details can be found in [6].

5. VERIFICATION OF THE FINITE ELEMENT ALGORITHM

To validate the viscoelastic algorithm we run several problems with closed-solution. We will present here the analysis of an uniaxial compression test and of a reinforced concrete tube under internal pressure solved analytically by Arutyunyan [11]. Of course, the formulation presented can approximate any constitutive relation for linear creep.

a. Uniaxial Compression Test

The creep function proposed for Arutyunyan, in $(\text{kgf/cm}^2)^{-1}$, is

$$C(t,\tau) = \frac{1}{E(\tau)} + \phi(\tau)\left(1- e^{-\gamma(t-\tau)} \right) \tag{20}$$

where $E(\tau) = 2.10^5(1-e^{-0,03\tau})$ [kgf/cm^2], $\phi(\tau) = 0,90.10^{-5} + 4,82.10^{-5}/\tau$ [cm^2/kgf] and $\gamma = 0,026$ day^{-1}. The rheological model used is a spring of modulus $E = E(\tau)$ in series with a Kelvin model with the properties $1/\phi(\tau)$ for the spring and $1/(\phi(\tau)\ \gamma)$ for the dashpot. The same retardation time is used for hydrostatic and deviatoric terms in order to make the Poisson modulus ν = 1/6 constant. A constant compression stress of 10 kgf/cm^2 is applied at the age of 28 days, and removed at 128 days. The absolute values of the strains obtained are plotted in Fig. 3.

b. Reinforced Concrete Pipe Under InternaL Pressure.

A concrete tube with internal and external radius of 40 and 44 cm, respectively, has an external steel reinforcement with 0,088 cm of thickness, and is submitted to an internal pressure of 10 kgf/cm^2 at 28 days of age. The creep function for the concrete is the same of the previous problem, but with $E(\tau) = 2.10^5$ kgf/cm^2 constant. The modulus for the steel reinforcement is $E_a = 2.10^6$ kgf/cm^2. The result of this problem is shown if Fig.3 through the rate $H_\phi = \sigma_\phi(\tau)/\sigma_\phi(28)$, where σ_ϕ is the circumferencial stress in the concrete tube.

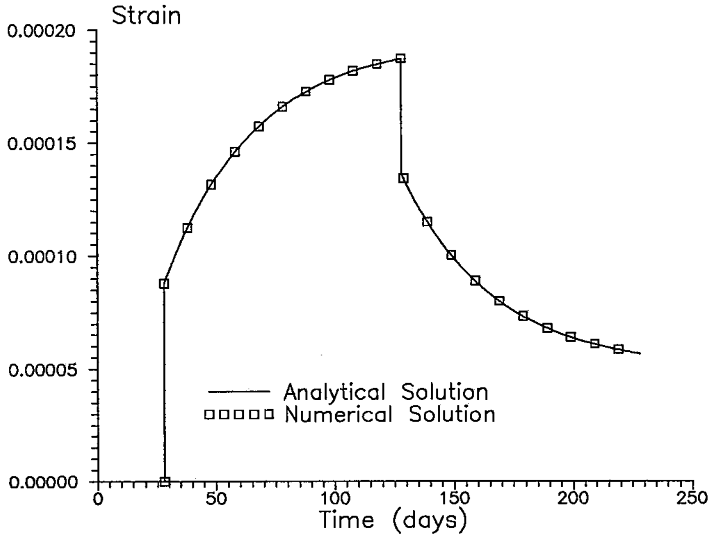

Figure 2 - Uniaxial tension test results for Arutyunyan model

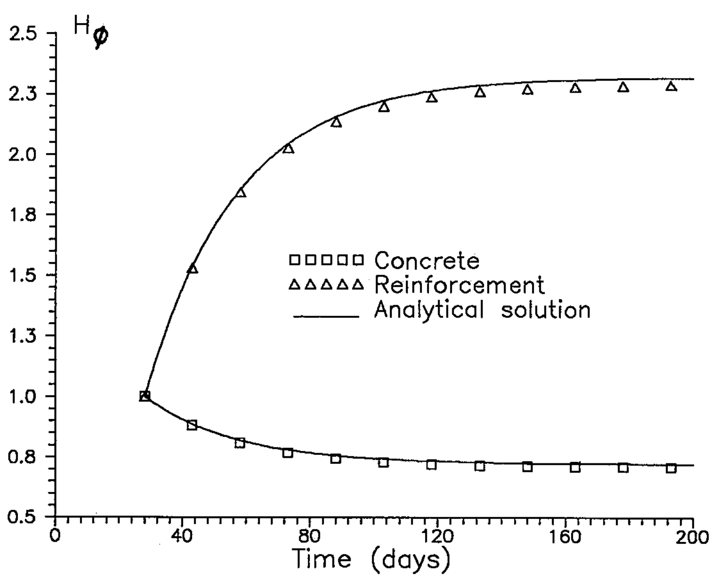

Figure 3 - Reinforced concrete tube under internal pressure: Arutyunyan model

6. NUMERICAL EVALUATION OF CRACK GROWTH INITIATION CRITERIA

Equations (8) can be written for the linear elastic case as:

$$\Phi_e^o = \frac{1}{6}\,\sigma^o\;\varepsilon^o \qquad \Phi_e^s = \frac{1}{2}\,s_{ij}\;e_{ij} \tag{21}$$

