

## EARTHQUAKE RESPONSE OF SYSTEM WITH ASYMMETRIC CHARACTERISTIC

S. Aoki

Tokyo Metropolitan Technical College, 1-10-40 Higashi-Ohi, Shinagawa-ku 140, Japan

### ABSTRACT

When structural systems are subjected to excess seismic loading, hysteretic restoring force-deformation relation is seen. In this paper, earthquake response of system with asymmetric restoring force-deformation relation is examined by time history analysis using actual earthquake records. First, effect of asymmetric characteristic on the maximum response is examined. Second, the maximum response of system with hysteretic characteristic is compared with that of linear system. Finally, energy absorbed by hysteresis loop is obtained.

### 1. INTRODUCTION

When systems are subjected to excess seismic loading, plastic deformation is seen. It is an important problem to analyze response characteristics of such systems in order to obtain response considering plastic deformation and to study process of system destruction. For analysis considering plastic deformation, many models are proposed for representing hysteretic restoring force-deformation relations (Ang 1998). Many studies are carried out using these models (Zaiming, Katukura and Izumi 1991). On the other hand, reduction of earthquake response by energy absorption is studied (Lee 1979). Application of this study to development of reliability is examined (Aoki 1991).

In many studies, symmetric hysteresis loop models are used. However, when hysteresis loop is asymmetric, response characteristics are different from those of symmetric case (Aoki 1993). In this paper, effect of asymmetric characteristic on earthquake response is examined by analysis using actual earthquake records. First, effect on the maximum response is examined. Next, response of nonlinear system is compared with that of linear system. Finally, relation between absorbed energy by hysteresis loop and earthquake response is examined.

### 2. ANALYTICAL MODEL AND INPUT EXCITATIONS

As an analytical model, single-degree-of-freedom system shown in Fig.1 is used for simplicity. Equation of motion with respect to displacement of mass relative to input point  $z(=x-y)$  is expressed as follows.

$$\ddot{z} + 2\zeta\omega_n \dot{z} + f = -\ddot{y} \quad (1)$$

Where  $\zeta$  is damping ratio,  $\omega_n$  is natural circular frequency and  $f$  is restoring force. Restoring force-deformation relation is represented by perfectly-elasto-plastic model shown in Fig.2. In this model, it is assumed that yielding forces are asymmetric as shown in Fig.2. One yielding force  $f_1$  is

determined by using the maximum restoring force of linear system  $|R|_{max}$  as follows.

$$f_1 = \alpha |R|_{max} \tag{2}$$

Where  $\alpha$  is a parameter which represents yielding effect.  $|R|_{max}$  is obtained by the maximum displacement response of linear system  $|z_1|_{max}$  as follows.

$$|R|_{max} = \omega_n^2 |z_1|_{max} \tag{3}$$

The other yielding force  $f_2$  is determined by using  $f_1$  as follows.

$$f_2 = \beta f_1 \tag{4}$$

Where  $\beta$  is a parameter which represents asymmetric effect.

Absorbed energy by plastic deformation is also obtained, which is expected to be great effect on response characteristics. As shown in Fig.3, sum of plastic deformation on positive side  $Z_+$  and that on negative side  $Z_-$  are obtained. Then, absorbed energy  $E_n$  is obtained by the following equation.

$$E_n = f_1 Z_+ + f_2 Z_- \tag{5}$$

Value of  $\zeta$  is selected as 0.01. Values of natural period are selected from 0.05s to 5.0s.  $\alpha=0.7$  is selected for the case where yielding effect is relatively small and 0.3 for relatively great case.  $\beta=0.8$  is selected for the case where asymmetric effect is relatively small and 0.5 for relatively great case.  $\beta=1.0$  which means symmetric case is adopted for comparison.

As input excitations, El Centro(1940) NS component and Hachinohe(1968) EW component are used.

### 3. MAXIMUM RESPONSE

Effect of hysteretic restoring force-deformation relation on the response spectrum which represents the maximum response is examined. The maximum response of absolute acceleration  $S_A(|\ddot{x}|_{max})$ , that of relative velocity of mass to input point  $S_V(|\dot{z}|_{max})$  and relative displacement  $S_D(|z|_{max})$  are obtained.

