CYLINDRICAL SHELL UNDER IMPACT LOAD INCLUDING TRANSVERSE SHEAR AND NORMAL STRESS

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Abstract
The general governing equations of shell of revolution under shock loads are reduced to equations describing the elastic behavior of cylindrical shell under axisymmetric impact load. The effect of lateral normal stress, transverse shear, and rotary inertia are included, and the equations are solved by Galerkin finite element method. The results are compared with the previous works of authors.

Introduction
Due to many industrial applications, the impact mechanics of shells have received considerable attention in recent years. In most of these analysis the behavior of shell against the applied dynamic load is modeled by conventional formulations of shell. Tene [1] considered the dynamic behavior of shell of revolution and included in his analysis the effect of transverse shear and rotary inertia. In this analysis Hobolt method is used along with the finite difference technique to solve the dynamic equations. Shakeri and Eslami [2] formulated the cylindrical shell with the same assumptions, and solved the problem with Galerkin finite element method.

The Galerkin finite element formulations have been used by Eslami to solve the coupled thermoelastic problems of beams, thick spheres and shells [3-5]. Eslami and Shakeri also found the response of cylindrical shell under coupled thermoelastic shocks, using the same method [6].

In this paper the Galerkin finite element method is used to analyse a cylindrical shell under impact load. The effect of lateral normal stress along with the effects of shear deformation and rotary inertia are included, and the results are compared with the previous works of authors.

Theoretical Formulations
A cylindrical shell of thickness h and radius R is considered. The axis of cylinder is denoted by x and radial axis by z. The displacement components in x and z directions are shown by u and w. Due to axisymmetric loading, the displacement component in circumferential direction vanishes, i.e. \( v = 0 \). Denoting the shell element rotation by \( \psi \), the general equations of motion of a cylindrical shell under axisymmetric loading, including the effect of lateral normal stress, transverse shear and rotary inertia is derived through the principal of virtual work and are

\[
\frac{d^2 u}{dx^2} (ah) + \frac{dw}{dx} \left( \frac{v h}{R} \right) + \frac{d^2 \psi}{dx^2} \left( \frac{ah^3}{12R} \right) + p_1 = J_0 \ddot{u}
\]
\[ \frac{bh}{2} \frac{d^2 w}{dx^2} - \left( \frac{\nu h}{R} \right) \frac{du}{dx} + \left( \frac{bh}{2} \right) \frac{d \psi}{dx} + \left( \frac{a}{R} \ln \frac{L+1}{L+1} \right) + P_2 = J_0 \dot{w} \] (1)
\[ \frac{ah^3}{12R} \frac{d^2 u}{dx^2} + \left( \frac{ah^3}{12} \right) \frac{d^2 \psi}{dx^2} - \left( \frac{bh}{2} \right) \frac{dw}{dx} - \left( \frac{bh}{2} \right) \psi = J_2 \ddot{\psi} \]

In these equations the constants are defined as
\[ E_1 = \frac{E}{(1+\nu)(1-2\nu)} \quad P_2 = P_z / E_1 \]
\[ J_i = I_{i} / E_1 \quad b = (1-2\nu) \quad a = 1-\nu \] (2)

where
\[ I_{i} = \int \rho z^i d\zeta \quad i=0,2 \] (3)
are inertia terms, \( E \) is the modulus of elasticity, \( \nu \) is the Poisson's ratio, \( \rho \) is the mass density, and \( P_1 \) and \( P_2 \) are the components of external force along the \( x \) and \( z \) directions. In Eqs.(1) \( . \) denotes derivative with respect to time.

Approximating the field variables \( u, w, \) and \( \psi \) with identical shape functions
\[ N_i = (x_j - x) / (x_j - x_i) \]
\[ N_j = (x - x_i) / (x_j - x_i) \] (4)
where \( i \) and \( j \) are the end nodes of the base element \( e \), the formal Galerkin approximation of Eqs.(1) follows to be
\[ \int f(e) \left[ \left( \frac{ah^3}{12R} \right) \frac{d^2 u}{dx^2} - \left( \frac{\nu h}{R^2} \right) \frac{dw}{dx} + \left( \frac{a}{R} \ln \frac{L+1}{L+1} \right) + P_2 - J_0 \dot{w} \right] N_m dx = 0 \]
\[ \int f(e) \left[ \left( \frac{bh}{2} \right) \frac{d^2 w}{dx^2} - \left( \frac{\nu h}{R^2} \right) \frac{du}{dx} + \left( \frac{b}{R} \right) \frac{d \psi}{dx} + \left( \frac{a}{R} \ln \frac{L+1}{L+1} \right) + P_2 - J_0 \dot{\psi} \right] N_m dx = 0 \]
\[ \int f(e) \left[ \left( \frac{ah^3}{12R} \right) \frac{d^2 u}{dx^2} + \left( \frac{ah^3}{12} \right) \frac{d^2 \psi}{dx^2} - \left( \frac{bh}{2} \right) \frac{dw}{dx} - \left( \frac{bh}{2} \right) \psi - J_2 \ddot{\psi} \right] N_m dx = 0 \] (5)

where integrations are carried over the entire length of the base element \( e \). Considering the following shape functions
\[ u(e) = \langle N \rangle \{u\}(e) \]
\[ w(e) = \langle N \rangle \{w\}(e) \]
\[ \psi(e) = \langle N \rangle \{\psi\}(e) \] (6)

and substituting into Eqs.(5) and applying weak formulation to the terms with second order derivatives yield the following finite element equilibrium equation describing the dynamic behavior of the shell
\[ [M] \{\ddot{X}\} + [K] \{X\} = \{F\} \] (7)
where the definition of submatrices of the base element \((e)\) are as follows