These quantities are evaluated through the stresses, which are a standard output of the finite element program. So, expressions (21) can be written as

$$\Phi_e^o = \frac{1}{18\,K_R}\,(\sigma^o)^2 \qquad \Phi_e^s = \frac{1}{4\,G_R}\,s_{ij}^2 \tag{22}$$

Applying (22) in (7) and evaluating the derivative $\partial\Phi/\partial t$, it results

$$\dot\Phi(t) = \frac{K_R\dot\Phi_e^o(t)}{K_o(t)} + K_R\int_o^t \dot K(t,\tau)\;\dot\Phi_e^o(\tau)\;d\tau \;+\; \frac{G_R\dot\Phi_e^s(t)}{G_o(t)} + G_R\int_o^t \dot G(t,\tau)\;\dot\Phi_e^s(\tau)\;d\tau \tag{23}$$

where $\dot K(t,\tau)$ and $\dot G(t,\tau)$ are the derivatives of the creep compliance functions for the hydrostatic and deviatoric parts, respectively, K_R and G_R are the bulk and shear modulus in the reference body (elastic, non-ageing), and

$$\dot\Phi_e^o(t) = \frac{1}{9\,K_R}\;\sigma^o(t)\;\dot\sigma^o(t) \quad : \quad \dot\Phi_e^s(t) = \frac{1}{2\,G_R}\;s_{ij}(t)\;\dot s_{ij}(t) \tag{24}$$

Approximating the derivatives of the creep compliance functions through Dirichlet-Prony series, it is possible to define state variables for $\dot\Phi^o(t)$ and $\dot\Phi^s(t)$ in the form

$$p_j^o(t) = \int_0^t \frac{\dot\Phi_e^o(\tau)}{\eta_j^o(\tau)}\;e^{-(t-\tau)/\theta_j^o}\;d\tau \quad ; \quad p_k^s(t) = \int_0^t \frac{\dot\Phi_e^s(\tau)}{\eta_k^s(\tau)}\;e^{-(t-\tau)/\theta_k^s}\;d\tau \tag{24}$$

Thus, equation (23) can be expressed as

$$\dot\Phi(t) = \dot\Phi_e^o(t) + K_R\sum_{j=1}^{n} p_j^o(t) \;+\; \dot\Phi_e^s(t) + G_R\sum_{k=1}^{m} p_k^s\;(t) \tag{25}$$

Assuming a linear variation of $\dot\Phi^o(\tau)$ and $\dot\Phi^s(\tau)$ in the time interval $\{t \le \tau \le t+\Delta t\}$, it is possible to obtain expressions similar to (13):

$$p_j^o(t+\Delta t) = p_j^o(t)\;e^{-\Delta t/\theta_j^o} + \frac{\dot\Phi_e^o(t)}{3K_j(t)}\left(1-e^{-\Delta t/\theta_j^o}\right) + \Delta\left(\frac{\dot\Phi_e^o(t)}{3K_j(t)}\right)\left(\frac{\Delta t-\theta_j^o(1-e^{-\Delta t/\theta_j^o})}{\Delta t}\right) \tag{27}$$

$$\underset{\sim}{p}_k^s(t+\Delta t) = \underset{\sim}{p}_k^s(t)\, e^{-\Delta t/\theta_k^s} + \frac{\dot{\underset{\sim}{\Phi}}{}_e^s(t)}{2G_k(t)}\left(1-e^{-\Delta t/\theta_k^s}\right) + \Delta\left(\frac{\dot{\underset{\sim}{\Phi}}{}_e^s(t)}{2G_k(t)}\right)\left(\frac{\Delta t-\theta_k^s(1-e^{-\Delta t/\theta_k^s})}{\Delta \bar{t}}\right) \tag{28}$$

Using this approach, the state variables for the viscoelastic strains and for the pseudo-strain energy are evaluated in the same routine.

Viscoelastic crack growth initiation is evaluated through the analysis of two bodies with identical load history, material and boundary conditions, having crack lengths a and $a + \Delta a$. The fracture parameter $W_f(t)$ is, at each time, given by

$$\dot{W}_f(t) = \frac{\dot{\Phi}_1(t) - \dot{\Phi}_2(t)}{\Delta a} \tag{29}$$

The crack growth initiation occurs when the condition (6) is fulfilled.

7. VISCOELASTIC CENTER-CRACKED PLATE

We consider a viscoelastic plate of width 2W under uniform tension with a crack of length 2a perpendicular to the direction of loading, as indicated in Figure 4. The length 2L was considered to be 5W in order to approximate a plate of infinite length. It was considered 2a = 8 cm, 2W = 20 cm, 2L = 50 cm, thickness t=1 cm and the applied tensile traction as 10 kgf/cm^2; Δa was taken as a/4000. Due to the lack of an adequate creep function for tension, it was considered the Aratyunyan model for concrete in compression, with the creep function of equation (20), which is expected to be close to the real behaviour at small stress level. We used the same rheological model employed in item *a* of Section 5.