Acceleration, velocity and displacement response spectrum for  $\alpha=0.7$  using El Centro NS as input are shown in Fig.4(a),(b) and (c), respectively. In these figures, results for linear system which does not have hysteretic characteristic are shown for comparison. Acceleration response of system with hysteretic characteristic is smaller than that of linear system. Response decreases as asymmetric effect becomes great. For velocity response, when system has hysteretic characteristic, response is smaller than that of linear system except for short period range. Response decreases as asymmetric effect becomes great. For displacement response, when hysteretic characteristic is symmetric, response is smaller than that of linear system near peak response. On the other hand, response increases as asymmetric effect becomes great.

Same characteristics can be seen for Hachinohe EW input. When  $\alpha=0.3$ ,

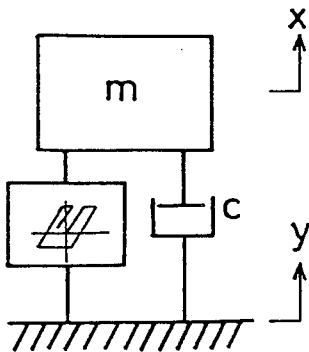


Fig.1 Analytical model used in this study

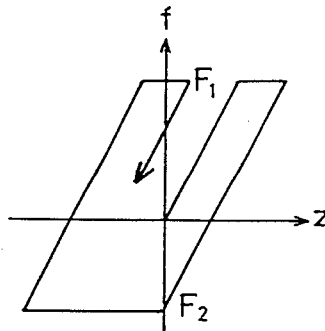


Fig.2 Perfectly-elasto-plastic restoring force-deformation relation

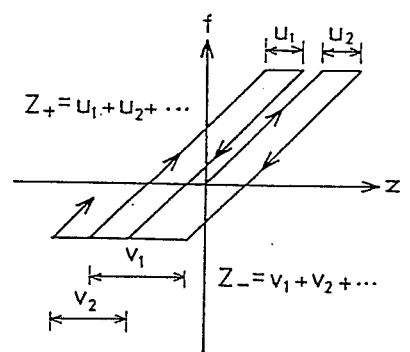


Fig.3 Sum of plastic deformation

although reduction rate is different, almost same characteristics are obtained.

4. RESPONSE REDUCTION FACTOR

In order to examine results obtained in Chapter 3 in detail, response reduction rate is obtained. The response reduction factor for acceleration  $\gamma_A$ , velocity  $\gamma_V$  and displacement  $\gamma_D$  defined as following equations are obtained.

$$\begin{cases} \ddot{x}|_{max} = (1-\gamma_A) |\ddot{x}_l|_{max} \\ \dot{z}|_{max} = (1-\gamma_V) |\dot{z}_l|_{max} \\ z|_{max} = (1-\gamma_D) |z_l|_{max} \end{cases} \quad (6)$$

where  $|\ddot{x}_l|_{max}$ ,  $|\dot{z}_l|_{max}$  and  $|z_l|_{max}$  are the maximum response of linear system for acceleration, velocity and displacement, respectively.

In Table1, response reduction factors for  $\alpha=0.7$  using El Centro NS as input

