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IMPEDANCE FUNCTIONS OF TIME DOMAIN FINITE ELEMENT SOLUTIONS

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Abstract

It is shown that ideal impedance functions of the halfspace and those of Finite Element Models may correspond to one another under defined conditions. This results in the application of impedance functions used as a criterion for the quality of FEM-models, /1/.

1 Introduction

This paper deals with the analysis and verification of stiffness functions generated by numerical integration in the time domain and by subsequent transform of time-dependent vectors into frequency domain. The applied structural models are focussed on the elastic and viscoelastic halfspace with an ideal, massless, rigid plate on the surface excited harmonically. This paper only deals with the rocking mode:

$$
\begin{aligned}
k_{MM} &= \pi \cdot 3\cdot(1-\nu)/(8\cdot G) \cdot RE\left\{ \mathbf{P}^f / \mathbf{v}_V{}^f \right\} \\
c_{MM} &= \pi\cdot c_S \cdot 3\cdot(1-\nu)/(8\cdot G\cdot R) \cdot IM\left\{ \mathbf{P}^f / \mathbf{v}_V{}^f \right\} \cdot 1/\omega .
\end{aligned}
$$

k_{MM} and c_{MM} are the real and imaginary part in terms of the dimensionless frequency a_0 for axisymmetrical models.
In reference /1/ more details for the frequency domain impulse P^f are given and the influence of plane strain, plane stress and axisymmetric models on impedances is evaluated for the rocking, vertical and horizontal mode.

2 Influence of the Soil Section Simulating the Halfspace

The displacements of the plane strain and axisymmetric FEM models are calculated by the codes ADINA /2/ and ASHSD /3/. Fig.1 simultaneously shows that depth and radius little influences the impedance functions when using different models providing a sufficient soil section discretized. In Fig.2 the influence of the soil depth, namely a shallow soil model of 30 m and a deeper one of 100 m, can be seen.

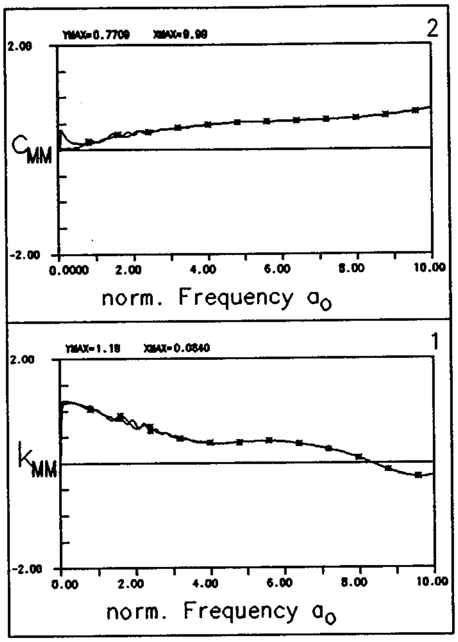

Fig.1
Impedance Functions of Two Soil Models with Viscous Dampers at Both Sides and at the Base in horizontal and Vertical Direction.

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4920/5/271/100

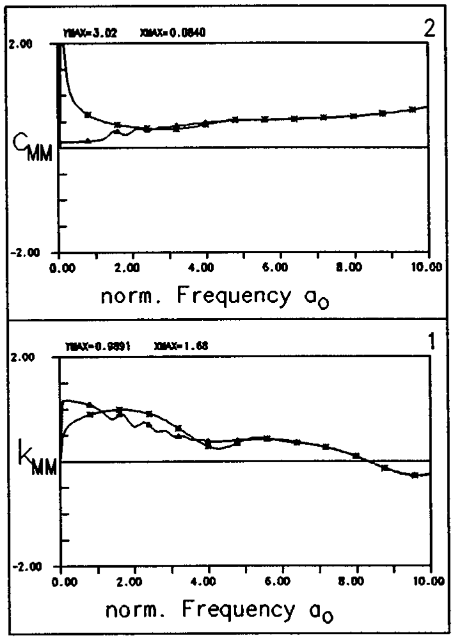

Fig.2
Impedance Functions of a 30 m and a 100 m Deep Soil Model.

