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IMPROVED GENERATION OF PSD FUNCTIONS, ARTIFICIAL ACCELEROGRAMS AND SPECTRA, FULLY COMPATIBLE WITH A DESIGN RESPONSE SPECTRUM

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ABSTRACT

The paper presents preliminary results of a new technique for improved generation of PSD functions, artificial accelerograms and spectra, free of the current ZPA and damping inconsistencies and artificial artefacts. It is demonstrated by a numerical experiment using the USNRC RG 1.60 design spectrum that the new inversion equation that has been developed is able to solve the ZPA paradox. This also explains that the mathematical iteration algorithm already in use during more than 20 years is deficient, and not the NRC RG 1.60 spectrum itself.

A. INTRODUCTION

A design response spectrum such as e.g. the NRC RG 1.60 ground acceleration spectra (Ref. [1]), or floor or in-structure design response spectra are the basic input for seismic analysis or qualification testing of equipment. This regulatory and engineering type of input is only sufficient for crude response spectrum (RS) analysis. Additional information are needed such as:

- an artificial accelerogram matching this design RS for time history integration (THI) analysis of systems or qualification testing of equipment.
- Power Spectral Density (PSD) functions for stationary (RV) or non-stationary (TRV) random vibration theory analysis, or analysis in the frequency domain (Refs. [2], [3], [4]).
- Advanced response spectrum techniques which need other spectra than the design RS, such as the relative velocity and relative acceleration RS (Ref. [5]) and other PSD derived parameters (Ref. [6]).

The determination of the PSDF compatible with the design RS is usually the basic operation to generate such informations, but there are two fundamental paradoxes that remain unsolved since more than 20 years, they are (Ref. [2]):
- the ZPA inconsistency
- the damping inconsistency.

A research program is underway (Ref. [7]) which has theoretical ambitions to solve the above paradoxes, and the practical ambitions to simplify the current algorithms, and to avoid the introduction of artificial artefacts to cure the paradoxes.

The present paper reports only the very encouraging results that have been obtained, at the date of writing the paper, about solving the first paradox, being the ZPA inconsistency, which has always been explained in the past by many authorities in the field (Refs. [2], [4], [8] to [12]) as being a deficiency in the design RS input instead of a deficiency in the algorithm to derive the PSD function.

B. THEORETICAL BASIS FOR IMPROVED PSD FUNCTION GENERATION

1. Simplified Problem Statement

All three problems: the generation of PSD functions, artificial accelerograms, and other spectra have as a starting point the stationary RV theory relationship between the design RS S_{pAn}, (ω_n, ξ_n) and the one sided (non-evolutionary) PSD $G(\omega)$ given by Eq. (1):

$$S^2_{pAn}\ (\omega_n, \xi_n) = \eta_n^2\ \omega_n^4 \cdot \int_0^\infty |H_n|^2\ G(\omega)\ d\omega \tag{1}$$

in which:

$$|H_n|^2 = \left[(\omega_n^2 - \omega^2)^2 + (2\xi_n\omega_n\omega)^2\right]^{-1} \tag{2}$$

η_n = the response peak to rms ratio (being a function of ω_n, ξ_n, an equivalent duration, the probability of non-exeedance, and implicidly of $G(\omega)$).

2. Current Solution

Up till now no analytical solution exists to solve exactly the mathematical inverse problem i.e. to find $G(\omega)$ as an integral equation function of S^2_{pAn}.

The mathematical techniques that have been used so far (Refs. [8] to [13]) rely on an iteration scheme, with rare exceptions such as the collocation method used in Ref. [14] (the latter explicitly does not report any improvements).

Although the iterative scheme developed in Ref. [13] is of a more sophisticated nature, the algorithm in use basically consists of the following equation:

$$G(\omega)(J+1) = G(\omega)(J)\left[\frac{S^T_{pAn}(\omega, \xi_n)}{S_{pAn}(\omega, \xi_n)(J)}\right]^2 \tag{3}$$

in which:

S^T_{pAn} = the target design RS.