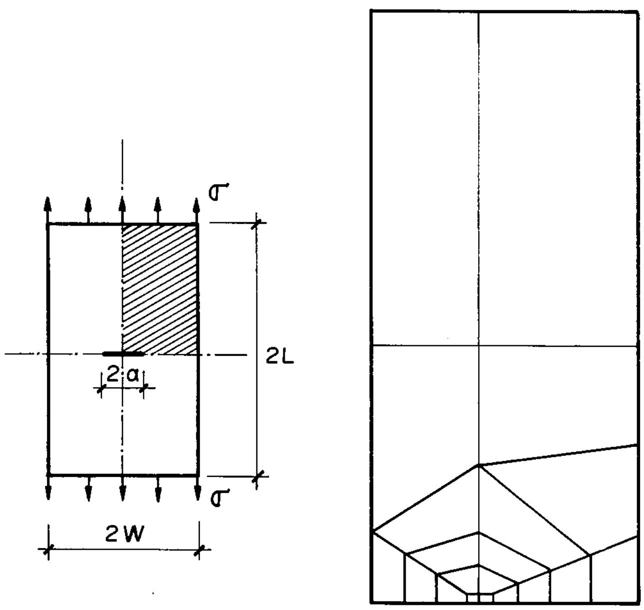

Figure 4 - Center-cracked plate geometry and mesh

Only a quarter of the plate was discretized in finite elements. It was considered four constant load histories, applied at 2, 7, 28 and 90 days of age, respectively. The results obtained are shown in Figure 5, where the numerical and analytical solutions are plotted. The error observed in the numerical solution is 4.6% and constant over all the process. This suggests that the error originates in the elastic solution due to the coarse mesh used, and not in the lack of accuracy of the viscoelastic algorithm.

The results in Fig.5 show the importance of taking account of deferred effects in the evaluation of crack growth parameters. Through the comparison of curves like these to experimental curves of critical fracture parameters (K, G, J) at several ages, it is possible to determine if a crack will have deferred growth or not (Fig.6). The results obtained also show the greater sensitivity to cracking of early age concrete that is observed experimentally.

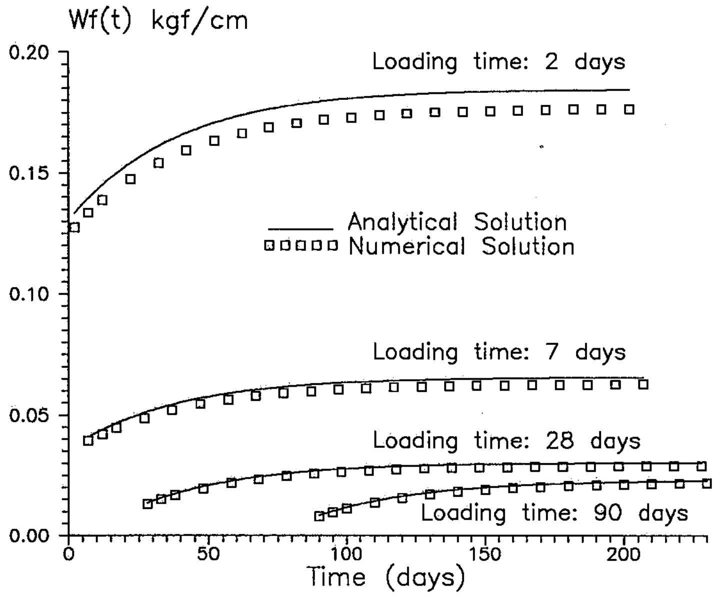

Figure 5. Center-cracked plate results.

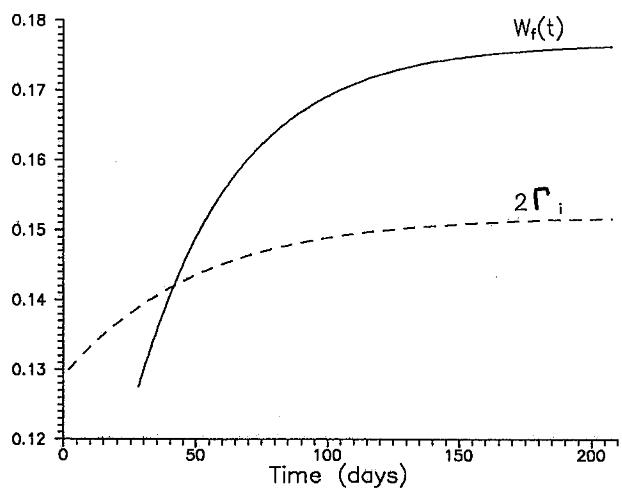

Figure 6 - Critical condition for deferred crack growth

8. FINAL REMARKS

The criteria for the determination of the critical state of cracks due to Schapery's formulation has been implemented numerically for an ageing viscoelastic material as concrete. This is a first step towards the analysis of a crack propagation phenomena. An example shows the application of the procedure. A better representation may be obtained using a continuum damage model to represent the nonlinear behaviour of the material close to the crack tip, as indicated in [6,7].

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