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DYNAMIC RESPONSE OF A COMPOSED SHELL UNDER EXPLOSIVE LOAD

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1. INTRODUCTION

The response of a nuclear containment which is a shell structure composed of a cylindrical shell and a spherical one under a chemical explosive load is presented in this paper. The shell is separated into finite ring-elements. For every element the unknown displacements are expressed with Fourier series expanded in circumferential direction of the shell and with Hermite interpolation in shell axis (meridional direction). By means of Hamilton's principle the dynamic equation of an element can be formed. The total dynamic equations are solved by Wilsion $-\theta$ method. A numerical example for containment is analysed. The computational results are compared with the engineering design values under the SSE (safe shutdown earthquake) load.

2. FORMATION OF COMPUTATIONAL MODEL

2.1 Structural Model

It is supposed that a composed shell is considered as a cylindrical shell connected with a half spherical shell on its head and fixed with rigid base at its bottom, as shown in Fig.2.1. Assume that the shell is made of isotropic materials and belongs to the range of linear elastic small deformation.

2.2 Load Model

In general, dynamic pressure may be neglected when the strength of a shock wave is weak. We consider only overpressure load in this paper. Fig. 2. 2 shows the relation between reflected overpressure and incident overpressure when the incident shock wave is with various angles.

When the shock wave impinges on a cylindrical shell and a spherical one, the load is a function both of space and of time. To cylindrical shell, when $\theta < \pi/2$, the relation of overpressure and time shows as in Fig.2.4.

$$t_d = R(1-\cos\theta)/U_o$$

$$t_e = \frac{4R}{C_r}(\pi/2-\theta)/\pi/2 \tag{2.1}$$

$$C_r = 128.6\sqrt{\frac{1.088P_{so}^2+4.83\times10^8P_{so}+3.43\times10^{10}}{4.14\times10^4P_{so}+4.9\times10^9}}$$

In which R is the radius of cylindrical shell, Uo is the transmitted velocity of shock wave, Cr is the velocity of reflected wave, P_{to} in Fig. 2.3 is defined as follows

$$P_{to} = P_{so}\left(1 - \frac{t_o}{t_+}\right)e^{-\frac{t_o}{t_+}} \qquad (2.2)$$

when $\theta > \pi/2$, the relation of overpressure and time shows in Fig.2.5.

$$P_w = P_{so}(1.5 - \theta/\pi)$$
$$t_o = \frac{4R}{C_r}(\theta - \pi/2)/\pi/2 \qquad (2.3)$$

To the spherical shell as shown in Fig.2.3, ψ is an angle between the normal line of it and the incident shock wave front inverse direction I. When we use ψ instead of θ in formulae (2.1), (2.2) and (2.3), the relation of overpressure and time can be obtained.

3.COMPRTATIONAL FORMULAE

3.1 Basic Formulae

The coordinate system of a revolution shell is indicated in Fig.3.1, in which (i, j, k) are the global coordinate vectors, (R, V, Z) are the cylindrical coordinate vectors, and (S, V, N) are the local coordinate vectors. The geometric relation, the relation between internal force and strain of the shell and the expression of strain energy can be obtained according to the thin shell theory in the local coordinate system.

$$\varepsilon_1 = \frac{\partial u}{\partial s} + \frac{w}{R}$$
$$\varepsilon_2 = \frac{1}{r}\frac{\partial v}{\partial \theta} + \frac{u}{r}\cos\phi + \frac{w}{r}\sin\phi$$
$$\gamma = \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial s} - \frac{v}{r}\cos\phi$$
$$\chi_1 = -\frac{\partial^2 w}{\partial s^2} + \frac{1}{R}\frac{\partial u}{\partial s} \qquad (3.1)$$
$$\chi_2 = -\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} - \frac{\cos\phi}{r}\frac{\partial w}{\partial s} + u\frac{\cos\phi}{rR} + \frac{\sin\phi}{r^2}\frac{\partial v}{\partial \theta}$$
$$\chi_{12} = -\frac{1}{r}\frac{\partial^2 w}{\partial \theta^2} + \frac{\cos\phi}{r}\frac{\partial w}{\partial \theta} + \frac{1}{rR}\frac{\partial u}{\partial \theta} + \frac{\sin\phi}{r}\frac{\partial v}{\partial s} - \frac{\cos\phi\sin\phi}{r^2}v$$

$$N_1 = \frac{Eh}{1-\mu_2}(\varepsilon_1 + \mu\varepsilon_2) \qquad M_1 = \frac{Eh^3}{12(1-\mu^2)}(\chi_1 + \mu\chi_2)$$
$$N_2 = \frac{Eh}{1-\mu^2}(\varepsilon_2 + \mu\varepsilon_1) \qquad M_2 = \frac{Eh^3}{12(1-\mu^2)}(\chi_2 + \mu\chi_1) \qquad (3.2)$$

$$N_{12} = N_{21} = \frac{Eh}{2(1+\mu)} \gamma \qquad M_{12} = M_{21} = \frac{Eh^3}{12(1-\mu)} \chi_{12}$$

$$W = \frac{1}{2}(N_1 \varepsilon_1 + N_2 \varepsilon_2 + N_{12}\gamma + M_1 \chi_1 + M_2 \chi_2 + 2M_{12} \chi_{12}) \tag{3.3}$$