$S_{pAn}\ (J)$ = the RS obtained from Eq. (1) using the result of $G(\omega)\ (J)$ of the J'th iteration.

Usually as a first guess G(ω) (o), the white noise assumption G(ω) (o) = g_o is used for which we have the relation:

$$S^2_{pAn}(\omega, \xi_n)\ (o) = \frac{\pi}{4}\ g_o\ \frac{\omega}{\xi_n} \tag{4}$$

and from which G(ω) (1) can be computed from Eq. (3), and so forth until G(ω) (J+1) is nearly equal to G(ω) (J) at all frequencies, usually after 10 iterations or more.

3. The New Method

Using an approximate form of Eq. (1), and using matrix inversion, the principal author has found an inversion integral equation for Eq. (1) (Ref. [7]). If Eq. (1) is symbolized in a matrix format operation:

$$\left\{\left(S^T_{pA}\right)^2\right\} = [M_{pA}]\ \{G(\omega)\} \tag{5}$$

in which $[M_{pA}]$ is the operator on G(ω) to obtain the target design spectrum S^T_{pAn}, the inverse integral equation can be symbolized as:

$$\{G(\omega)\} = \left[M_{pA}\right]^{-1}\left\{\left(S^T_{pA}\right)^2\right\} \tag{6}$$

Since the analytical format of (6) that was obtained is an integral equation in function of S^T_{pAn}, **no iterations are involved** in the operation to find G (ω) corresponding to the target $S^T_{pA}(\omega_n, \xi_n)$, and therefore no convergence problems nor physical limitations could exist on the idealized mathematical formulation of the inverse problem (the only physical limitation is that G (ω) should remain positive or equal to zero).

C. THEORETICAL BASIS FOR OTHER SPECTRA GENERATION

1. Current Solution

The current solution to find **the relative velocity spectrum** $S_{vn}(\omega_n, \xi_n)$ corresponding to the design RS $S^T_{pAn}(\omega_n, \xi_n)$, consists of using the stationary RV theory relationship:

$$\omega_n^2 S^2_{vn}(\omega_n, \xi_n) = \eta_n'^2 .\ \omega_n^2 \int_0^\infty \omega^2 |H_n|^2 G(\omega) d\omega \tag{7}$$

Assuming with good accuracy that $\eta'_n \cong \eta_n$, and from the knowledge of G(ω) as determined according to B-2) above, it is therefore straight forward to obtain $S^2_{vn}(\omega_n, \xi_n)$ from Eq. (7).

The **relative acceleration spectrum** $S_{an}(\omega_n, \xi_n)$ could also be obtained from the stationary RV theory from:

$$S_{an}^2(\omega_n, \xi_n) = \eta_n''^2 \int_0^\infty \omega^4 |H_n|^2 G(\omega)\, d\omega \tag{8}$$

If we assume the same constant peak factor $\eta''_n = \eta'_n = \eta_n = \eta$ for the response as well as for the excitation, the following stationary RV theory relationship holds between S_{an} on one hand, and S_{vn}, S_{pAn} and the peak acceleration A_o on the other hand:

$$S_{an}^2 = 2\omega_n^2(1 - 2\xi_n^2)S_{vn}^2 - S_{pAn}^2 + A_o^2 \tag{9}$$

2. The New Method

Using a similar method as indicated in B–3), we can symbolise for the approximate Eq. (7):

$$\{\omega^2 S_v^2\} = [M_v]\{G(\omega)\} \tag{10}$$

hence with Eq. (6), it is possible to find a direct **integral equation relationship** between **the relative velocity response spectrum** S_{vn} and the design S_{pAn}, which can therefore be symbolized as:

$$\{\omega^2 S_v^2\} = [M_v]\left[M_{pA}\right]^{-1}\left\{\left(S_{pA}^T\right)^2\right\} \tag{11}$$

The equation does not need iterations, nor does it need to determine first the PSD to generate the relative velocity spectrum.

