SMiRT-12 / K. Kussmaul (Editor)

H08/5

AN ENDOCHRONIC-DAMAGE CONSTITUTIVE MODEL OF CONCRETE EXTENDED TO BIAXIAL STRESSES

R.D. Zhao and H.M. Che

Research Institute of Structural Engineering, Southwest Jiaotong University, Chengdu, Sichuan 610031, P.R. China

ABSTRACT

In the paper, some existing constitutive response behavior test results were investigated by the elasto—plastic—damage method. Through analyzing, an endochronic—damage constitutive equation for uniaxial stresses was developed. Compared with the elastic damage theory, it broadens the range of application. Some unloading features can be predicted by the equation. By the concept of effective strain, the endochronic—damage constitutive equation under uniaxial stresses was extended to biaxial stresses. Checked by different researchers' test results, it is shown that the extension is successful. The equation is also able to predict the softening branch of stress—strain curve under biaxial stresses.

1 INTRODUCTION

With the developments in computer science and analysis method, the nonlinear behavior of structures has been investigated world wide. However, a constitutive model which is provided with clear physical concept, simple form, wide application range and good computational precision is pressing need. It has become a branch of solid mechanics. Up to now, the constitutive models available to describe the nonlinear behavior of concrete material can be classified roughly into two groups: a nonlinear elastic model and an elasto—plastic model. However, these two models can not exactly mirror the real physical properties of concrete. As is well known the failure of many materials such as concrete, geomaterials and composites is due to the propagation and coalescence of microcracks. This phenomenon, called damage, is most often treated as strain softening in structural analysis. Herein, the authors attempt to introduce concepts of the damage and the intrinsic time into nonlinear analysis of concrete material, which are fundamentally different from both a plastic model and a nonlinear elastic model. At last, agreement with experimental data is also demonstrated.

2 ANALYSIS OF SOME EXISTING CONSTITUTIVE RESPONSE BEHAVIOR TEST RESULTS

In the most of existing damage models of concrete, concrete was regarded as an elastic—brittle material and nonlinearity just only resulted from deterioration of elastic stiffness. The material returns to the original state as long as load is removal. This is not consistent with test results of concrete. The real response of concrete displays elasto—plastic coupling. Therefore, we must take this physical property into account when constitutive equation is developed. In general, we can write out following equations:

$\sigma = E\varepsilon^e = E(\varepsilon - \varepsilon^p)$, for undamaged elastic—plastic material (1)

$\sigma=E\varepsilon^{e}(1-D)=E(\varepsilon-\varepsilon^{p})(1-D)$, for damaged elastic—plastic material (2)

where E=modulus of elasticity; ε^{e}=elastic strain; σ=stress; ε^{p}=plastic strain, ε=total strain, D=damage variable.

Because authors have not personally completed the cyclic loadings tests at high strain level. Some existing results accomplished by Sinha[1], Karsan[2] and Guo[3] are quoted and analyzed. After making an investigation on test results, the following functions are selected to fit data of plastic strain and damage variable:

$D=1.0-a\exp(b\varepsilon/\varepsilon_c)$ (3)

$\varepsilon^{p}=\varepsilon[1-c\exp(d\varepsilon/\varepsilon_c)]$ (4)

where a,b,c,d are material constants listed in Table 1; ε_c=peak strain.

Table 1 Some material constants

Researchers	a	b	c	d	a • c	b+d	σ_o(MPa)
Sinha	1.5008	−0.5540	0.7878	−0.1958	1.1823	−0.7498	−26.4
Guo−1	1.1020	−0.4456	0.8324	−0.3055	0.9173	−0.7511	−21.5
Guo−2	1.0490	−0.4471	0.8339	−0.3084	0.8748	−0.7555	−22.5
Karsan−1	0.9337	−0.5899	0.8250	−0.2072	0.7703	−0.7971	−23.7
Karsan−2	1.6553	−0.9071	0.9340	−0.2381	1.5461	−1.1452	−35.2

Recalling Eq. (2), we can get:

$\sigma=(a\cdot c)E\cdot\varepsilon\cdot\exp[(b+d)\varepsilon/\varepsilon_c]$ (5)

The above equation makes clear that uniaxial constitutive law of concrete can be expressed in the form of exponential function. According to elastic damage theory and applying the hypothesis of energy identity, a constitutive equation was given by Gao[4]:

$\sigma=E\varepsilon$, for $\varepsilon/\varepsilon_c\leqslant k_o$

$\sigma=E\cdot\varepsilon\cdot\exp(-\varepsilon/\varepsilon_c+k_o)$, for $\varepsilon/\varepsilon_c>k_o$ (6)

where k_o=threshold of damage.

