SMiRT-12 / K. Kussmaul (Editor)

L02/2

A VISCOELASTOPLASTIC STUDY OF THIN SHELLS SUBJECTED TO A THERMAL CYCLIC LOADING

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1) INTRODUCTION.

In this paper we will study all kind of structures considered as an assembly of elementary mechanisms: springs, sliders, dash-pots. The structure is supposed to be subjected to a mechanical initial load (primary load), and a thermal cyclic load (secondary load) which implies a non-linear behaviour of parts of the stucture. The secondary load can also results from cyclic given displacements.

In a first step, we will remind the classical elastoplastic study of a structure composed of two thin cylinders made of a plastic kinematic hardening material (fig 1). This kind of structure has already been studied by several authors (Zarka [1], Jiang and Leckie [2]) and it is well known that one can obtain various modes of behaviour depending on the values of the couple composed of the primary (P) and secondary (S) loads. This behaviour can be purely elastic, elastic shakedown, plastic shakedown or ratchetting. For the two-cylinder example of figure 1, one can obtain the interaction diagram of figure 2.

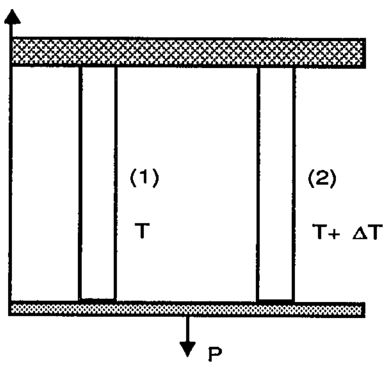


fig 1

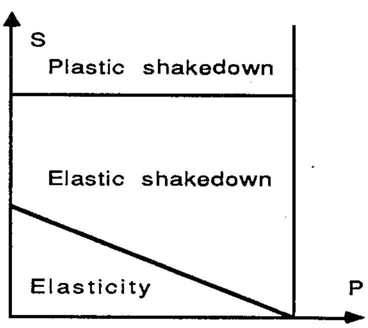


fig 2

In this paper we will extend the field of application of the standard elastoplastic theory and include viscous effects such as viscoelasticity and creep. We have therefore constructed an incremental approach to take into account these effects and also a time dependance of the constitutive law. The Zarka's method has been used because this approach is efficient to give directly the final state of the structure with a few elastic calculations. Unfortunately this method suppose that the mechanisms induring plasticity are well known. Errors can appear when this assumption is not satisfied. We have therefore developped a new chronological method instead of the classical step-by-step solution in order to compute exactly the plastification order.

2) PROBLEM FORMULATION.

We will use Débordes and Nayroles's notations [3].
Let us denote by E the linear space of strains and by I the subspace of E of compatible strains. If ε^0 is any given strain, the strain equation can be written as:

$$\varepsilon \in \varepsilon^0 + I$$

In this paper ε^0 will be identified to thermal strains or linked to given displacements.

If Σ is the linear space of stresses, J the subspace of Σ of self-equilibrating stresses and σ^* any particular solution of the equilibrium equation for the external load P, the equilibrium equation is

$$\sigma \in \sigma^* + J$$

The constitutive law is represented by an operator c from E into Σ; c is linear and self-adjoint for elastic problems and non-linear in case of viscoelacticity, plasticity and creep. Any mechanical problem can be summarized by:

Problem 1: given a strain ε^0 and a load P, find the displacement v, the strain ε and the stress σ solution of:

$$\varepsilon \in \varepsilon^0 + I \qquad \sigma \in \sigma^* + J \qquad \sigma \in c(I)$$

2.1) Elastoplastic problems.

In case of elasticity Problem 1 becomes

(1) $$\varepsilon^e \in \varepsilon^0 + I \qquad \sigma^e \in \sigma^* + J \qquad \sigma^e = k\,\varepsilon^e$$

where the elastic stiffness k replace c and the symbol ^{e} denotes elasticity.
For plastic problems we have:

(2) $$\varepsilon^t \in \varepsilon^0 + I \qquad \sigma \in \sigma^* + J$$

$$\sigma = k\,\varepsilon^e \qquad \varepsilon^t = \varepsilon^e + \varepsilon^p \qquad \varepsilon^p \in \partial\Psi_C(\sigma)$$

where the total strain ε^t is the sum of elastic ε^e and plastic ε^p strains and the plastic strain ε^p satisfies the plastic flow rule $\varepsilon^p \in \partial\Psi_C(\sigma)$.

