SMiRT-12 / K. Kussmaul (Editor)

L11/1

ELASTO/VISCO-PLASTIC DEFORMATION OF MODERATELY THICK SHELLS OF REVOLUTION UNDER THERMAL LOADING DUE TO FLUID

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ABSTRACT

This paper is concerned with an analytical formulation and a numerical solution of the thermo-elasto/visco-plastic deformation of general, moderately thick shells of revolution subjected to thermal loads due to fluid. At first the temperature distribution through the thickness is supposed to be curves of second order, and the temperature field in the shell under the appropriate initial and boundary conditions is determined by using the equations of heat conduction and heat transfer. Secondly the stresses and deformations are derived from the thermal stress equations. The equations of equilibrium and the relations between the strains and displacements are developed by extending the Reissner-Naghdi theory for elastic shells. For the constitutive relations, the Perzyna elasto/visco-plastic equations including the temperature effect are employed. The fundamental equations derived are numerically solved by the finite difference method. As a numerical example, the simply supported cylindrical shell made of mild steel under thermal loading due to fluid is analyzed, and the results are compared with those from classical theory which neglects the effect of shear deformations.

1 INTRODUCTION

The reduction of heat conduction equations for shell structures from three dimensional to two dimensional ones is generally desirable in the aspect of mathematical treatment. Up to now, this kind of analytical method has been proposed by Bolotin (1960), Steele and Yang(1969), Updike and Kalnins(1987) and so on. By solving these equations, the solutions for thermal elastic problems on thin shells have been obtained (e.g. Shirakawa and Ochiai(1979), Endo(1979)). Among the analytical methods Bolotin's equation, assumed the temperature distribution through the thickness as linear, is so simple and convenient that it is often used to solve the heat conduction of thin shells. There are, however, two problems; one is the problem of accuracy of solutions at initial response stages, and another is for the way to introduce the boundary conditions of temperature on the inner and outer surfaces of the shell.

In the present paper the authors suppose the temperature distribution through the thickness to be curves of second order, and develop the analytical method for thermo-elasto/visco-plastic deformation of moderately thick shells of revolution, which has been scarcely reported yet.

2 FUNDAMENTAL EQUATIONS

If the middle surface of axisymmetrical shells is given by $r=r(s)$, where r is the distance from the axis and s is the meridional distance measured from a boundary along the middle surface, the relations among the non-dimensional

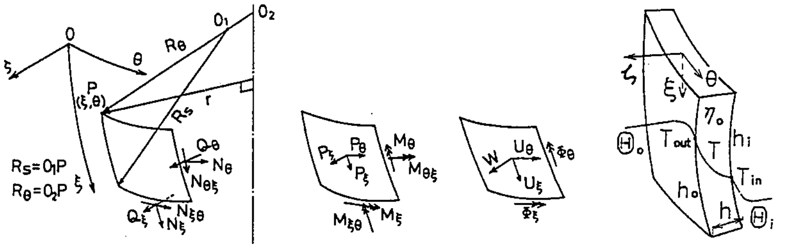

Fig.1 Coordinates and notations

curvatures $\omega_\xi(=a/R_s)$, $\omega_\theta(=a/R_\theta)$ and the non-dimensional radius $\rho(=r/a)$ become:

$$\omega_\xi=-(\gamma'+\gamma^2)/\omega_\theta,\ \omega_\theta=\sqrt{1-(\rho')^2}/\rho,\ \omega'_\theta=\gamma(\omega_\xi-\omega_\theta),\ \rho''/\rho=-\omega_\xi\omega_\theta,\ \gamma=\rho'/\rho,\ \xi=s/a,\ (\ )'=d(\ )/d\xi \tag{1}$$

where a is the reference length. An arbitrary point in the shell can be expressed by the orthogonal coordinate system (ξ,θ,ζ) as shown in Fig.1.