The revolution shell is separated into finite ring-elements. For every element the unknown displacements are expressed with Fourier series in circumferential direction and with Hermite interpolation in meridional direction.

$$\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \sum_{k=0}^{\infty} \begin{bmatrix} \cos K\theta & & \\ & \sin K\theta & \\ & & \cos K\theta \end{bmatrix} \begin{bmatrix} [x]_1 & & \\ & [x]_1 & \\ & & [x]_2 \end{bmatrix} \begin{bmatrix} [G]_1 & & \\ & [G]_1 & \\ & & [G]_2 \end{bmatrix} [a_k] \tag{3.4}$$

The load on every ring-element also can be expressed with Fourier series in circumferential direction and with Hermite interpolation in meridional direction.

$$\begin{bmatrix} P_u \\ P_v \\ p_w \end{bmatrix} = \sum_{k=0}^{\infty}[S_k^2] \begin{bmatrix} [x]_0[G]_0 & & \\ & [x]_0[G]_0 & \\ & & [x]_0[G]_0 \end{bmatrix} \cdot$$

$$[P_u(-1),\ P_u(1),\ P_v(-1),\ P_v(1),\ P_w(-1),\ P_w(1)]^T \tag{3.5}$$

Then we add the inertial force $-\rho h\ddot{v}$, the total force is therefore expressed as follows

$$[P] = \sum_{k=0}^{\infty}[S_k^2](-\rho h[x][A][\ddot{a}_k] + [P_k]) \tag{3.6}$$

In accordance with Hamiltion's principle and from formulae (3.3) and (3. 6) , the reaction at every ring - element's joint circle can be obtained. Because the resultant of two neighbouring ring - elements at their joint should be zero, the total dynamic equations for k grade Fourier series may be acquired as follows

$$[M][a_k] + [K_k][\ddot{a}_k] = [P_k] \tag{3.7}$$

4. NUMERICAL EXAMPLE AND RESULTANT DISCUSSION

The computational model is shown in Fig.2.1. In the geometric relation equations (3 .1) there is a singularity as r=0. In order to avoid this situation , we may image that a very small hole as $\psi=0.01°$ is cut out from the top of the shell. Therefore it would be considered as a free boundary condition. The botton of the cylinder containment is built - in in a rigid fundation. An incident shock wave's overpressure is taken to be 1.1×10^4Pa, reflected overpressure is 2.36×10^4 Pa , positive phase duration $t_+=0.19$ sec. The total equations are integrated by Wilson-θ method taking θ=1.4.From k=0 to k=9,10 expanded terms of Fourier series are taken here. The displacement—time relations for point 1 and point 10 on the shell(Fig.3.1) are shown in Fig.4.1a and Fig.4.1b. From these Figures, we can find that the displacements in horzontal direction are much larger than the displacements in vertical direction.

Fig.4.2 shows the differentive states in various time for a cross section of the cylinder (a circle) at 34m from its bottom. The shell is in a state of free vibration after t=0.29sec.From Fig.4.2, we can find that the dynamic response of the shell is very complicated. The main motion is in the shock wave incident direction. In a horizontal cross-section plane the displacement which is in the shook wave direction is the largest one , and the displacements at θ=0 and at θ=

π are larger than that one at $\theta = \pi/2$.
It is known from the analysis of containment under shock wave that the displacement related to low grade model plays a main role.
Some of the internal force-time curves on a circle (a cross section at H=10.8m) for points at $\theta=0$, $\theta=\pi/2$ and $\theta=\pi$ are shown in Fig.4. 3. It is known from the analysis that the membranous forces in the shell play an important role. The relations between horizontal acceleration and time on a circle (H=34.8m) at points of $\theta=0$ and $\theta=\pi$ are shown in Fig.4.4a and Fig 4.4b respectively. There is an acceleration jump in the impinged area of the shell and its peak value is always larger than the others.
The maximum horizontal displacements and accelerations of the containment under shock wave load induced by chemical explosion of four boxear TNT in 488 meters distance far from the shell are compared with those results computed out under SSE in Fig.4.5 and Fig.4.6. Most of the responses of the containment under four boxcar TNT shock load are larger than that under SSE load. It agrees very well with the conclusions of reference 3.