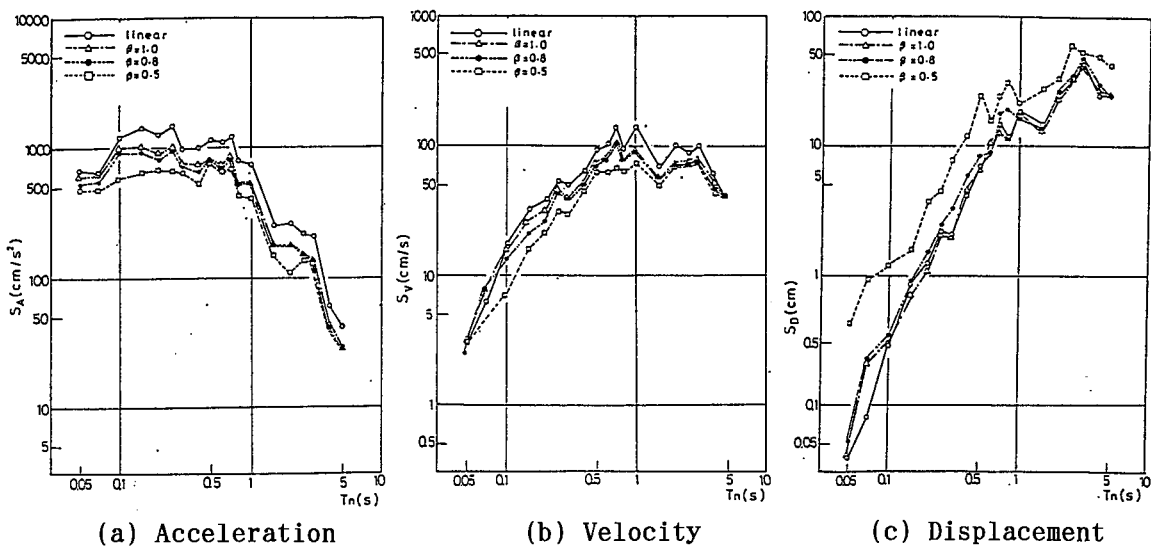


Fig.4 Response spectrum(El Centro NS,  $\zeta=0.01$ ,  $\alpha=0.7$ )

Table 1 Response reduction factor (El Centro NS,  $\zeta=0.01$ ,  $\alpha=0.7$ )

$T_n$ (s)	Acceleration			Velocity			Displacement		
	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$
0.05	0.05	0.12	0.21	0.0	0.21	0.09	-0.04	-0.35	-9.52
0.07	0.01	0.14	0.27	-0.21	-0.18	0.26	-1.49	-0.19	-10.70
0.10	0.13	0.22	-0.40	0.08	0.19	0.47	-0.09	-0.19	-1.25
0.15	0.22	0.32	0.50	0.21	0.34	0.41	-0.11	-0.23	-0.98
0.20	0.18	0.27	0.43	0.17	0.31	0.40	0.07	-0.29	-2.11
0.25	0.24	0.33	0.51	0.10	0.15	0.34	0.08	-0.17	-1.01
0.30	0.21	0.27	0.35	0.21	0.21	0.32	0.06	-0.48	-2.36
0.40	0.25	0.33	0.48	0.18	0.17	0.31	-0.13	-0.49	-1.82
0.50	0.25	0.28	0.28	0.19	0.19	0.21	0.06	-0.21	-2.28
0.60	0.28	0.29	0.28	0.19	0.17	0.31	-0.08	0.02	-0.58
0.70	0.26	0.29	0.43	0.21	0.23	0.42	0.22	-0.13	-0.47
0.80	0.26	0.29	0.40	0.23	0.21	0.28	0.07	-0.61	-1.49
1.00	0.28	0.29	0.44	0.28	0.28	0.43	0.0	0.10	-0.13
1.50	0.28	0.28	0.41	0.13	0.18	0.25	0.15	0.06	-0.88
2.00	0.28	0.30	0.55	0.20	0.29	0.46	0.19	-0.02	-1.63
2.50	0.27	0.29	0.38	0.03	0.31	0.07	0.04	-0.02	-0.71
3.00	0.29	0.30	0.35	0.10	0.09	0.14	0.09	-0.03	-0.14
4.00	0.28	0.29	0.29	0.10	0.20	0.20	-0.05	-0.18	-0.92
5.00	0.27	0.28	0.28	0.0	0.0	0.0	0.04	0.04	-0.60

are shown. In this table, when  $\beta=1.0$ ,  $\gamma_v$  is almost constant independent of natural period of 0.25s-2.0s. In the region where natural period is shorter or greater than this region, reduction rate is small. Thus, in this paper, natural period of 0.05s-0.2s is referred to short period range, that of 0.25s-2.0s middle period range and that of 2.5s-5.0s long period range. In Table 2, expected values of response reduction factor for each range are shown. In middle period range, acceleration and velocity response decrease and displacement response increases as asymmetric effect becomes great. This characteristics can be seen in other ranges. In Table 3, response reduction factors for  $\alpha=0.3$  are shown. Acceleration and velocity response decrease as  $\alpha$  decreases. When hysteretic characteristic is symmetric, that is,  $\beta=1.0$  in middle period range, displacement response decreases as  $\alpha$  decreases. When hysteretic characteristic is asymmetric, displacement response increases as  $\alpha$  decreases. In Table 4 and Table 5, response reduction factors using Hachinohe EW as input are shown. Displacement response is reduced in long period range since predominant period of Hachinohe is longer than that of El Centro. However, same characteristics as Table 2 and Table 3 can be seen.

From above shown results, acceleration and velocity response decrease as yielding force decreases and asymmetric effect becomes great. When hysteretic characteristic is symmetric, displacement response decreases as yielding force decreases near predominant period. On the other hand, when hysteretic characteristic is asymmetric, displacement response increases as yielding force decreases and asymmetric effect becomes great.