The **relative acceleration spectrum** can then be obtained from Eq. (9).

D. PRESENT NUMERICAL RESULTS

To test the new method for generating a PSDF compatible with a target design RS at all frequencies, the following numerical experiment was conducted on the USNRC R.G. 1.60 horizontal ground design spectrum of Ref. [1] shown in Figure 1 for a damping ξ_n of 5% by the full line ——.

- Assuming a constant $\eta_n = \eta = 3.0$ model in the computer program to generate $G(\omega)$ according to Ref. [13], the corresponding response spectrum to this $G(\omega)$ was recalculated using Eq. (1).

It is shown in Figure 1 as the dot dot (.....) line.

As expected we find the ZPA paradox inconsistency reported also in Refs. [11] to [13].

Using the new inverse integral equation (6) another $G'(\omega)$ was generated (which remains positive and very close to $G(\omega)$), and the corresponding spectrum was also recalculated using Eq. (1).

It is shown in Figure 1 as the dash-dash line (——-).

It is seen that this spectrum nearly coincides with the target NRC spectrum **without** the ZPA inconsistency paradox.

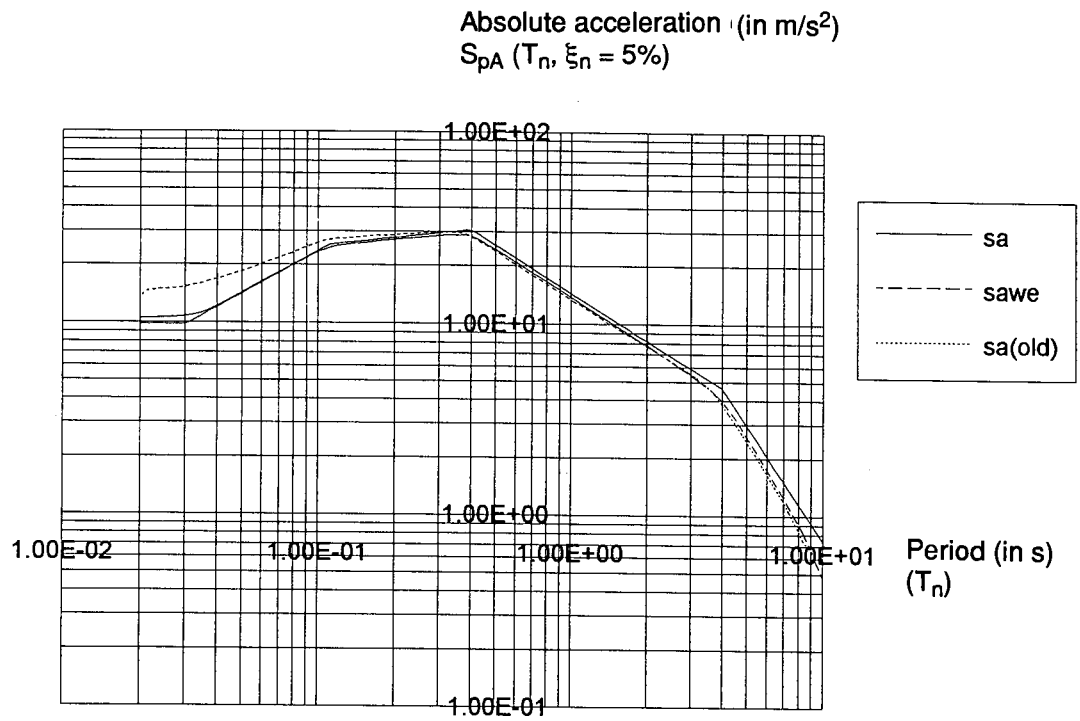

Figure 1: **Comparison of the old and new method spectra with the NRC R.G. 1.60 (5% damping) spectrum, showing resolution of the ZPA paradox.**

E. SUMMARY AND CONCLUSIONS

A project partially funded by the Commission of European Communities DG XII is underway for improved generation of PSD functions, artificial accelerograms and spectra, fully compatible with a design RS (Ref. [7]).