5. CONCLUSIONS

5.1 The pulse phenomenon of acceleration is remarkable when the revolution shell is impinged by shock wave induced by chemical explosion. On the contrary, the pulse phenomenon of displacement is less clear.
5.2 The movement of comtainment belongs to a whole oscillation.
5.3 The membranous forces of the shell under the related load play an important role.
5.4 Most of the responses of the shell under the load induced by four boxcar TNT in 488 meters distance far from it are larger than that under the load of SSE.

6. REFERENCES

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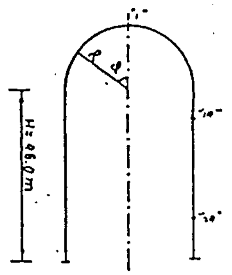

Fig.2.1 Composed shell model

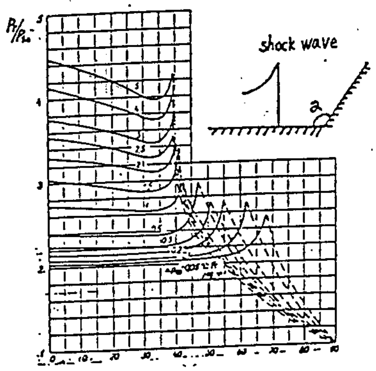

Fig.2.2 Relation between reflected overpressure and incident overpressure

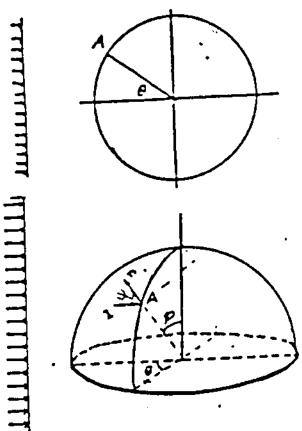

Fig.2.3

Cylinder and Spherical shell

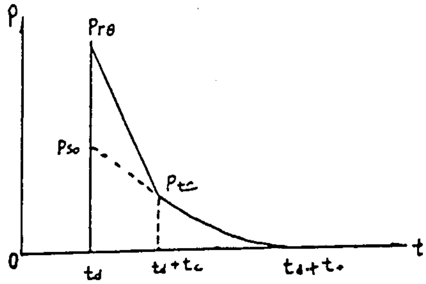

Fig.2.4

Overpressure—time when $\theta < \pi/2$

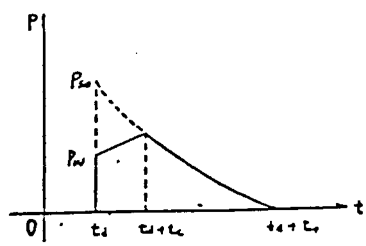

Fig.2.5

Overpressure—time when $\theta > \pi/2$

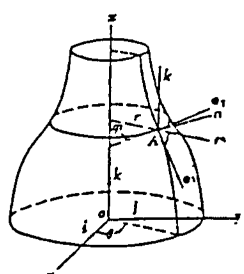

Fig.3.1 Coordinate system of the revolution shell

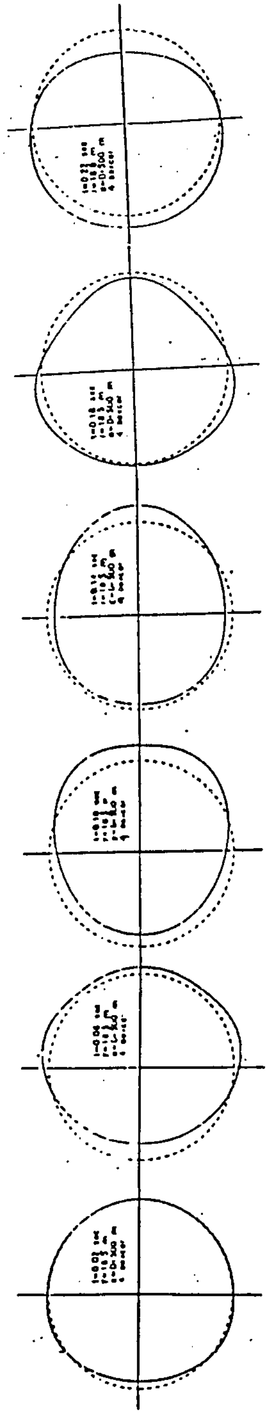

Fig.4.2 Different deformative states in varions time of a olrole at H= 34m from bottom