Let us denote the non elastic strain ε^{ne} and the self-equilibrating stress ρ such as:

$$\varepsilon^{ne} = \varepsilon^t - \varepsilon^e \qquad \rho = \sigma - \sigma^e$$

The difference between elastic and plastic problems with the choice $\sigma^* = \sigma^e$ gives the Zarka's formulation:

Problem 2: on the assumptions of Problem 1, find v, ε^{ne} and ρ solutions of

(3) $$\varepsilon^{ne} = \varepsilon^p + k^{-1}\rho \in I \qquad \rho \in J \qquad \varepsilon^p \in \partial\Psi_C(\sigma)$$

The two first equations of (3) represent a classical elastic problem where the plastic strain ε^p is assumed to be a given strain. Suppose ε^p known; we have therefore:

(4) $$\rho = a\,\varepsilon^p$$

where a is a stiffness operator obtained from the elastic parameters of the mechanisms of the whole structure.
The third equation of (3) will be used to find the plastic strain. According to Zarka's method, let us introduce a new hardening parameter

(5) $$y = b\,\varepsilon^p - \rho$$

where b represents initial hardening parameters. With these new notations the plastic flow rule can be written as

(6) $$\varepsilon^{p'} \in \partial\Psi_{C'}(\sigma^e + \rho)$$

where C' is a new yield surface whose center is the elastic solution σ^e.

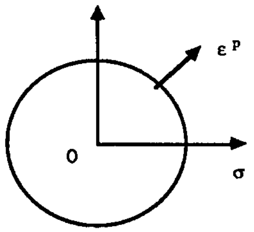


fig 3

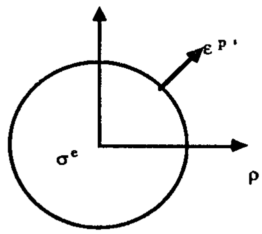


fig 4

Equations (4) and (5) give (with $t = b - a$ and $\varepsilon^p = x$)

(7) $$y = t\,x$$

The computation algorithm is therefore

- computation of σ^e
- moving of the yield surface C'
- computation of y and x
- computation of $\rho = a\,x$
- computation of $\sigma = \sigma^e + \rho$
- computation of $\varepsilon^{ne} = x + k^{-1}\rho$
- computation of $\varepsilon^t = \varepsilon^{ne} + \varepsilon^0$

2.2) Viscoelastic problems.

The incremental viscoelastic constitutive law is given by:

$$d\sigma = k_v\, d\varepsilon^v$$

where k_v is an operator including linear and viscous effects. We have now to replace the elastic problem (1) by an incremental viscoelastic problem and Problem 2 becomes:

(8) $$d\varepsilon^{ne} = d\varepsilon^p + k_v^{-1}\, d\sigma \in I \qquad d\rho = d\sigma - d\sigma^v \in J \qquad d\varepsilon^p \in \partial\Psi_C(\sigma)$$

The two first equations describe now an incremental elastic problem whose solution can be written as:

$$d\rho = a_v\, d\varepsilon^p$$

where a_v is a "stiffness" operator including the viscous effects. Finally, as well as in the plastic problem we obtain:

(9) $$dy = t_v\, dx$$

where $t_v = b - a_v$, and the new algorithm is nearly similar to the one described in § 21.

2.3) Creep problems.

During this step we suppose that no plasticity occurs and that temperature is constant. The incremental steady-state creep constitutive law is given by:

$$d\varepsilon^f = A\sigma^n\, dt$$

where A,n are given constants and t is time.The elastic problem (1) is always the same and Problem 2 becomes:

(10) $$d\varepsilon^{ne} = d\varepsilon^f + k^{-1}\, d\rho \in I \qquad d\rho \in J$$

which is an incremental elastic problem that we can easily solve because the creep strain is known. Its solution can be written as:

$$d\rho = a_f\, d\varepsilon^f$$

and finally we obtain once again:

(11) $$dy = t_f\, dx$$

where $t_f = b - a_f$, and the algorithm is always similar to the one described in § 21.

3) COMPUTATIONAL PROCESS.

In each of the preceeding cases we have to solve a linear set of equations such as

(12) $$Y = TX$$

where vectors Y and X are put intead of the unknowns y or dy and x or dx and T is the matrix which represents operator t or t_v either t_f (eq. 7 or 9 either 11). The plasticity conditions give an element of X or the value of the projection of an element of Y onto the yield surface C'. The ways to solve this problem are detailed below.

3.1) Step-by-step approach.

At the begining of a step, the locations of the yield surface and the Y "stress point" is known. Computation of the linear problem (3) or (8) either (10) gives the new position of the yield surface C'.

