STUDY ON SIMPLIFIED ESTIMATION OF J-INTEGRAL UNDER THERMAL LOADING

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1. Introduction

For assessing structural integrity or safety of nuclear power plants, strength of structures under the presence of flaws sometimes needs to be evaluated. Because relative large inelastic deformation is anticipated in the liquid metal reactor components even without flaws due to high operating temperature and large temperature gradients, inelastic effects should be properly taken into account in the flaw assessment procedures.

It is widely recognized that J-integral and its variations - e.g. fatigue J-integral range and creep J-integral - play substantial roles in the flaw assessment under the presence of large inelastic deformation. Therefore their utilization has been promoted in the recent flaw assessment procedure both for low and high temperature plants. However, it is not very practical to conduct a detailed numerical computation for cracked structures to estimate the values of these parameters for the purpose of trailing crack growth history. Thus development of simplified estimation methods which do not require full numerical calculation for cracked structures is desirable.

A method using normalized J-integral solutions tabulated in the handbook [1] is a direct extension of linear fracture mechanics counterpart and it can be used for standard specimen and simple structural configurations subjected to specified loading type. The reference stress method [2] has also been developed but in this case limit load solutions, which are often difficult to obtain for general stress distribution, are necessary instead of nonlinear J-integral solutions. However, both methods have been developed mainly for mechanical loading and thus applying these techniques to thermal stress problem is rather difficult except the cases where the thermal stress can be properly substituted by equivalent mechanical loading as in the case of simple thermal expansion loading. Therefore alternative approach should be pursued for estimating J-integral and their variations in thermal stress problems.

On the other hand, a method utilizing the stress intensity factor (SIF) solutions for crack surface loading, called influence function method, has been developed for linear elastic problem. This method is very versatile in the sense that it can be applied for estimating the SIF for arbitrary loading using the stress distribution for an identical but uncracked body, which is much easier to analyze. Therefore utilization of this method in the nonlinear thermal stress problem should be promising if succeeded. Sakon and Kaneko [3] showed that purely elastic solutions give reasonably good estimates of J-integral even for elastic-plastic cracked bodies subjected to deformation controlled loading. On the other hand, Budden [4] studied procedures for estimating J-integral based on stress/strain distribution obtained by elastic-plastic analysis for uncracked bodies. To assess the validity of these approaches as well as their variations in a broader range of conditions, detailed finite element analysis for cracked and uncracked cylinders was conducted in this study.
2. Computational Model and Result of Uncracked Body Analysis

Objects of analysis are cylinders with an fully circumferential constant-depth crack shown in Figure 1. Three depths of crack were considered, varying its ratio to the cylinder wall thickness as 0.25, 0.5 and 0.75. For each size of the crack, three types of temperature distribution were applied. They were linear through-wall, nonlinear through-wall and axial temperature distributions, illustrated in Figure 1. 20 increments were spent to reach the final temperature distribution with equal temperature increments. Magnitudes of temperature changes were determined so that proper amount of plastic deformation takes place at the final stage of calculation.

For simplicity, temperature dependency of the material parameters was not taken into account. Bilinear stress-strain model was used with the isotropic hardening hypothesis. The following values were assumed for mechanical and thermal properties of the material.

Young's modulus $E=153.86$ GPa
Poisson's ratio $\nu=0.3$
Thermal expansion coefficient $\alpha=2\times10^{-5}$/°C
Yield Stress $\sigma_y=196$ MPa
Plastic hardening modulus $H=d\sigma/de_p=15.386$ GPa=$E/10$

General-purpose finite element program, MARC (ver. K4), was used in this study. An example of finite element subdivision with 8-noded axisymmetric isotropic elements for three crack depths is shown in Figure 2. Because of temperature gradient along the crack direction, conventional J-integral loses path-independency. Therefore extended J-integral was calculated by virtual crack extension approach, using a Parameter Card, LORENZI. Thermal strains are treated correctly for maintaining path-dependency in this approach.

Radial distributions of stress component normal to the crack plane at the final steps of uncracked body analyses are shown in Figure 3 in the case of linear through-wall temperature distribution as an example. Maximum elastic stresses generated at the inner surface are about 2.3, 3.1 and 2.0 in each temperature distribution. From the distributions of stress by elastic-plastic analysis, it can be seen that 25-30 per cent of the thickness is yielded.

3. Simplified Estimation Methods for J-integral

Totally six simplified approaches were studied in this study. All of them employed the influence function method to estimate values of the stress intensity factor from uncracked body solutions. Influence functions developed by Buchalet and Bamford [5] based on their finite element calculations were utilized.

In the first three methods, uncracked body stress obtained by purely elastic analysis was utilized, while the latter three used the results by elastic-plastic uncracked body analysis. These methods are refereed to E1, E2, E3, EP1, EP2 and EP3 method, respectively thereafter. Each method can be described as follows:

(1) Methods based on elastic uncracked body analysis

**Method E1** [1]---- Estimate the SIF, $K$, from stress distribution obtained by purely elastic analysis for uncracked body (neglecting plasticity) and convert it to the J-integral by a linear elastic relationship as

$$J_{E1} = \frac{(1-\nu^2)}{E} K(a)^2$$

(1)

where $a$ denotes the crack depth.
Method E2 ---- Apply the Irwin's crack length correction and re-estimate the stress intensity factor for effective crack depth, \(a_{\text{eff}}\), and convert it to the J-integral as

\[
J_{E2} = \frac{(1-v^2)}{E} K(a_{\text{eff}})^2
\]

\[
a_{\text{eff}} = a + \frac{1}{6\pi} \frac{(K_a)}{\sigma_y}
\]

(2)

It should be mentioned that \(a_{\text{eff}}\) was set to a when \(a_{\text{eff}}\) calculated by eq.(2) exceeded the cylinder wall thickness, following the suggestion in [4].