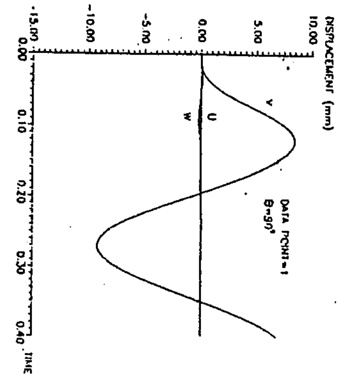

Fig.4.1a Displacement—time at point 1 on the shell

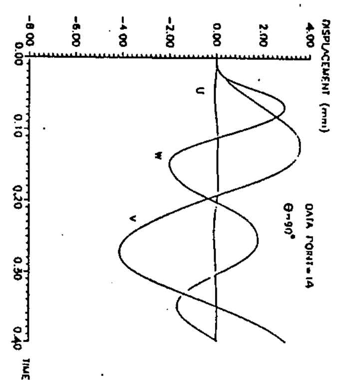

Fig.4.1b Displacement—time at point 14 on the shell

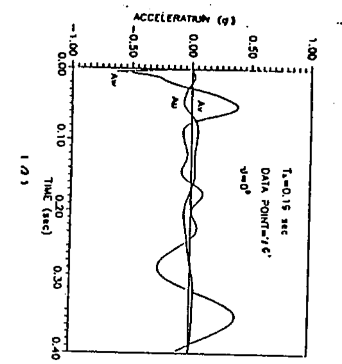

Fig.4.4a Horizontal acceleration for H=34.8m θ= 0

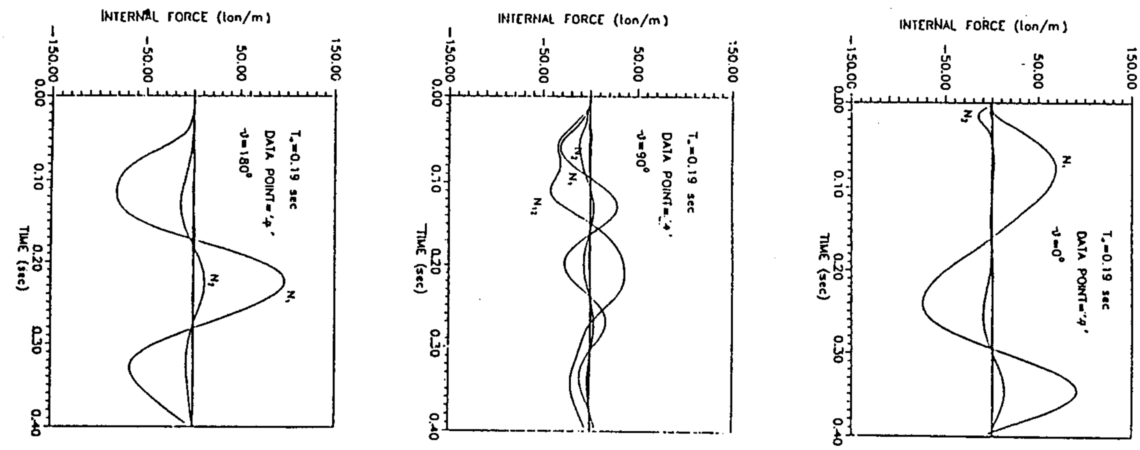

Fig.4.3 Internal force—time of a circle at H=10.8m for θ=0, π/2, π

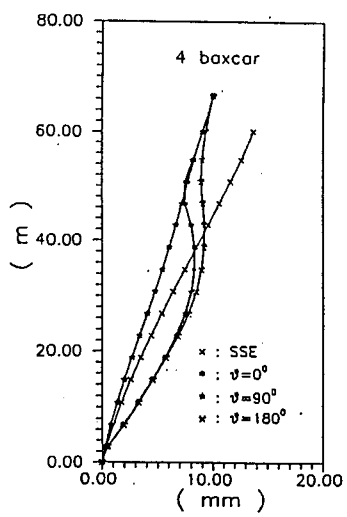

Fig.4.5 Comparation of horizontal displacement

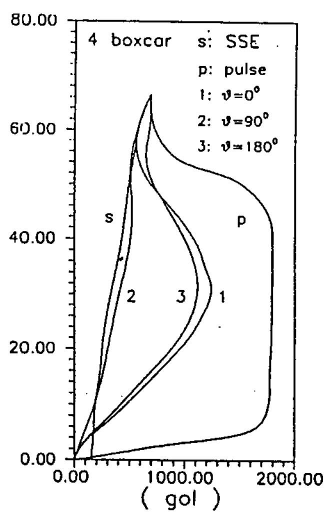

Fig.4.6 Comparation of horizontal acceleration

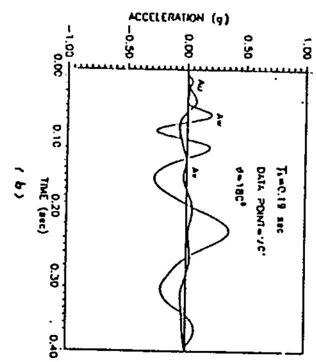

Fig.4.4b Horizontal acceleration for H=34.8m θ= π