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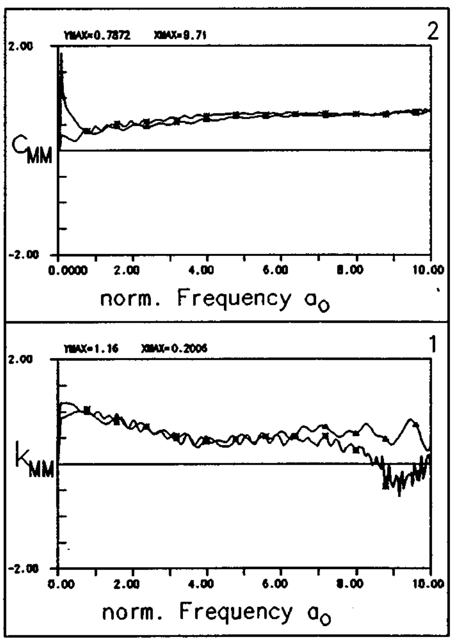

Fig.3
Impedance Functions Effected by Different Element Sizes:

System 3020SYB: 10.0m x 10.0m
System 6020Y8 : 2.5m x 2.5m

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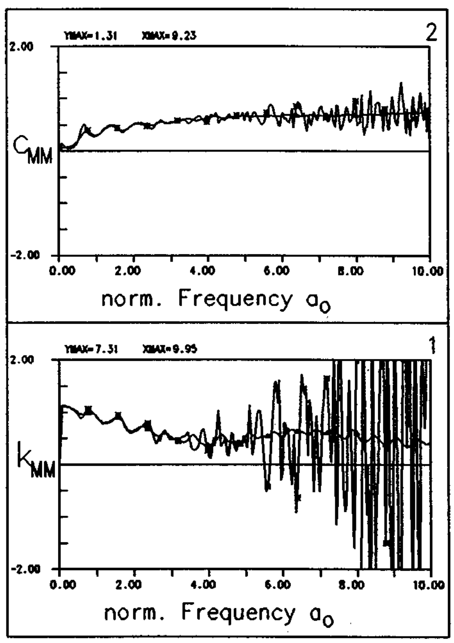

Fig.4
Impedance Functions Effected by Different Boundary Conditions.

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3 Influence of the Element Size on the Frequency Behaviour

Referring to a given soil section, the element size may change the frequency behaviour, s.Fig.3 for the upper frequency range.

4 Influence of Different Boundary Conditions

Fig.4 points out the results of two different boundary conditions for the same system:

- The strongly ruffled line is the result of a system which has a rigid base but viscous dampers on both sides;
- the lightly ruffled line belongs to a system with viscous dampers at the base and on both sides.

These dampers on both sides and at the base nearly exactly simulate the energy-absorbing boundary conditions of the halfspace in spite of being frequency independent. Each figure of this paper is related to viscous dampers at the lower and the lateral boundaries, except Fig.4. The influence of other boundary-conditions can be seen in /1/.

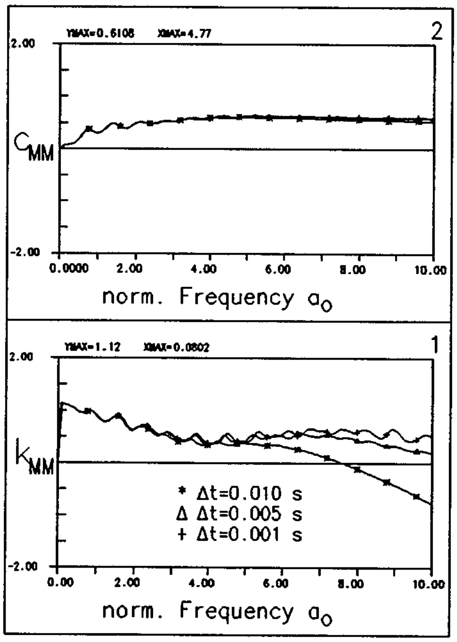

Fig.5
Impedance Functions of a Soil Model Effected by Different Time Increments Δt, WILSON operator.

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0.01-0.005-0.001

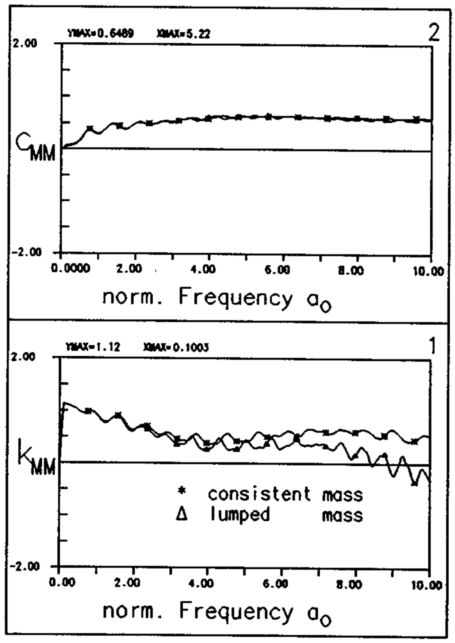

Fig.6
Impedance Functions of a Soil Model Effected by Different Mass Configurations.

