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IDENTIFICATION OF DYNAMIC CHARACTERISTICS OF JOINTS BETWEEN STRUCTURAL AND MECHANICAL ELEMENTS

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ABSTRACT

The paper presents an identification technique in frequency domain based on the Fourier series,FFT and harmonic balance for to determine the unknown dynamic characteristics of the elastic nonlinear joints encountered in the nuclear installations.An example concerning a pump transmission is given in order to illustrate the procedure.Using this method of high accuracy we can identify also more elaborate models for systems with many degrees of freedom.

1 INTRODUCTION

The using of joins in mechanical and structural elements is very important in the nuclear industry as well as for the equipment that for the isolation of structures.Any dynamic analysis by FEM or other method requires to know the parameter characteristics (inertial,stiffness and damping parameters).
The joint mechanical parameters are nonlinear and dependent upon different conditions,such as preload,surface mating position,external load temperature an other conditions,so that characteristics determined in a test environment normally can not be applied in other enviroment.Conventional methods for to determine these parameters proposed in [1] , [2] , [3] consist of isolating the joint from the system and testing it with static load or sinusoidal excitation.However,often these joints can not be separated from the system in which they are incorporated.
Furthermore,if some joints could be separated, it is difficult to install them into a test structure without any additional joint. Recently J.Wang,P.Sas [4] proposed an identification method based on the experimentally determination of parametrical characteristics of joints in systems,a multi degrees of freedom system being converted into several single degree of freedom systems by imposing selected eigenvectors on it.Such a single degrees of freedom system is recognized as an alternative form of the Rayleigh's quotient and is represented by both physical parameters of the original system and model parameters of the selected mode.
The paper uses an identification method in frequency domain based on the expressing the nonlinear terms in Fourier series and

applying the principle of harmonic balance. The dynamical data are measured for a periodic force applied to the system and for periodic steady-state responses induced by it. Also, our procedure can be easily implemented on microcomputers. As an example we consider a block system encountered in the pump transmission consisting from two mechanical mass elements connected by an unknown elastic nonlinear joint.

2 IDENTIFICATION METHOD OF NONLINEAR SYSTEMS BASED ON HARMONIC BALANCE

We consider the following vibrating system having nonlinearity in spring characteristics

$$m\ddot{x} + c\dot{x} + kx + g(x) = F \tag{1}$$

In the first time we assume that the nonlinearity g(x) is approximated by a polynomial of x of the form

$$g(x) = ax^2 + bx^3 + \ldots \tag{2}$$

where a,b,... are unknown coefficients. Thus, the identification is reduced to the determination of parameters m,c,k in eq.(1) and coefficients a,b...in eq.(2). We suppose an harmonic external force F and we induce a steady-state response x with the same period as that of the external force. Then, both the external force F and the response x are measured in the time interval of a period, and they are developed in Fourier series as follows

$$\begin{aligned} x &= X_0 + X_1\cos\omega t + X_2\cos 2\omega t + X_1^*\sin\omega t + X_2^*\sin 2\omega t + \ldots \\ F &= F_0 + F_1\cos\omega t + F_2\cos 2\omega t + F_1^*\sin\omega t + F_2^*\sin 2\omega t + \ldots \end{aligned} \tag{3}$$

where $T = 2\pi/\omega$ is the period. Expressing x and F as above can be done easily by using, for example, a FFT algorithm, and thus the Fourier coefficients $X_0, X_1, \ldots, F_0, F_1, \ldots$ appearing in the above expressions are known quantities. Similarly, after having been determined by the operation of x, the terms $x^2, x^3 \ldots$ appearing in eq.2 are developed in Fourier series as follows

$$\begin{aligned} x^2 &= X_0^2 + X_1^2\cos\omega t + X_2^2\cos 2\omega t + (X_1^2)^*\sin\omega t + (X_2^2)^*\sin 2\omega t + \ldots \\ x^3 &= X_0^3 + X_1^3\cos\omega t + X_2^3\cos 2\omega t + (X_1^3)^*\sin\omega t + (X_2^3)^*\sin 2\omega t + \ldots \end{aligned} \tag{4}$$

where the Fourier coefficients $X_0^2, X_1^2, \ldots, X_0^3, X_1^3, \ldots$ are also known quantities .