Table 2 Response reduction factor (El Centro NS,  $\zeta=0.01$ ,  $\alpha=0.7$ )

$T_n$ (s)	Acceleration			Velocity			Displacement		
	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$
0.05-0.20	0.12	0.21	0.20	0.05	0.17	0.33	-0.33	-0.25	-4.91
0.25-2.00	0.26	0.30	0.41	0.19	0.21	0.33	0.06	-0.19	-1.27
2.50-5.00	0.28	0.29	0.33	0.06	0.15	0.10	0.03	-0.05	-0.59
0.05-5.00	0.22	0.27	0.31	0.10	0.18	0.25	-0.08	-0.16	-2.26

Table 3 Response reduction factor (El Centro NS,  $\zeta=0.01$ ,  $\alpha=0.3$ )

$T_n$ (s)	Acceleration			Velocity			Displacement		
	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$
0.05-0.20	0.41	0.47	0.08	-1.24	-0.60	-13.89	-8.55	-14.37	-775.14
0.25-2.00	0.65	0.67	0.67	0.37	0.50	0.52	0.22	-1.28	-6.97
2.50-5.00	0.68	0.68	0.69	0.15	0.21	0.42	-0.02	-0.17	-1.80
0.05-5.00	0.58	0.61	0.48	-0.24	0.04	-4.32	-2.78	-5.27	-261.30

Table 4 Response reduction factor (Hachinohe EW,  $\zeta=0.01$ ,  $\alpha=0.7$ )

$T_n$ (s)	Acceleration			Velocity			Displacement		
	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$
0.05-0.20	0.05	0.12	0.12	-1.29	-4.62	-12.59	-8.19	-30.52	-239.75
0.25-2.00	0.25	0.29	0.36	0.12	0.19	0.30	-0.04	-0.42	-2.38
2.50-5.00	0.28	0.30	0.39	0.10	0.11	0.17	0.05	-0.11	-1.07
0.05-5.00	0.19	0.24	0.29	-0.36	-1.44	-4.04	-2.73	-10.35	-81.07

Table 5 Response reduction factor (Hachinohe EW,  $\zeta=0.01$ ,  $\alpha=0.3$ )

$T_n$ (s)	Acceleration			Velocity			Displacement		
	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$	$\beta=1.0$	$\beta=0.8$	$\beta=0.5$
0.05-0.20	0.25	0.24	0.32	-14.80	-17.72	-22.32	-82.75	-209.42	-671.24
0.25-2.00	0.65	0.66	0.67	0.41	0.40	0.22	-0.65	-2.29	-10.02
2.50-5.00	0.68	0.68	0.68	0.27	0.28	0.35	0.28	-0.13	-1.29
0.05-5.00	0.53	0.53	0.56	-4.71	-5.68	-7.25	-27.71	-70.61	-227.52

## 5. ABSORBED ENERGY BY HYSTERESIS LOOP

In Fig.5(a) and (b), absorbed energy for  $\alpha=0.7$  and  $\alpha=0.3$ , respectively are shown using El Centro NS as input. Energy increases as asymmetric effect becomes great. Energy increases as  $\alpha$  decreases. From comparison to results obtained in Chapter 4, acceleration and velocity response decrease as energy increases. When hysteretic characteristic is symmetric, displacement response decreases as energy increases near predominant period. For other natural periods, increase of energy is not always related to reduction of displacement response. When hysteretic characteristic is asymmetric, displacement response increases as energy increases.

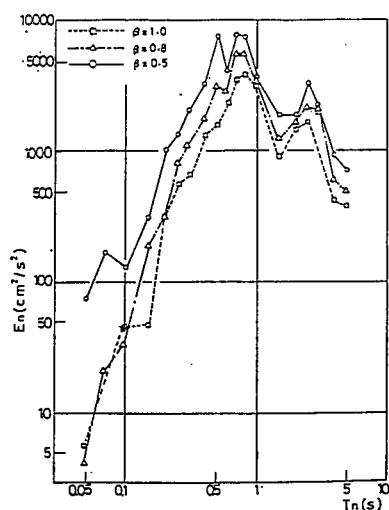
In order to examine characteristics of displacement response from view point of permanent deformation, sum of plastic deformation on positive side  $Z_+$  and that on negative side  $Z_-$  are shown in Table 6. For constant value of  $\alpha$ ,  $Z_+$  is almost constant and  $Z_-$  increases as  $\beta$  decreases.  $Z_+$  and  $Z_-$  increases as  $\alpha$  decreases. Therefore, displacement response is significantly related to difference between  $Z_+$  and  $Z_-$ , that is, permanent deformation.