If Y still belongs to the moving yield surface C', there is no plastification and X is known. If Y is outside C', plasticity occured during the step and the new Y location will be calculated by projection onto C'. If more than one mechanisms plastifies, the stress point Y is projected as in figure 5 and this projection process may lead to anticipate the plastification state of the whole structure. In fact the directions of projection during this step can be like those shown in figure 6 which implies existence of a "risky area" where the projection process can differ from the real solution. This area is shown in grey on figure 7.

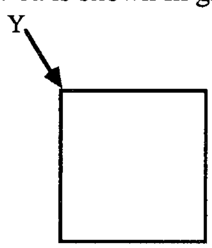


fig 5

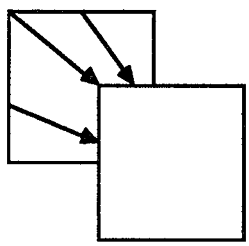


fig 6

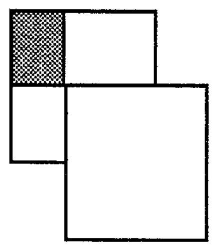
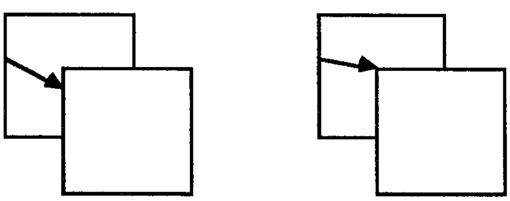


fig 7

We can increase the number of iterations to limitate this risky area or use the method detailed below.

3.2) Chronological approach.

The aim of this method is to compute exactly the plastification order of the various mechanisms and to take it into account during the projection process. Thus, for each step, we will calculate the plastification priorities. One can prove that if the Y point is located as in fig 9, it will be pulled in the 1 direction as ratio a/A is bigger than b/B.



exact theory

step-by-step method

fig 8

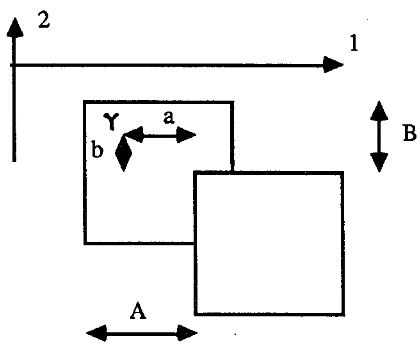


fig 9

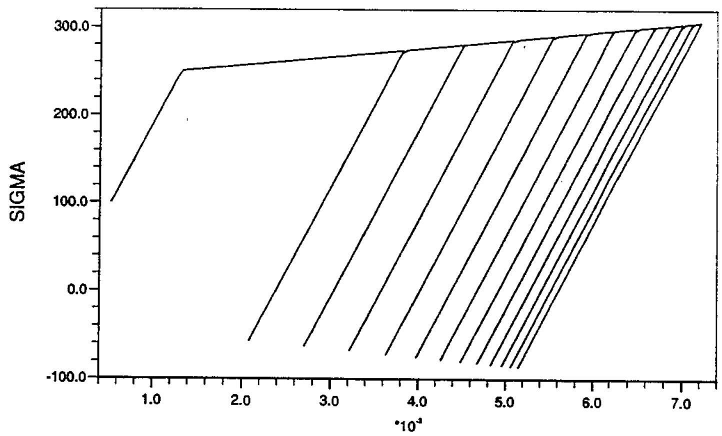
The standard algorithm is therefore modified as follows:
- study of possible plastifications (with the step-by-step method)
- calculation of the order of plastifications
- projections according to this order till Y belongs to the yield surface

The advantage of this method is to optmize computation time: for instance, in case the geometrical and mechanical parameters of the structure are constant, only one step is needed to get the analytical solution.

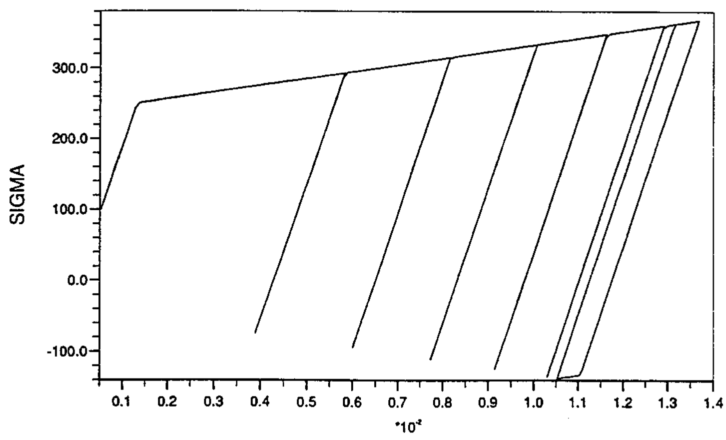
4) NUMERICAL EXAMPLES.