Now ,we substitute eq.(3) and (4) into eq.(1) and apply the principle of harmonic balance and we obtain the following equation

$$[A]\cdot\{S\} = \{F\} \tag{5}$$

where

$$[A] = \begin{bmatrix} 0 & 0 & -X_0 & X_0^2 & X_0^3 \\ -\omega^2 X_1 & \omega X_1^* & X_1 & X_1^2 & X_1^3 \\ -\omega^2 X_1^* & -\omega X_1 & X_1^* & (X_1^2)^* & (X_1^3)^* \\ -4\omega^2 X_2 & 2\omega X_2^* & X_2 & X_2^2 & X_2^3 \\ -4\omega^2 X_2^* & -2\omega X_2 & X_2^* & (X_2^2)^* & (X_2^3)^* \end{bmatrix} \quad (6)$$

$$\{S\} = \{ m \; c \; k \; a \; b \; \ldots \}$$

$$\{F\} = \{F_0 \; F_1 \; F_1^* \; F_2 \; F_2^* \ldots\}$$

If various values are given to ω properly, and if some appropriate terms are retained in the Fourier series in equation (3) and (4), the number of equations contained in eq.(5) exceeds that of unknown cuantities. Then, eq.(5) are solved using the least square method to obtain

$$\{S\} = \left([A]^T [A]\right)^{-1} [A]^T \{F\} \quad (7)$$

Some times, a weighting matrix can be used to improve accuracy. When the weighting matrix $[W]$ is used, $\{S\}$ is given, instead of eq.(6) by

$$\{S\} = \left([A]^T [W] \; [A]\right)^{-1} [A]^T [W] \; \{F\}$$

If neither the qualitative nor quantitative character of the restoring force is known, it will be appropiate to express them in a piecewise manner. Hence the region of the displacement is divided into small subdomains at points $x_{-n}, x_{-n+1}, \ldots, 0, x_1, \ldots, x_{n'}$, and the values of g in these points $g_{-n}, g_{-n+1}, \ldots, g_0, g_1 \ldots, g_{n'}$, are suposed to be unknown. Here (-n) and (n') are integers which are determined depending on how many subdomains are considered to express g. An appropiate interpolating function $\phi_i(x)$, for which $\phi_i(x_i) = 1$ and $\phi_i(x_j) = 0 (i \neq j)$, is introduced. Then g can be approximated as follows, similarly to FEM

$$g = \sum_{i=-n}^{n'} \phi_i(x) g_i \quad (8)$$

Substituting the data of x from (3) into (8) we obtain the variation of g in a period in which unknown coefficients g_i appear linearly. Then, g is developed in a Fourier series with the unknown coefficients g_i again appearing linearly , of the form

$$g = \sum_{i=-n}^{n'} g_i \left\{ (g_i)_0 + (g_i)_1 \cos\omega t + (g_i)_2 \cos 2\omega t + \ldots + (g_i)_1^* \sin\omega t + (g_i)_2^* \sin 2\omega t + \ldots \right\} \quad (9)$$

where the Fourier coefficients $(g_i)_0, (g_i)_1 \ldots$ are known quantities. As the final step, eq.(3) and (9) are substituted into eq.(1)

and the principle of harmonic balance is used. We obtain for the vector S the expression

$$\{S\} = \{m \ c \ g_{-n} \cdots g_0 \cdots g_n\}$$

We can generalize the procedure and consider the viscous damping force expended in the form

$$f = \sum_{i=-n}^{n'} \phi_i(\dot{x}) f_i$$

3 IDENTIFICATION OF A NONLINEAR JOINT

We consider the mechanical system with two degrees of freedom encountered in the pump transmission presented in Fig.1 coupled by an elastic nonlinear joint

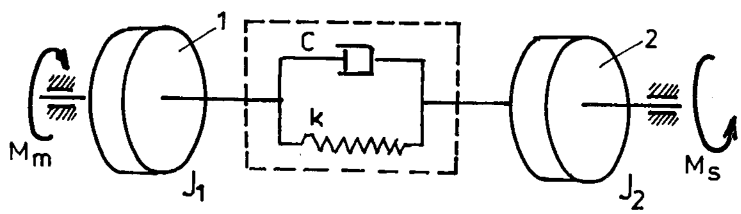

Fig.1 Flywheels coupling with a nonlinear joint

If φ_1 and φ_2 are the rotations of two flywheels, the relative rotation is $\varphi = \varphi_1 - \varphi_2$ and respectively $M_m(\varphi)$ the instantaneous variable torsion moment through joint.
The motion differential equation of the dynamical system has the form