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5 Frequency Behaviour of the Integration Operator

Fig.5 shows the impedance function of a given FEM model computed with the Wilson operator; the parameter is the time increment Δt. The line for Δt = 0.001 s - marked by (+) - fairly corresponds to the line of the more exact Newmark operator for Δt = 0.01 s, which is not shown here.

6 Consistent Mass and Lumped Mass

Fig.6 demonstrates the different impedance functions of a soil model with lumped mass or consistent mass. Referring to the same system, the results differ in the upper frequency range.

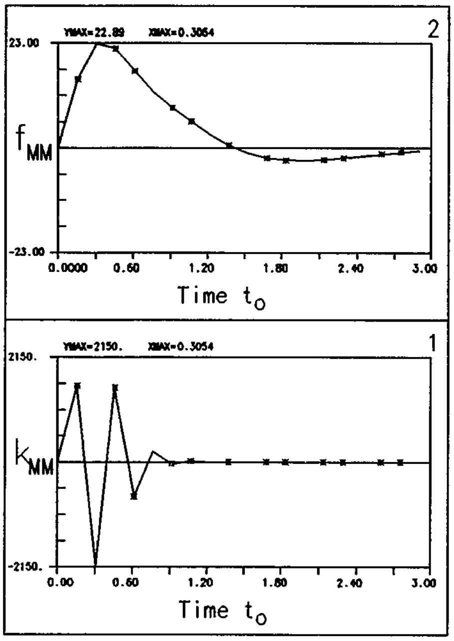

Fig.7
Time-Normalized Impedance and Compliance Function in Time Domain.

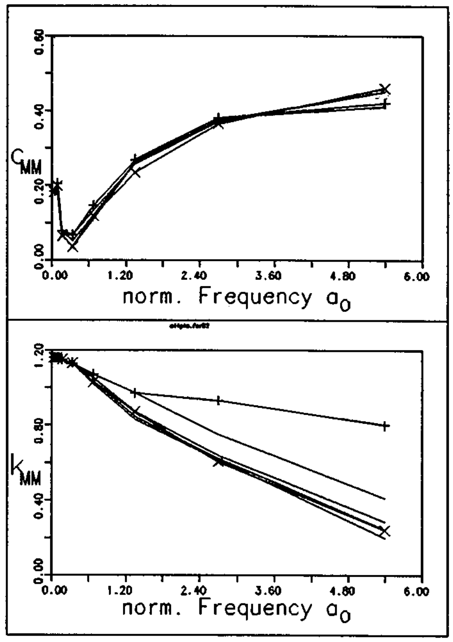

Fig.8
Impedance Functions of Mono-Frequency Excitation without Damping by Evaluating Different Time Ranges.

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7 Evaluation of Impedance by means of Compliance

The numerical computed impedance functions are verified in the time domain by applying the corresponding compliance functions to the convolution operator. Investigated is the coincidence for the deformation computed by direct integration of the differential equation and by the convolution operator /1/. Fig.7 points out a typical impedance and compliance function in the time domain.

8 Impedances of Mono-Frequency Excitation without Damping

In relation to the evaluation of different time ranges of the displacement history, the impedance functions due to monofrequency excitation may show the influence of the starting from rest , i.e. the influence of the homogeneous solution of the differential equation, see Fig.8. Especially the real part k_{MM} of the impedance demonstrates this influence when evaluating only the first period of the excitation time history, see the line marked by "+". On the other side the lower line - marked by "x" - shows the impedance due to harmonic excitation which follows from the evaluation of the second period of excitation time, i.e. the starting from rest has no influence anymore. The other lines represent different evaluations of time ranges for starting from rest.