3 ENDOCHRONIC—DAMAGE CONSTITUTIVE MODEL OF CONCRETE

Following Bazant's train of thought[5], let's consider the uniaxial stress—strain relation for the Maxwell solid, $d\varepsilon=d\sigma/E+\sigma dt/(EZ_1)$, in which Z_1 is a constant; t=time; and try what happens if dt is replaced with $d\varepsilon$. Then multiplying by $E/d\varepsilon$, one can get a linear differential equation for ε as a function of ε: $d\sigma/d\varepsilon+\sigma/Z_1=E$. The solution for the initial condition $\sigma=0$ at $\varepsilon=0$ is $\sigma=EZ_1[1-\exp(-\varepsilon/Z_1)]$, which is a curve resembling the inelastic stress—strain diagram. And now Z_1 can be called intrinsic time. Generally speaking, it is a function of strain tensor ε_{ij}. Further, if let Z_1 be ε_c(peak strain of concrete under uniaxial compression), the above equation can be changed into $\sigma=E\varepsilon_c[1-\exp(-\varepsilon/\varepsilon_c)]$.

It can be concluded that the above equation can model nonlinear hardening stress—strain relation. But we must realize that it is not suitable to model constitutive relation of concrete. The test results show that before loading there exist microcracks at interfaces between coarse aggregates and mortar in concrete. When the loading is small, the microcracks are stable and concrete behaves, essentially, as a linear elastic material up to about 30% of its uniaxial compressive strength. Taking it into consideration, the above equation can be modified as:

$\sigma=E\varepsilon$, for $\varepsilon/\varepsilon_c\leqslant k_o$

$$\sigma=(1/k_1)E\cdot\varepsilon_c\cdot[1-\exp(-\varepsilon/\varepsilon_c)], \quad \text{for } \varepsilon/\varepsilon_c>k_o \tag{7}$$

where k_o=the proportionality limit (i. e. limit of elasticity) expressed in relative strain. Set the continuity at $\varepsilon=k_o\varepsilon_c$, we can get:

$$1/k_1=k_o/[1-\exp(-k_o)] \tag{8}$$

Eqs. (7) and (8) are just the constitutive laws for undamaged concrete. In the case of damaged material, applying the hypothesis of strain equivalence, one can get:

$$\sigma=E\varepsilon, \quad \text{for } \varepsilon/\varepsilon_c\leqslant k_o$$
$$\sigma=k_o/[1-\exp(-k_o)]E\varepsilon_c[1-\exp(-\varepsilon/\varepsilon_c)](1-D) \text{ for } \varepsilon/\varepsilon_c>k_o \tag{9}$$

In order to determine the evolution function of damage D, we can use Eq. (6), then:

$$D=1-[\varepsilon/(k_o\varepsilon_c)][\exp(k_o)-1]/[\exp(\varepsilon/\varepsilon_c)-1] \tag{10}$$

Eq. (9) is just the endochronic—damage constitutive equation of concrete.

If one compares Eq. (6) with Eq. (9), it can be found that these two possess the identical form. But we must remember that they are very different in modelling physical properties. Eq. (6) only applies to modelling the deterioration of elastic stiffness while Eq. (9) applies to modelling both plasticity and deterioration of modulus of elasticity, and consequently it broadens the range of application. From the definition of plasticity strain, we can get:

$$\varepsilon^p=\varepsilon-k_o\varepsilon_c[1-\exp(-\varepsilon/\varepsilon_c)]/[1-\exp(-k_o)], \text{ for } \varepsilon/\varepsilon_c>k_o \tag{11}$$

We must point out that the above model applies both uniaxial compression and tension. But thresholds of damage are different. As for uniaxial tension, damage process starts at about unity of relative strain so that concrete can be regarded as a linear elastic material in ascending portion, while in descending portion there exists also elasto—plastic coupling. ε_c should be substituded by ε_t (ultimate tensile strain).

Comparisons between theoretical and experimental results are shown in Fig. 1. It can be seen that the model gives satisfactory estimation.