2.1 Heat conduction equation

The equation of heat conduction at a point in the shell body is given by the orthogonal coordinates (ξ,θ,ζ) as follows;

$$\frac{\partial T}{\partial t}-\frac{\chi}{L_\xi L_\theta a^2\rho}\{\frac{\partial}{\partial\xi}(\rho\frac{L_\theta}{L_\xi}\frac{\partial T}{\partial\xi})+\frac{L_\xi}{\rho L_\theta}\frac{\partial^2T}{\partial\theta^2}+\frac{\partial}{\partial\zeta}(a^2\rho L_\xi L_\theta\frac{\partial T}{\partial\zeta})\}-\frac{\eta_o}{c\rho_o}=0 \tag{2}$$

where $L_\xi=1+\zeta/R_s$, $L_\theta=1+\zeta/R_\theta$, T is the temperature at (ξ,θ,ζ,t) , $\chi(=\lambda_o/c\rho_o)$ is the thermal diffusivity, c is the specific heat, ρ_o is the mass density, λ_o is the coefficient of thermal conductivity and η_o is the heat generation per unit volume and per unit time.

The boundary conditions of the temperature on the inner and outer surfaces $(\zeta=\mp h/2)$ of the shell are;

$$\left[\frac{\partial T}{\partial\zeta}\right]_{\zeta=-h/2}=h_i(T_{in}-\Theta_i),\qquad \left[\frac{\partial T}{\partial\zeta}\right]_{\zeta=h/2}=-h_o(T_{out}-\Theta_o) \tag{3}$$

where $h_i=k_i/\lambda_o$, $h_o=k_o/\lambda_o$ and k is the heat transfer coefficient. T_{in} , T_{out} are the temperature on the inner and outer surfaces of the shell, Θ_i , Θ_o are ambient fluid temperature of the shell and h is the thickness of the shell.

The method, assumed the temperature distribution through the thickness as linear, has been often adopted by Bolotin (1960), Shirakawa and Ochiai (1979), Mizoguchi (1966) and so on. If this assumption is used, the boundary conditions of eqs.(3) can not be well introduced in the derivation of eqs.(6), and the resulting definite integral term becomes zero. In the present paper, to remove the above difficulty and to improve the accuracy of initial response stages, the temperature distribution through the thickness is supposed to be curves of second order by using coefficients T_o, T_1 and T_2 as follows:

$$T(\xi,\theta,\zeta,t)=T_o(\xi,\theta,t)+T_1(\xi,\theta,t)\zeta+T_2(\xi,\theta,t)\zeta^2 \tag{4}$$

After substituting eq.(4) into eq.(2), integrating eq.(2) across the thickness, integrating eq.(2) multiplied by ζ through the thickness, and integrating eq.(2) multiplied by ζ^2 through thickness, with consideration of the surface boundary conditions (eqs.(3)) and the next approximation;

$$\frac{L_\theta}{L_\xi}\simeq1-(\frac{1}{R_s}-\frac{1}{R_\theta})\zeta+\frac{1}{R_s}(\frac{1}{R_s}-\frac{1}{R_\theta})\zeta^2 \tag{5}$$

we have the following three equations;

$$\left.\begin{aligned}&(1+\frac{C_1}{12})\frac{\partial T_o}{\partial t}+\frac{h}{12}C_2\frac{\partial T_1}{\partial t}+\frac{h^2}{4}(\frac{1}{3}+\frac{C_1}{20})\frac{\partial T_2}{\partial t}-\frac{\chi}{a^2}\{(\gamma-\frac{C_3}{12})\frac{\partial T_o}{\partial\xi}-\frac{h}{12}\lambda\omega'_\xi(1+\frac{3}{20}C_4)\frac{\partial T_1}{\partial\xi}+\frac{h^2}{12}C_5\frac{\partial T_2}{\partial\xi}+(1+\frac{C_{15}}{12})\frac{\partial^2T_o}{\partial\xi^2}+\frac{h}{12}C_6\frac{\partial^2T_1}{\partial\xi^2}\\&+\frac{h^2}{12}(1+C_7)\frac{\partial^2T_2}{\partial\xi^2}+\frac{1}{\rho^2}(1+\frac{\lambda}{12}\omega_\theta C_6)\frac{\partial^2T_o}{\partial\theta^2}-\frac{h}{12\rho^2}C_6\frac{\partial^2T_1}{\partial\theta^2}+\frac{h^2C_8}{12\rho^2}\frac{\partial^2T_2}{\partial\theta^2}\}-\frac{C_{12}}{h}T_o-\frac{C_{11}}{2}T_1-\frac{h}{4}C_{12}T_2-C_{13}h_o\Theta_o-C_{14}h_i\Theta_i\\&-C_{16}\{Q_o+\frac{C_2}{h}Q_1+\frac{C_1}{h^2}Q_2\}=0\end{aligned}\right\} \tag{6}$$

and so on. The boundary conditions eqs.(3) are introduced into the terms $[\partial T/\partial\zeta]_{-h/2}^{h/2}$ to appear in the integrations. $c_1\sim c_{16}$ are constants depending on shell geometries, materials and h_i , h_o . And $Q_0\sim Q_4$ are given as:

$$Q_n=\int_{-h/2}^{h/2}\eta_o\zeta^n d\zeta\quad(n=0,1,2,3,4) \tag{7}$$

For three independent variables; T_o, T_1, T_2 , eqs.(6) are given, and by solving these relations, the temperature field in the shell can be determined.

