SMiRT-12 / K. Kussmaul (Editor)
1993 Elsevier Science Publishers B.V.

J01/1

A DYNAMIC MODEL FOR FLOW-INDUCED VIBRATION OF PARALLEL-PLATE FUEL ASSEMBLIES

C.Q. Guo, R.H. Peng and D.L. Sun

Hengyang Institute of Technology, Hunan, P.R. China

ABSTRACT

In this paper, a dynamic model for flow-induced vibration of parallel-plate fuel assemblies is established which is based on the assumption that all the plates have the same deflections at any instant. The coupled differential equation for the plates and the coolant is developed. By solving this equation with Fourier transformation, the relation between the natural frequences and the flow velocity is obtained. On this basis, the critical velocity for statical instability and the vortex shedding resonant velocity are given.

1 INTRODUCTION

The design of the parallel-plate fuel assemblies of some research and power nuclear reactors has been greatly concerned in recent years. In this type of fuel assemblies, as many as 20 flat-thin fuel-bearing plates are stacked in parallel and the coolant flows through the narrow channels between the plates. The length-to-width ratios of these rectangular plates are usually as great as 10 to 1. The longitudinal edges of the plates are built-in along the assembly walls, and the lateral edges at the entrance and exit are structurally free (Fig. 1). The principal problem in this type of fuel assemblies is the flow-induced vibration of the plates, which would cause some severe consequences.

Many attempts have been made to describe the flow-induced vibration and instability of parallel-plate fuel assemblies, that is summarized by Davis and Kim, 1991. To improve the analytical results, Davis and Kim developed a fluid-structural interaction model which consists only a single fuel plate. With this model, They gave a static critical velocity and a resonat velocity. However, this model did not consider the interactions between two adjacent plates, which should not be ignored for the channels between the plates are very narrow.

In this paper, a new model for the analysis of flow-induced vibration and instability of parallel-plate fuel assemblies is developed. In this model, it is assumed that all the plates have the same deflections at any instant. Because the fuel plates are very thin and the channels between them are very narrow, this assumption is much reasonable, especially for the plates near the centre of the assembly. From this assumption, it can be readly derived that the flow velocity of the coolant in the narrow channels remains costant in the longitudinal direction.

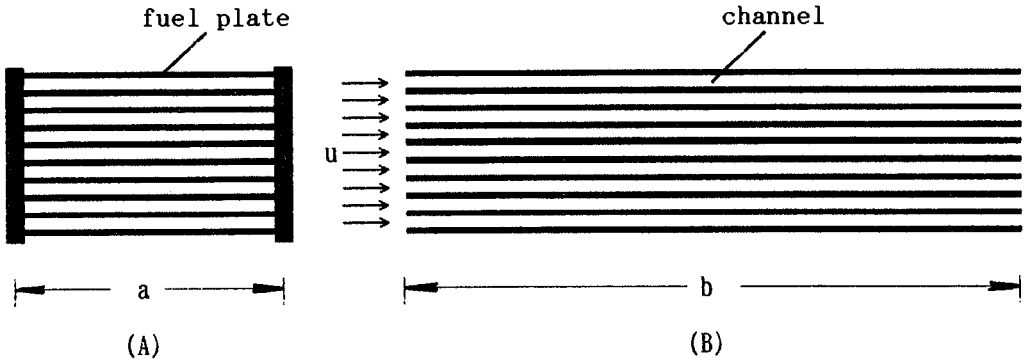

Fig. 1 A parallel-plate fuel assembly; (A) front view, (B) side crose section view

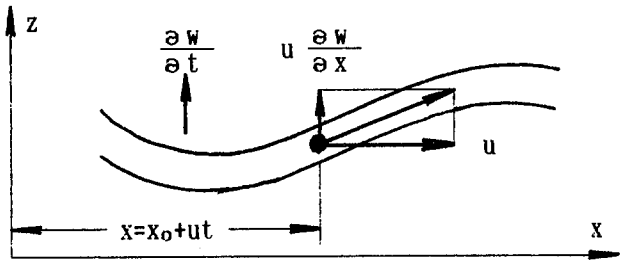

Fig. 2 Motion of coolant in the channel

2 EQUATIONS FOR PLATES AND CHANNEL COOLANT

The equation for one of the plates to the fluid pressure difference is given by the classical elastic-thin-plate theory as

$$D\nabla^4 w + M\frac{\partial^2 w}{\partial t^2} = -P \tag{1}$$

where D is the flexural bending stiffness of a single plate, w is the deflection of the plate, M is the plate mass per unit area, p is the pressure difference between the two surfaces of the plate.

By using the momentum theorem (Fig. 2), the equation of motion for the coolant flowing through the channels between the plates can be developed as

$$p = M_c\left(\frac{\partial}{\partial t} + u\frac{\partial}{\partial x}\right)^2 w \tag{2}$$

where M_c is the fluid mass per unit area, u is the velocity in the longitudinal direction of the coolant flow in the channels.

By substituting Eq.(2) to Eq.(1), the coupled equation for the plates and coolant is developed as follows

$$D\nabla^4 w + (M+M_c)\frac{\partial^2 w}{\partial t^2} + 2M_c u\frac{\partial^2 w}{\partial x \partial t} + M_c u^2\frac{\partial^2 w}{\partial x^2} = 0 \tag{3}$$

3 SOLUTION FOR NATURAL FREQUENCES

By using Fourier transformation in Eq.(3) on x, y, t, the eigen-equation is obtained as

$$D(k_m^2+k_n^2)^2 - (M+M_c)\omega_{mn}^2 - 2M_c u k_m \omega_{mn} - M_c k_m^2 u^2 = 0 \tag{4}$$

where k_m is the modal eigenvalue of the longitudinal direction, k_n is the modal eigenvalue of the lateral direction, ω_{mn} is the natural frequency.