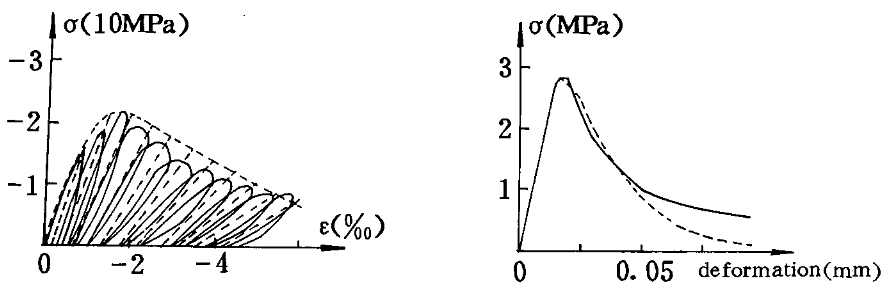


Fig. 1 Stress—strain curves under uniaxial stresses(----- calculated —— tested)

4 THE CONSTITUTIVE MODEL EXTENDED TO BIAXIAL STRESSES

As it is shown that under both uniaxial compression and tension loading, the constitutive models keep same form. Under biaxial stresses, we can define stress space as:

$$\{\sigma\}=[\sigma_1\ \sigma_2]^T \qquad (\sigma_1>\sigma_2) \tag{12}$$

Corresponding strain space is:

$$\{\varepsilon\}=[\varepsilon_1\ \varepsilon_2]^T \qquad (\varepsilon_1>\varepsilon_2) \tag{13}$$

The effective strain under biaxial compression loading:

$$\bar{\varepsilon}=\sqrt{<-\varepsilon_1>^2+<-\varepsilon_2>^2} \tag{14}$$

The evolution equation of damage for biaxial compression is:

$$D=1-[\exp(k_{ob})-1]/k_{ob}\cdot(\bar{\varepsilon}/\bar{\varepsilon}_{max})/[\exp(\bar{\varepsilon}/\bar{\varepsilon}_{max})-1],\ \text{for}\ \bar{\varepsilon}/\bar{\varepsilon}_{max}>k_{ob} \quad (15)$$

The plastic strains are:

$$\underline{\varepsilon}^{p}=\underline{\varepsilon}[1-\exp(k_{ob}-\bar{\varepsilon}/\bar{\varepsilon}_{max})/(1-D)],\ \text{for}\ \bar{\varepsilon}/\bar{\varepsilon}_{max}>k_{ob} \quad (16)$$

where k_{ob}=the threshold of damage expressed in relative strain.

The constitutive equation can be written as:

$$\underline{\sigma}=\underline{\underline{C}}:(\underline{\varepsilon}-\underline{\varepsilon}^{p})(1-D)=\underline{\underline{C}}:\underline{\varepsilon}\cdot\exp(k_{ob}-\bar{\varepsilon}/\bar{\varepsilon}_{max}),\ \text{for}\ \bar{\varepsilon}/\bar{\varepsilon}_{max}>k_{ob} \quad (17)$$

where $\underline{\underline{C}}$=elastic stiffness tensor.

Under biaxial tension—compression loading, the behavior becomes more linear because of tensile stress. In order to take this into account, the total damage is taken as a combination of tension and compression damage:

$$D=\alpha_t D_t+\alpha_c D_c \quad (18)$$

where α_t, α_c are weighting factors corresponding to tension and compression loading respectively; D_t and D_c are tension and compression damage.

Because the nonlinearity is controlled by the level of compressive stress, the compressive weighting factor is selected as:

$$\alpha_c=\sigma_{2p}/\sigma_o \quad (19)$$

where σ_o=ultimate compressive strength for uniaxial stress; σ_{2p}=ultimate compressive strength for tension—compression loading.

The tensile weighting factor is expressed as:

$$\alpha_t=1-\alpha_c \quad (20)$$

The total damage is:

$$D=\alpha_c D_c+(1-\alpha_c)D_t \quad (21)$$

From Eq. (21), it can be seen that when $\alpha_c=1$ (i.e. uniaxial compression), $D=D_c$, and if $\alpha_c=0$ (i.e. uniaxial tension), D=Dt . D_t and D_c can be found from the following:

$$D_t=1-(\varepsilon_1/\varepsilon_{1max})\cdot[\exp(k_{ot})-1]/\{k_{ot}[\exp(\varepsilon_1/\varepsilon_{1max})-1]\}$$
$$D_c=1-(\varepsilon_2/\varepsilon_{2max})\cdot[\exp(k_{oc})-1]/\{k_{oc}[\exp(\varepsilon_2/\varepsilon_{2max})-1]\} \quad (22)$$

where k_{ot}=the threshold of damage under uniaxial stress; k_{oc}=the threshold of damage under uniaxial compression; ε_{1max} and ε_{2max} are ultimate tensile and compressive strain under tension—compression loading.