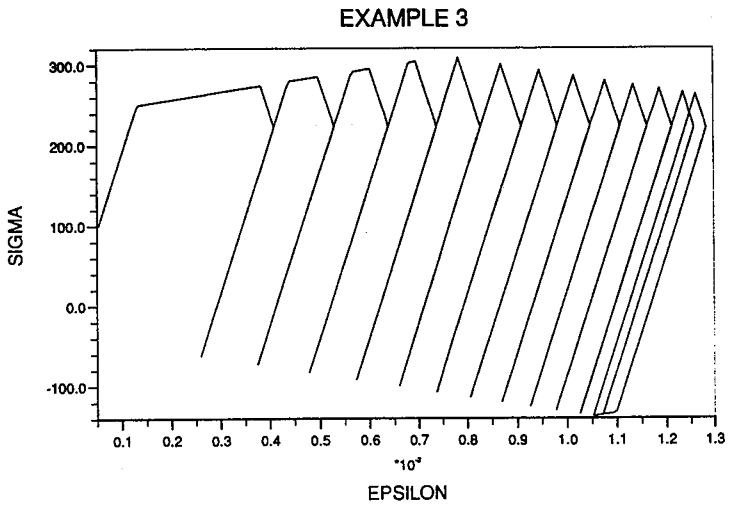
The structure of fig 1 is made of two mechanisms with the same geometrical parameters and the same constitutive law. Tube 2 is subjected to thermal cyclic loading while the temperature of tube 1 remain constant. In the first example the primary and secondary loads P,S are choosen in order to obtain elastic shakedown after 12 cycles. In example 2, S is increased to obtain plastic shakedown after 6 cycles. Examples 3 an 4 are similar to examples 1 and 2, but creep occurs in the two tubes. The next figures show the behaviour of tube 1.

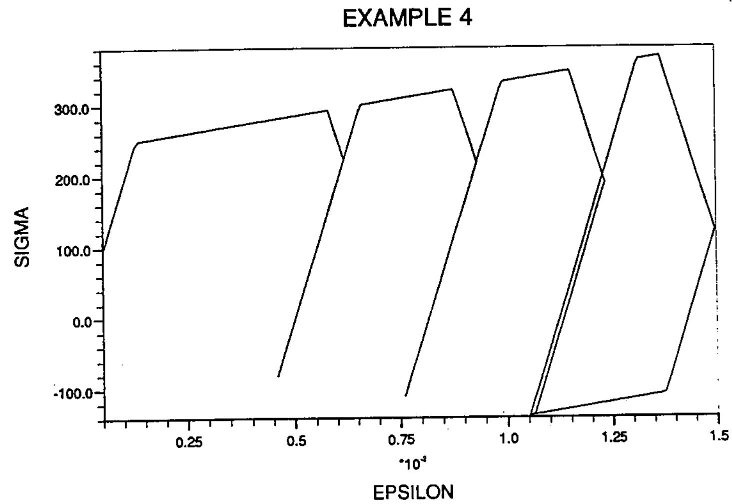
EXAMPLE 1



EXAMPLE 2







5) CONCLUSION.

In this paper we have first studied the response of a structure submitted to a mechanical load and a thermal cyclic load when the behaviour of each mechanism is plastic with kinematical hardening. This approach leads to solve only elastic problems when using Zarka's method.
In case of more complicated constitutive laws such as viscoelasticity and creep, we have converted the initial problem into an incremental formulation based on the principles used in the standard plastic case. These non linear problems are solved by a new chronological plastic theory which leads to exact computation of the basis equations. This theory was applied to a simple structure made of two thin shells but the Fortran program was written in order to solve any kind of structure considered as an assembly of elementary mechanisms.

REFERENCES

[1] ZARKA. J. 1979. Sur l'étude du comportement global des matériaux soumis à un chargement cyclique. Journal de Mécanique Appliquée. Vol 3, n°3. p 291-326
[2] JIANG. W., LECKIE. F. 1992. A. A direct method for the shakedown analysis of structures under sustained and cyclic loads. Journal of Applied Mechanics. Vol 59. p251-260.
[3] DEBORDES. O., NAYROLES. B. 1977. Sur la théorie et le calcul à l'adaptation des structures élastoplastiques. Journal de Mécanique. Vol 15, n°1. p 1-53