Method E3 ---- Multiply \(J_{E1}\) by the ratio of the effective crack length to the original crack length as

\[
J_{E3} = \frac{a_{\text{eff}}}{a} \frac{(1-v^2)}{E} K(a)^2
\]

(3)

It should be noted that re-evaluation of SIF for corrected crack depth is not made in this case but that the result coincides with that of eq.(2) in a special case of the crack in an infinite solid.

(2) Methods based on elastic-plastic uncracked body analysis

Method EP1[2] ---- Uses stress and strain distributions obtained by elastic-plastic analysis for uncracked body and estimate J-integral as

\[
J_{EP1} = \frac{(1-v^2)}{E} K_\sigma(a)K_\varepsilon(a)
\]

(4)

where \(K_\sigma\) and \(K_\varepsilon\) are SIFs obtained from stress and strain distributions in the uncracked body, respectively. For the latter, 'fictitious' stress component normal to crack plane is estimated by

\[
\sigma_z = \frac{E}{(1+v)(1-2v)} \left\{ (1-v)\varepsilon_z + v(\varepsilon_\theta + \varepsilon_r) \right\}
\]

(5)

Method EP2[2] ---- As in method E2, apply the Irwin's crack length correction to both \(K_\sigma\) and \(K_\varepsilon\) and re-evaluate them by using the effective crack lengths, \(a_{\text{eff}}\) and \(a_{\text{eff}}\), respectively, as

\[
J_{EP2} = \frac{(1-v^2)}{E} K_\sigma(a_{\text{eff}})K_\varepsilon(a_{\text{eff}})
\]

\[
a_{\text{eff}} = a + \frac{1}{6\pi} \frac{(K_\sigma)}{\sigma_y}, \quad a_{\text{eff}} = a + \frac{1}{6\pi} \frac{(K_\varepsilon)}{\sigma_y}
\]

(5)

Method EP3 ---- Analogously to method E3, multiply \(J_{EP1}\) by the square-root of the ratios of the effective crack lengths to the original crack length as

\[
J_{EP3} = \frac{(1-v^2)}{E} \sqrt{\frac{a_{\text{eff}}}{a}} K_\sigma(a)\sqrt{\frac{a_{\text{eff}}}{a}} K_\varepsilon(a)
\]

(7)

It should be noted that re-evaluation of SIFs for corrected crack depth is not made in this case but that the result coincides with that of eq.(6) in a special case of the crack in an infinite solid.

4. Result of Cracked Body Analysis

Path-independency of J-integral by LORENZI option was accurately maintained except the values from the innermost path, which suffered numerical inaccuracy due to crack-tip singularity. Averages
of the values from all paths except the innermost one were used as the reference values for evaluating the simplified estimations.

Figure 4 shows the comparison of simplified estimations of J-integral normalized by the reference values for all calculated cases. In the purely elastic condition, simplified estimations are in a good agreement with each other as well as with the reference values, as expected. This demonstrates the effectiveness of the influence function method used in this study. In the elastic-plastic state, especially after the maximum elastic stress generated in uncracked body exceeding the yield stress, J-integral values estimated by the six simplified estimation methods show large difference. The following observations can be made in reference to Figure 4.

1. Method E1 gave generally reasonable estimates for elastic-plastic J-integral in spite of its simplicity. This tends to give conservative prediction of J-integral (larger than the reference values) in many cases but makes somewhat unconservative prediction for the shallow crack with axial temperature gradient.

2. Method E2 made conservative (maybe excessively) prediction for the shallow crack cases but unconservative for the medium and deep crack cases. This must be related to the stress distribution: crack length correction makes the J-integral values smaller than those without correction in the latter cases where the crack tip is located in the neutral axis or compressive side.

3. Method E3 predicted the largest values of J-integral in most cases. It should be noted that this method gets rid of the above-mentioned problem of E2 and always gave conservative estimations even though seemingly too much in some cases.

4. Method EP1 tends to give unconservative prediction in most cases, although the ratio to the reference values did not go bellow 0.5.

5. Method EP2 showed a similar trend as Method E2, suffering the same problem.

6. Method EP3 gave most conservative values among the three methods based on elastic-plastic uncracked body stress analysis. Moreover their ratio to the reference values is rather stable without respect to crack depth and temperature distribution (between 0.87 and 1.35).

5. Recommendation

The above observations might allow one to make the following recommendations on the use of simplified estimation of J-integral for thermally loaded structures:

1. If only the result of purely elastic analysis for an uncracked structure is available, it is suggested to use Method E1 for the most accurate estimation and Method E3 for conservative estimation, depending on the situation one faces.

2. If the result of elastic-plastic analysis for an uncracked structure is available, it is recommended to use Method EP3 for accurate and conservative estimation of J-integral.

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(References)


**Figure 1** Object of Analysis

**Figure 2** Example of Finite Element Modeling

**Figure 3** Example of Stress Distributions in Uncracked Cylinder (Case of Linear Through-Wall Temperature)
Figure 4 Comparison of Simplified versus Detailed Estimations of J-integral