A new inversion integral equation to find the PSD from the design RS has been developed which is already able to solve the ZPA paradox. This paradox is therefore not due to the deficiency in the design RS or a peak factor formula but is due to the deficiency of the iteration algorithm that have been used the last 20 years in nearly all PSDF and artificial accelerogram generating programs. This was demonstrated in a numerical experiment on the USNRC R.G. 1.60 design spectrum and using a constant peak factor.

Also a new integral equation was found which permits to generate very efficiently the relative velocity spectrum directly in function of the design RS without a need to derive the PSD as an intermediate step.

The next phases of the research program consist of applying the new PSDF-RS inversion equation to:

- Hopefully Solve the damping inconsistency paradox using a **new** non-stationary TRV model.
- Improve the generation of artificial accelerograms fully compatible with the design RS at all frequencies and dampings without using artificial artefacts such as peak correction or clipping and adding corrective time histories.

REFERENCES

[1] USAEC Regulatory Guide 1.60 – Rev. 1 (1973) "Design Response Spectra for Seismic Design of Nuclear Power Plants".

[2] Preumont A. "Application of the Random Vibration Approach in Seismic Analysis of LMFBR Structures" – CEC report EUR 11369-EN, Nuclear Science and Technology (1988).

[3] Preumont A, Shihab S., Cornaggia L., Reale M., Labbé P., Noé H. "Application of the Random Vibration Approach in the Seismic Analysis of LMFBR Structures" – Benchmark Calculations – CEC report EUR 14153-EN, Nuclear Science and Technology (1992).

[4] Shihab S., Preumont A. "Non-Stationary Random Vibration in Seismic Analysis" – CEC report RAI 0135-B (1991).

[5] Mertens P.G. "Generalised Algebraic Modal Response Spectrum Combination Using the Modified Relative Acceleration Spectrum" – Paper K13/1 – SMIRT 12 – (1993).

[6] Mertens P.G. "A New Generalised Algebraic Multiple Support Combination Rule" – Paper K18/6 – SMIRT 12 – (1993).

[7] Mertens P.G., Preumont A. "Improved Generation of Evolutionary PSD Functions, Artificial Accelerograms and Spectra, fully compatible with a Design Response Spectrum" – CEC Contract (1992) (presently unpublished research).

[8] Vanmarcke E., Gasparini D. "Simulated Earthquake Ground Motions" – SMIRT 4 – San Francisco (August 1977) (Paper K1/9).

[9] Sundararajan C. "An Iterative Method for the Generation of Seismic Power Spectral Density Functions" – Nuclear Engineering and Design 61 (1980) – pp 13-23.

[10] Unruh J., Kana D.D. "An Iterative Procedure for the Generation of Consistent Power/Response Spectrum" – Nuclear Engineering and Design 66 (1981) 427-435.

[11] Christian J.T. "Generating Seismic Design Power Spectral Density Functions" – Earthquake Spectra, Vol. 5, N° 2 (1989).

[12] Kaul M.K. "Stochastic Characterization of Earthquakes Through Their Response Spectrum" – Earthquake Engineering and Structural Dynamics, Vol. 6, 497-509 (1978).

[13] Preumont A. "On the Connection Between the Response Spectrum and Spectral Properties of Earthquake Accelerograms" Fourth International Conference on Application of Statistics in Soil and Structural Engineering – University of Firenze (Italy) 1983, Pitagore Editrice.

[14] Sundararajan C., Gupta G.D. "An Integral Equation Approach for the Generation of Seismic Power Spectral Density Functions" – Transactions ASME, Vol. 102 (August 1980), pp 264-270.