If $k_m \geqslant 0$, $k_n \geqslant 0$, $\omega_{mn} \geqslant 0$ and $u \geqslant 0$ are prescribed, Eq.(4) can be rewritten as

$$\omega_{mn}^2 \pm \frac{2M_c}{M+M_c} k_m u \omega_{mn} + \frac{M_c}{M+M_c} k_m^2 u^2 - \frac{M}{M+M_c} \omega_{mn}^{*2} = 0 \tag{5}$$

where

$$\omega_{mn}^{*} = \sqrt{\frac{D}{M}} (k_m^2 + k_n^2) \tag{6}$$

is the natural frequency of the plate in vacuum.

Eq.(5) discribes the relation between the natural frequences and the flow velocity of the coolant.

4 CRITICAL VELOCITY FOR STATIC INSTABILITY

By setting $\omega_{mn}=0$ in Eq.(5), the critical velocity for static instability is obtained as

$$u_{cr} = \sqrt{\frac{M}{M_c}} \frac{\omega_{mn}^{*}}{k_m} \tag{7}$$

Substituting (6) to (7) gives

$$u_{cr} = \sqrt{\frac{D}{M_c}} (k_m + \frac{k_n^2}{k_m}) \tag{8}$$

u_{cr} has its minimum when $k_m \approx k_n$ and k_n has its minimum. Then we have the approximate minimum static critical flow velocity

$$u_{cr} \approx 2\sqrt{\frac{D}{M_c}} k_n \tag{9}$$

At this critical flow velocity, the plate may experience large deflections while there is no vibration.

5 VORTEX SHEDDING RESONANT VELOCITY

In vortex-excited vibration, the relation between the vortex shedding frequency ω_{mn} and the flow velocity u is given as

$$\omega_s = 2\pi S/h \tag{10}$$

where S is Strouhal number, h is the thickness of the plate.

By equating ω_s in Eq.(10) to ω_{mn} in Eq.(5), the vortex shedding resonant velocity is obtained as

$$u_{vr} = \left(\frac{M}{4\pi^2 S^2 (M+M_c) \pm 4\pi S M_c k_m h + M_c k_m^2 h^2}\right)^{1/2} \omega_{mn}^{*} h \tag{11}$$

or

$$u_{vr} = \left(\frac{D}{4\pi^2 S^2 (M+M_c) \pm 4\pi S M_c k_m h + M_c k_m^2 h^2}\right)^{1/2} (k_m^2 + k_n^2) h \tag{12}$$

Because $(M+M_c)/M_c >> 1$ and for lower modal $k_m h << 1$, it can be taken approximately that u_{vr} has its minimum when k_m and k_n have their minima respectively. The minimum of k_m is zero, then the approximate minimum of vortex shedding velocity is

$$u_{vr} \approx \sqrt{\frac{D}{M+M_c}} \frac{k_n^2 h}{2\pi S} \tag{13}$$

6 RESULTS FOR ETR FUEL ASSEMBLY AND DISCUSSION

For the Engineering Test Reactor (ETR) (Doan,1958), the parallel-plate fuel assembly data are given as follows: plate width a=7.62cm, plate length b=76.2cm, plate thickness h=0.127cm, channel height h_c=0.254cm, plate density ρ=6.5×10^{-3}kg/cm^3, coolant density ρ_c=1.0×10^{-3}kg/cm^3, plate elastic modulus E=68.95Gpa, Poison's ratio ν=0.3, designed coolant flow velocity u=10m/s, Reynolds number Re≈60,000, Strouhal number S=0.155.

Substituting These data to the previous derived formula, we have the following results: the minimum static critical velocity u_{cr}=281m/s, the minimum vortex shedding resonant velocity is u_{vr}=5.7m/s for plates perfectly clamped to the assembly walls and u_{cr}=186m/s, u_{vr}=2.5m/s for plates simply-supported by the assembly walls.

In Davis' and Kim's single fuel plate model, the static critical velocities for ETR fuel assembly are about 22m/s for perfectly clamped plate and 10m/s for simply-supported plate, which are much less than those previous obtained in this paper. The conservativeness of single plate model in predicting static instability was also verified in a series of tests with mock assemblies (Ref. Davis and Kim, 1991).

The minimum vortex shedding velocities obtained in this paper are much less than the "leading edge resonant velocities" given by Davis and Kim (the ratios are about 0.2). It is concluded that vortex-excited vibrations occur much earlier.

7 CONCLUSIONS

1. The minimum static critical velocities are much higher than the design velocity. Therefore, the critical instability could not be considered in the design of parallel-plate fuel assemblies.
2. The minimum vortex shedding resonant velocities are less than the design velocity, that is, the design velocity is in the range of vortex shedding resonant velocities. Therefore, this phenomum must be considered in the design.
3. The static critical velocity and the vortex shedding resonant velocity for plates perfectly clamped to the assembly walls are higher than those for plates simply-supported by the assembly walls.
4. The single fuel plate model is much conservative in predicting static instability and unsafe in predicting dynamic instability.

REFERENCES

Davis,D.C., Kim,G. (1991). Design against Hydrodynamic Instabilities in Flat-Plate Type Fuel Element Assemblies. SMiRT 11 Trans. vol. J: pp. 105 -109.

Doan,R.L. (1958). The Engineering Test Reactor-A Status Report. Nucleonics. vol. 16: pp. 102-105