\[
X^{(e)} = \begin{bmatrix}
u_i & w_i & \psi_i & u_i & w_j & \psi_j \\
-J_0L/3 & 0 & 0 & -J_0L/6 & 0 & 0 \\
0 & -J_0L/3 & 0 & 0 & -J_0L/6 & 0 \\
0 & 0 & -J_2L/3 & 0 & 0 & -J_2L/6 \\
-J_0L/6 & 0 & 0 & -J_0L/3 & 0 & 0 \\
0 & -J_0L/6 & 0 & 0 & -J_0L/3 & 0 \\
0 & 0 & -J_2L/6 & 0 & 0 & -J_2L/3 \\
\end{bmatrix}
\]

\[
|M|^{(e)} = \begin{bmatrix}
\frac{ahl}{L} & \frac{vhl}{2R} & \frac{ahl^3}{12RL} & \frac{ahl}{2R} & \frac{vhl}{2R} & \frac{ahl^3}{12RL} \\
\frac{vhl}{2R} & \frac{-bhl}{2L-2HR} & \frac{-bhl}{4} & \frac{vhl}{2R} & \frac{bhl}{2L-2HR} & \frac{bhl}{4} \\
\frac{ahl^3}{12RL} & \frac{bhl^3}{12L} & \frac{ahl^3}{12RL} & \frac{ahl^3}{6} & \frac{ahl^3}{12RL} & \frac{ahl^3}{12L} \\
\frac{ahl}{L} & \frac{vhl}{2R} & \frac{ahl^3}{12RL} & \frac{ahl}{6} & \frac{vhl}{2R} & \frac{ahl^3}{12RL} \\
\frac{vhl}{2R} & \frac{-bhl}{2L-2HR} & \frac{-bhl}{4} & \frac{vhl}{2R} & \frac{bhl}{2L-2HR} & \frac{bhl}{4} \\
\frac{ahl^3}{12RL} & \frac{bhl^3}{12L} & \frac{ahl^3}{12RL} & \frac{ahl^3}{6} & \frac{ahl^3}{12L} & \frac{ahl^3}{6} \\
\end{bmatrix}
\]

\[
|K|^{(e)} = \begin{bmatrix}
\frac{P_1L}{2} + \frac{ahl^3}{12R} \left( \frac{d\psi}{dx} \right)_1 + \frac{ahl}{dx} \left( \frac{d\psi}{dx} \right)_1 \\
\frac{P_2L}{2} + \frac{bhl}{2} \left( \frac{dw}{dx} \right)_1 \\
\frac{ahl}{12} \left( \frac{d\psi}{dx} \right)_1 \\
\frac{P_1L}{2} - \frac{ahl^3}{12R} \left( \frac{d\psi}{dx} \right)_2 - \frac{ahl}{dx} \left( \frac{d\psi}{dx} \right)_2 \\
\frac{P_2L}{2} - \frac{bhl}{2} \left( \frac{dw}{dx} \right)_2 \\
\frac{ahl}{12} \left( \frac{d\psi}{dx} \right)_2 \\
\end{bmatrix}
\]

(10)

Where \(L\) is the length of base element \((e)\), and

\[
HR = \frac{aL}{6R} \frac{Ln}{1+h/2R} \frac{1}{1-h/2R}
\]

The boundary conditions imposed to the ends of the cylindrical shell are
either forced or kinematical conditions, where either one can be satisfied through the following relations

\[ u = u^* \text{ or } E_1 \left( ah \frac{d^2 u}{dx^2} + \frac{ah^3}{12R} d\psi + \frac{\psi h}{R} w \right) = N^* \]

\[ w = w^* \text{ or } E_1 \frac{bh}{2} \left( \frac{d^2 w}{dx^2} + \psi \right) = Q^* \]

\[ \psi = \psi^* \text{ or } E_1 \frac{ah}{12} \left( \frac{1}{R} \frac{du}{dx} + \frac{d\psi}{dx} \right) = M^* \]

where (*) denotes the known value of the function on the boundary.

The dynamic equilibrium equation (7) is solved the Newmark direct integration method. The algorithm is adapted to the "constant average acceleration" technique which is unconditionally stable.

Results

Consider a thin cylindrical shell under impulsive internal pressure. The pressure shock is assumed to be applied uniformly along the shell axis and is selected exponential as follows

\[ P_2(t) = 8 \times 10^6 (1 - \exp(-13100t)) \text{ (Pa)} \]

(12)

Let us consider a cylindrical shell with clamped ends. The dimensions and mechanical properties of the shell are as follows: length 40 cm, radius 10.85 cm, thickness 0.2 cm, the modulus of elasticity 200 GPa, Poisson's ratio 0.3, and mass density 8000 Kg/m³. Shell is divided into 14 elements and the duration of dynamic load is 1 msec., with time steps of 10⁻⁵ sec.

The lateral deflection of the shell at 0.2, 0.34, and 0.8 msec. are shown in Fig.1. The maximum deflection occurs near the boundary. This result is verified by Shakeri and Eslami [2], for dynamic loading of cylindrical shell. Fig.2 shows the deflection histories of nodes 2 and 8, next to the boundary and the middle of the shell, respectively.

The dynamic behavior of cylindrical shell with the transverse shear effect and rotary inertia have been studied by authors [2]. In this analysis the effect of normal stress as well as transverse shear and rotary inertia are considered. To find the effect of normal stress, the lateral deflection histories of these two formulations are shown in Fig. 3. It is seen that due to the effect of normal stress, the lateral deflection is diminished by about 17 percent.
Fig. 1 - Radial displacement along the length for different times.

Fig. 2 - Radial displacement histories for nodes 2 and 8.
Fig. 3 - Radial displacement histories for nodes 2 and 8, a comparison of two theories.

References