Plastic strains can be written as:

$$\underline{\varepsilon}^{p}=\underline{\varepsilon}[1-\beta/(1-D)]$$
$$\beta=\alpha_t\beta_t+\alpha_c\beta_c=\alpha_c\beta_c+(1-\alpha_c)\beta_t \quad (23)$$
$$\beta_i=\exp(k_{oi}-\varepsilon_i/\varepsilon_{imax})\leqslant 1,\qquad (i=t,1;c,2)$$

Now, one can obtain the constitutive equation of concrete:

$$\underline{\sigma}=\underline{\underline{C}}:(\underline{\varepsilon}-\underline{\varepsilon}^{p})(1-D)=\underline{\underline{C}}:\underline{\varepsilon}[\alpha_t\exp(k_{ot}-\varepsilon_1/\varepsilon_{1max})+\alpha_c\exp(k_{oc}-\varepsilon_2/\varepsilon_{2max})] \quad (24)$$

5 DETERMINATION OF PARAMETERS

5. 1 Ultimate strains under biaxial compression stresses

From the existing test results, ε_{1max} and ε_{2max} can be taken as:

$$\varepsilon_{2max}=-0.0025$$
$$\varepsilon_{1max}=\upsilon_c\varepsilon_{oc}+b\cdot R \quad (25)$$

where ε_{oc}=the ultimate strain for uniaxial compression; υ_c=Poisson' s ratio for uniaxial compression; R=stress ratio, $R\leqslant 1.0$; b=factor to be determined by use of $\varepsilon_{2max}=\varepsilon_{1max}$ at R=1.

5. 2 Ultimate strains under biaxial tension and compression loading

When tensile stress occurs, the ultimate strains can be calculated as follows:

$$\varepsilon_{2max}=\alpha_c\exp(\alpha_c-1)\varepsilon_{oc}+(1-\alpha_c)\exp(-\alpha_c)(-\upsilon_t\varepsilon_{ot})$$
$$\varepsilon_{1max}=\alpha_c\exp(\alpha_c-1)(-\upsilon_c\varepsilon_{oc})+(1-\alpha_c)\exp(-\alpha_c)\varepsilon_{ot} \quad (26)$$

where υ_t and ε_{ot} are tensile Poisson's ratio and ultimate strain at peak stress, respectively.

5. 3 Poisson's ratio

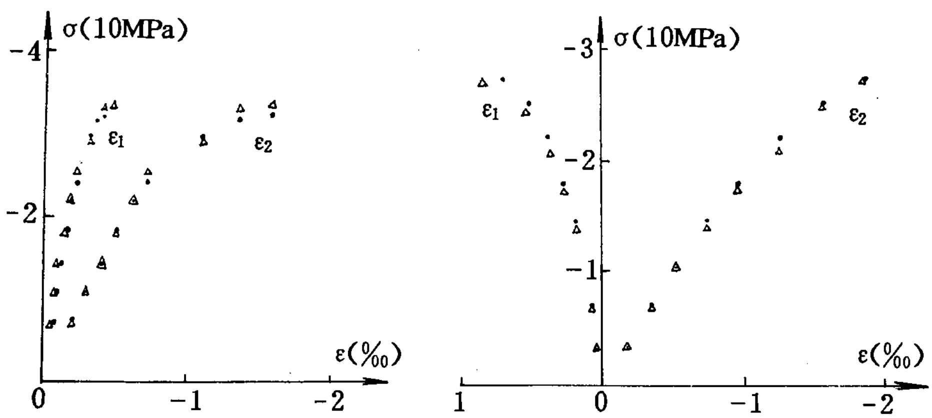
Poisson's ratio is a function of strain which can be expressed as:

$$\upsilon_c = 0.18 + 0.10(\varepsilon/\varepsilon_{oc}) + 0.11(\varepsilon/\varepsilon_{oc})^2 \tag{27}$$

Eq. (27) is directed against uniaxial compressive loading. Under uniaxial tension, Poisson's ratio is somewhat lower than in compression. Prior to peak stress, it almost keeps constant. The details yet remain to be studied. Very little is known of the increase of Poisson's ratio in the post—failure region. For the time being, one can set a limit to it. In effect, υ can be approximately taken as a constant corresponding to that at peak stress in the post—failure region.