2.2 Thermal deformation equations

An application of the Reissner shell theory (1941) to the shells of revolution and a differentiation of the equilibrium equation with time or load yield;

$$\left.\begin{array}{ll}\frac{\partial \dot{N}_\xi}{\partial \xi}+\gamma(\dot{N}_\xi-\dot{N}_\theta)+\frac{1}{\rho}\frac{\partial \dot{N}_{\theta\xi}}{\partial \theta}+\omega_\xi\dot{Q}_\xi+a\dot{P}_\xi=0, & \frac{\partial \dot{N}_{\xi\theta}}{\partial \xi}+\gamma(\dot{N}_{\xi\theta}+\dot{N}_{\theta\xi})+\frac{1}{\rho}\frac{\partial \dot{N}_\theta}{\partial \theta}+\omega_\theta\dot{Q}_\theta+a\dot{P}_\theta=0 \\ \frac{\partial \dot{Q}_\xi}{\partial \xi}+\gamma\dot{Q}_\xi+\frac{1}{\rho}\frac{\partial \dot{Q}_\theta}{\partial \theta}-(\omega_\xi\dot{N}_\xi+\omega_\theta\dot{N}_\theta)+a\dot{P}_\zeta=0, & \dot{Q}_\xi-\frac{1}{a}\left[\frac{\partial \dot{M}_\xi}{\partial \xi}+\gamma(\dot{M}_\xi-\dot{M}_\theta)+\frac{1}{\rho}\frac{\partial \dot{M}_{\theta\xi}}{\partial \theta}\right]=0 \\ \dot{Q}_\theta-\frac{1}{a}\left[\frac{\partial \dot{M}_{\xi\theta}}{\partial \xi}+\gamma(\dot{M}_{\xi\theta}+\dot{M}_{\theta\xi})+\frac{1}{\rho}\frac{\partial \dot{M}_\theta}{\partial \theta}\right]=0 & \end{array}\right\} \quad (8)$$

where the notations are shown in Fig.1.

The strains of the middle surface are given by (Naghdi,1957);

$$\dot{\varepsilon}_{\xi m}=\frac{1}{a}\left[\frac{\partial \dot{U}_\xi}{\partial \xi}+\omega_\xi\dot{W}\right],\quad \dot{\varepsilon}_{\theta m}=\frac{1}{a}\left[\frac{1}{\rho}\frac{\partial \dot{U}_\theta}{\partial \theta}+\gamma\dot{U}_\xi+\omega_\theta\dot{W}\right],\quad \dot{\varepsilon}_{\xi\theta m}=\frac{1}{2a}\left[\frac{1}{\rho}\frac{\partial \dot{U}_\xi}{\partial \theta}+\frac{\partial \dot{U}_\theta}{\partial \xi}-\gamma\dot{U}_\theta\right] \quad (9)$$

where $\varepsilon_{\xi\theta m}$ is half the usual engineering shear strain. The bending distortions χ_ξ , χ_θ , $\chi_{\xi\theta}$, $\chi_{\theta\xi}$ and the rotations Φ_ξ , Φ_θ , Φ_n are as follows (Naghdi,1957);

$$\dot{\chi}_\xi=\frac{1}{a}\frac{\partial \dot{\Phi}_\xi}{\partial \xi},\quad \dot{\chi}_\theta=\frac{1}{a}\left(\frac{1}{\rho}\frac{\partial \dot{\Phi}_\theta}{\partial \theta}+\gamma\dot{\Phi}_\xi\right),\quad \dot{\chi}_{\xi\theta}=\frac{1}{2a}\left(\frac{\partial \dot{\Phi}_\theta}{\partial \xi}-2\omega_\xi\dot{\Phi}_n\right),\quad \dot{\chi}_{\theta\xi}=\frac{1}{2a}\left(\frac{1}{\rho}\frac{\partial \dot{\Phi}_\xi}{\partial \theta}-\gamma\dot{\Phi}_\theta+2\omega_\theta\dot{\Phi}_n\right) \quad (10)$$