$$\ddot{\varphi} + \frac{M_v(\varphi)}{J_e} = \frac{M_o(t)}{J_e} \tag{10}$$

or in the particular case

$$\ddot{\varphi} + \frac{c}{J_e}\dot{\varphi} + \frac{k}{J_e}\varphi + \frac{\gamma}{J_e}\varphi^3 = M\sin(\omega t + \theta) \tag{11}$$

where $J_e = J_1 J_2/(J_1+J_2)$ is the equivalent moment of inertia.
The phase portrait φ, $\dot{\varphi}$ and the variation of the angle φ with respect to the time t is illustrated in the Fig.2.
Using the identification method in the frequency domain previously, for the numerical values, the joint being made by rubber.
m_1=100 kg; r_1= 0.2 m; J_1=2kgm^2; m_2=44.44 kg; r_2=0.3 m; J_2=2kgm^2; c=100 Ns/m; k=2000 N/m; M=200 kgm^2; γ=200 N/m^3
Eq.(9) becomes

$$\ddot{\varphi} + 100\dot{\varphi} + 2000\varphi + 200\varphi^3 = 200\cos t \tag{12}$$

$(\omega = 1 s^{-1})$

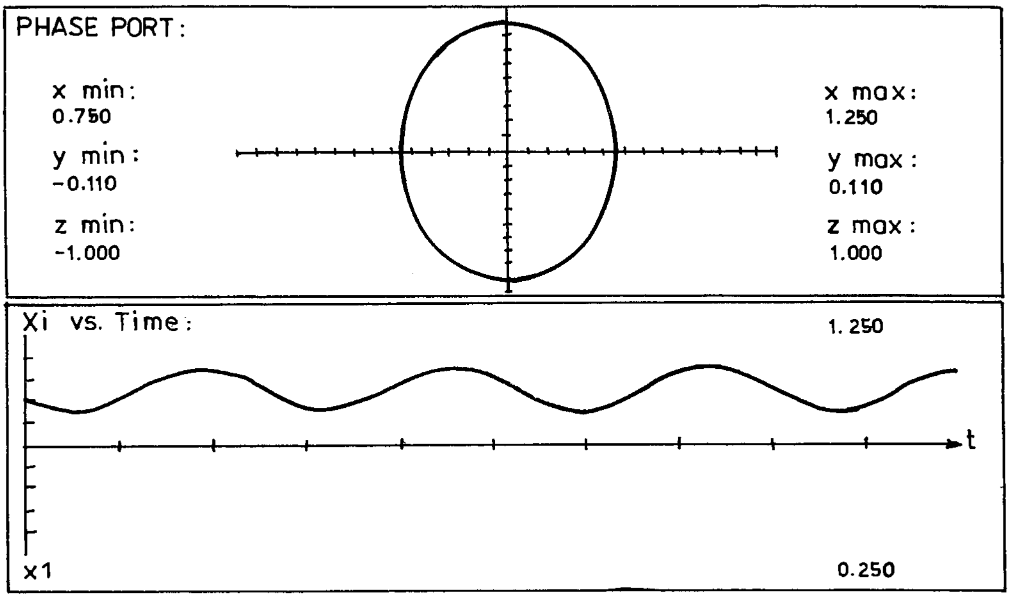

Fig.2 Phase portrait and angle φ variation

We introduce a new variable α by transformation $\varphi = \alpha - 1$ and we obtain the new differential equation

$$\ddot{\alpha} + 100\dot{\alpha} + 2600\alpha - 600\alpha^2 + 200\alpha^3 = 2200 + 200\cos t$$

Following the identification method it results

$$\{S\} = \{\ 1.148887957 \quad 100.0641277 \quad 2.59980183\ 10^3 \quad -599.70047474 \quad 199.80868402\ \}^T$$

Now, for the coefficients of eq.(10)using the inverse transformation $\alpha = 1 + \varphi$, we obtain

$$c/J_e = 100.0641277 \qquad k/J_e = 1999{,}826933 \qquad \gamma/J_e = 199.80868402$$

$$M = M_0/J_e = 199.8986447$$

4 CONCLUSIONS

As is result from the numerical example concerning the determination of the coefficients in nonlinear differential motion equation the identification method in frequency domain presents a very good accuracy for the practics and offers possibilities to be extended in the case of nonlinear dynamical systems with many degees of freedom.

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