Comparing Fig.5 with Fig.4, shape of velocity response spectrum is similar to that of absorbed energy. Thus, absorbed energy is related to velocity response spectrum. It may possible to evaluate absorbed energy using velocity response spectrum.

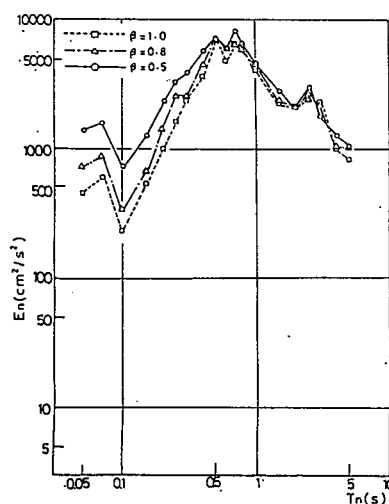
## 6. CONCLUSIONS

Effect of asymmetric characteristic of restoring force-deformation relation on earthquake response is examined. Obtained results are summarized as follows.

- 1) Acceleration and velocity response decrease as yielding force decreases.
- 2) When hysteretic characteristic is symmetric, displacement response decreases as yielding force decreases near predominant period. In other period range, response reduction effect is small. When hysteretic characteristic is asymmetric, displacement response increases as yielding force decreases.
- 3) Absorbed energy by hysteresis loop increases as yielding force decreases and asymmetric effect becomes great. Shape of graph for absorbed energy is similar to that for velocity response spectrum.
- 4) As damping ratio increases, absorbed energy decreases and reduction rate to response of linear system decreases.



(a)  $\alpha=0.7$



(b)  $\alpha=0.3$

Fig.5 Absorbed energy by hysteresis loop(El Centro NS,  $\zeta=0.01$ )

Table 6 Sum of Plastic Deformation (cm) (El Centro NS,  $\zeta=0.01$ )

T <sub>n</sub> (s)	$\alpha=0.7$				$\alpha=0.3$			
	$\beta=0.8$		$\beta=0.5$		$\beta=0.8$		$\beta=0.5$	
	Z <sub>+</sub>	Z <sub>-</sub>	Z <sub>+</sub>	Z <sub>-</sub>	Z <sub>+</sub>	Z <sub>-</sub>	Z <sub>+</sub>	Z <sub>-</sub>
0.05	0.0	-0.013	0.0	-0.334	1.44	-2.94	1.66	-10.9
0.07	0.0	-0.057	0.0	-0.751	1.66	-3.46	1.80	-11.8
0.10	0.0	-0.051	0.0	-0.318	0.064	-1.17	0.029	-4.03
0.15	0.0	-0.252	0.0	-0.666	0.175	-1.84	0.162	-5.95
0.20	0.0	-0.484	0.0	-2.45	0.934	-3.72	0.273	-12.6
0.25	0.0	-1.05	0.0	-2.72	0.470	-7.28	0.112	-15.9
0.30	0.202	-1.81	0.0	-6.03	2.27	-7.44	2.71	-20.4
0.40	0.0	-3.24	0.0	-9.41	4.23	-13.8	2.82	-32.7
0.50	1.11	-5.18	0.385	-19.4	6.47	-20.0	2.09	-40.2
0.60	2.91	-1.29	0.157	-11.5	4.28	-18.8	0.611	-36.7
0.70	0.037	-7.76	0.0	-15.8	4.62	-15.9	0.413	-43.4
0.80	0.641	-12.6	0.0	-25.1	6.51	-24.7	2.30	-49.7
1.00	1.06	-6.96	0.0	-13.7	7.23	-16.5	1.44	-38.5
1.50	1.65	-6.94	0.0	-21.7	5.81	-32.4	6.25	-62.0
2.00	0.175	-12.1	0.0	-21.1	4.58	-30.1	5.00	-42.5
2.50	1.12	-16.4	0.0	-45.9	13.2	-40.2	8.13	-76.2
3.00	0.0	-20.5	0.0	-34.6	11.5	-33.0	0.714	-60.5
4.00	0.861	-16.3	0.861	-40.1	33.1	-29.3	27.4	-78.6
5.00	6.50	-13.4	6.50	-38.1	44.3	-49.5	40.1	-98.2

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