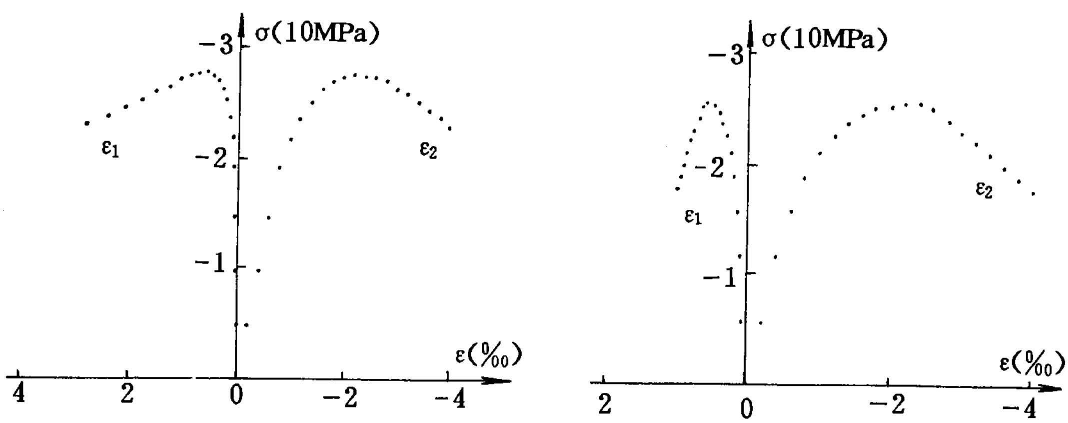
Fig. 2 shows comparison between theoretical and experimental results. Fig. 3 is complete stress—strain relations predicted by the model. Because no complete experimental data about biaxial stresses are available, it is impossible to make comparison.

The equation proposed here has been coded in nonlinear finite element analysis program for plane reinforced concrete structures (prestressed and reinforced concrete beams). The analytical and experimental results are close enough[6].



$\sigma_1/\sigma_2 = 0.50$ $\sigma_1/\sigma_2 = -0.05$

Fig. 2 Stress—strain curves under biaxial stresses(● calculated △ tested)



$\sigma_1/\sigma_2 = 0.15$ $\sigma_1/\sigma_2 = -0.052$

Fig. 3 Complete Stress—strain curves under biaxial stresses (calculated only)

6 CONCLUSIONS

In accordance with the damage theory, internal microcracks and structural defects of concrete caused by loading can be modeled as a continuous defect field. The propagation and coalescence of the microcracks leads to the failure of concrete. The deterioration of elastic stiffness can be described by damage variable.

Combining continuum damage mechanics with endochronic theory, one can rationally model elasto—plastic coupling of concrete. Investigating on test results, authors propose a comprehensive constitutive equation of concrete under uniaxial stresses, evolution laws of damage variable and plastic strain which are based on the damage concept and the intrinsic time concept. Compared with experimental data, the constitutive equation proposed by authors can model deformational response behavior of concrete very well, including descending portion and some unloading features. The elastic damage theory is inferior to our model in the above respects.

By the concept of effective strain, the endochronic—damage constitutive equation under uniaxial stresses has been extended to biaxial stresses. Checked by different researchers' (including authors') test results, it is shown that the extension is successful. The equation is also able to predict the softening branch of stress—strain curve under biaxial stresses.

REFERENCES

1. Sinha B P et al. 1964. Stress—Strain Relations for Concrete under Cyclic Loading. ACI Journal, Vol. 61, No. 2: 195—211.
2. Karsan I D and Jirsa J O. 1969. Behavior of Concrete Under Compressive Loadings. ASCE, Journal of the Structural Division, Vol. 95, No. ST12: 2543—2563.
3. Guo Z H et al. 1987. Investigation of Complete Stress—Deformation Curves for Concrete in Tension. ACI Materials Journal, Vol. 84, No. 4: 278—285.
4. Gao L B. 1988. Damage Constitutive Model and Its Verifications of Concrete (in Chinese), Proceedings of the Third Short—Term Course of Lectures and Symposium on Constitutive Relations of Materials, Chongqing.
5. Bazant Z P et al. 1976. Endochronic Theory of Inelasticity, Journal of the Engineering Mechanics, Division Vol. 102, No. EM4: 701—722.
6. Zhao R D. 1990. Investigation of Nonlinear Behavior on Concrete and Its Structures (in Chinese), A Doctoral Dissertation Presented to the Academic Committee of Southwest Jiaotong University, Chengdu, China.