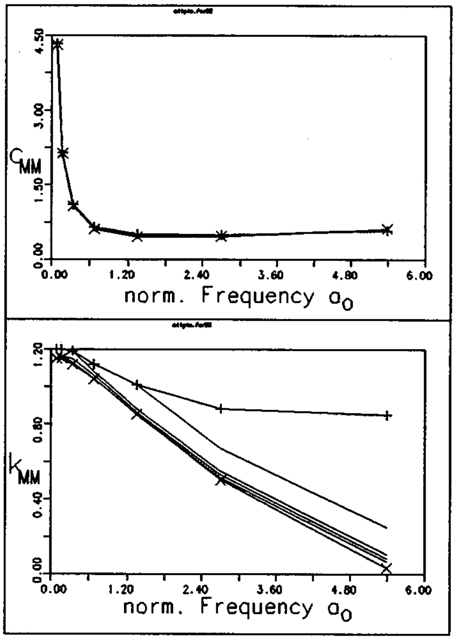

Fig.9
Impedance Functions of Mono-Frequency Excitation with Hysteretic Damping by Evaluating Different Time Ranges.

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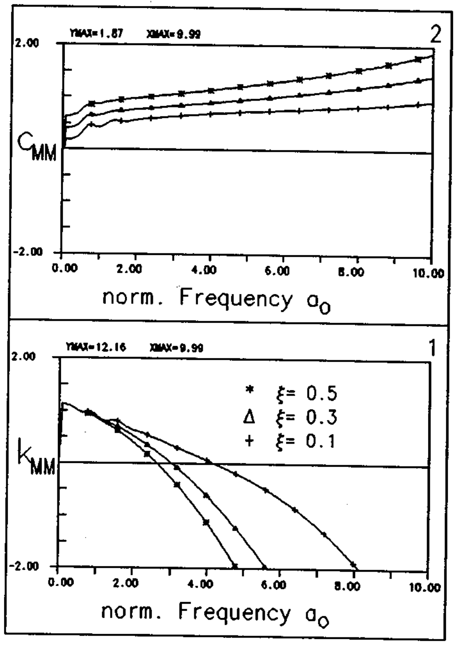

Fig.10
Impedance Functions of a Soil Model Effected by Different Viscous Damping Factors.

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9 Impedances of Mono-Frequency Excitation with Damping

Referring to the mono-frequency excitation without damping, the corresponding impedance lines in Fig.9 point out the influence of the starting from rest with damping included. The hysteretic damping in Fig.9 is related to tan $\Phi = \xi \cdot a_0 = 0.3$, /1/.

10 Conclusion

With the assumption of viscous dampers on both sides and at the base, it is obvious that suitable FEM models are able to simulate the frequency dependent behaviour of the halfspace in a wide frequency range.

The investigations show consequently that
- the displacement formulation of the elements (number of Gauss points) and
- the lumped and consistent mass representation and
- the choice of the integration operator including the

time increment used and the topography of the element model may dominate the impedance functions more than the very limited deviation according to frequency independent viscous dampers simulating the boundary conditions of the halfspace which yields to a certain overall oscillation.

As a result of the reflections at the boundaries, the rocking impedance functions especially point out only a small difference to the halfspace solution in form of a certain ruffled line. Certain systematic divergences between the halfspace solution and the FEM model solution are the result of the element behaviour exclusively.

Using the Rayleigh definition for viscous damping, the ruffling of the lines for both the real and the imaginary part of impedance is reduced, Fig.10. Simulating the halfspace by frequency independent means, the best and optimal boundary condition is a boundary without any bearing but with viscous dampers on both sides and at the base. Because of the singularity of the stiffness matrix no static solution exists; nevertheless dynamic problems may be solved by effective stiffness matrix.

Using integration in time domain, the impedance functions may illustrate the influence of the starting from rest. With respect to differences to the literature, short time loadings such as impact loads yield to different impedance functions.
Based upon impedances of the elastic and viscoelastic halfspace and upon numerical impedances of FEM models, the comparison of these two impedances shows that the solution of the FEM model using viscous dampers is equivalent to the halfspace solution. As a result, the advantage of time domain solutions especially the capability solving nonlinear problems keeps completely usable. The assumption is that the chosen structure has an equivalent coupling to the soil system.

The presented method for the computation of impedance functions is the intersection between analytic and discrete procedures. It can be used as a tool for the investigation of impedance functions derived from any topological site condition such as layers, embedments, depth depending soil properties and load depending properties for shear moduli and for damping factors. Impedances and compliances could be presented in a collection of different requirements, whereby frequency dependent compliances may be applied in the time domain with the convolution operator.

REFERENCES

/1/ Matthees,W.,Magiera,G.
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BAM-Forschungsbericht 189, 1992

/2/ ADINA, A Finite Element Program for Automatic Dynamic Incremental Nonlinear Analysis

/3/ ASHSD2, Dynamic Stress Analysis if Axisymmetric Structures Under Arbitrary Loading