$$\dot{\Phi}_\xi=\frac{1}{a}\left(-\frac{\partial \dot{W}}{\partial \xi}+\omega_\xi\dot{U}_\xi\right)+2\dot{\varepsilon}_{\xi\zeta m},\quad \dot{\Phi}_\theta=\frac{1}{a}\left(-\frac{1}{\rho}\frac{\partial \dot{W}}{\partial \theta}+\omega_\theta\dot{U}_\theta\right)+2\dot{\varepsilon}_{\theta\zeta m},\quad \dot{\Phi}_n=\frac{1}{2a}\left(-\frac{1}{\rho}\frac{\partial \dot{U}_\xi}{\partial \theta}+\frac{\partial \dot{U}_\theta}{\partial \xi}+\gamma\dot{U}_\theta\right) \quad (11)$$

The strains at the distance ζ from the middle surface are given as:

$$\left.\begin{array}{l}\dot{\varepsilon}_\xi=(\dot{\varepsilon}_{\xi m}+\zeta\dot{\chi}_\xi)/L_\xi,\quad \dot{\varepsilon}_\theta=(\dot{\varepsilon}_{\theta m}+\zeta\dot{\chi}_\theta)/L_\theta,\quad \dot{\varepsilon}_{\xi\theta}=\{(\dot{\varepsilon}_{\xi\theta m}+\dot{\Phi}_n)/2+\zeta(\dot{\chi}_{\xi\theta}+\dot{\Phi}_n/R_s)\}/L_\xi+\{(\dot{\varepsilon}_{\xi\theta m}-\dot{\Phi}_n)/2+\zeta(\dot{\chi}_{\theta\xi}-\dot{\Phi}_n/R_\theta)\}/L_\theta \\ \dot{\varepsilon}_{\xi\zeta}=\dot{\varepsilon}_{\xi\zeta m}/L_\xi,\quad \dot{\varepsilon}_{\theta\zeta}=\dot{\varepsilon}_{\theta\zeta m}/L_\theta\end{array}\right\} \quad (12)$$

Now, we shall use the elasto/visco-plastic equations by Perzyna (1966) for considering the temperature effect for constitutive relations. The visco-plastic strain rates $\dot{\varepsilon}_{ij}^{vp}$ are as follows;

$$\dot{\varepsilon}_{ij}^{vp}=\gamma_0(T)<\Psi(F)>S_{ij}J_2^{-1/2} \quad (13)$$

where the dot denotes partial differentiation with respect to time; S_{ij} , J_2 and $\gamma_o(T)$ are the deviatoric stress, the second invariant of the deviatoric stress tensor and a material constant, respectively, and γ_o is a function of absolute temperature T as well as σ^* in eq.(15). The symbol $<\Psi(F)>$ is defined as follows:

$$<\Psi(F)>=0 \ :\ F\leqq 0 \qquad <\Psi(F)>=\Psi(F)\ :\ F>0 \quad (14)$$

where function F is:

$$F=[\bar{\sigma}-\sigma^*(T)]/\sigma^*(T) \quad (15)$$

and $F=0$ denotes the von Mises yield surface, $\bar{\sigma}$ is the equivalent stress $(=\sqrt{3J_2})$ and $\sigma^*(T)$ is the static yielding stress obtained in the usual tension test.

In the present theory where the stress component σ_ζ , normal to the middle surface, can be assumed to be neglected, the constitutive equations are written as follows:

$$\{\dot{\varepsilon}\}=[D]^{-1}\{\dot{\sigma}\}+\{\dot{\varepsilon}^{vp}\}+\{\dot{\varepsilon}^t\} \quad (16)$$

where

$$\{\dot{\sigma}\}=\{\dot{\sigma}_\xi,\dot{\sigma}_\theta,\dot{\sigma}_{\xi\theta},\dot{\sigma}_{\xi\zeta},\dot{\sigma}_{\theta\zeta}\}^T,\quad \{\dot{\varepsilon}\}=\{\dot{\varepsilon}_\xi,\dot{\varepsilon}_\theta,\dot{\varepsilon}_{\xi\theta},\dot{\varepsilon}_{\xi\zeta},\dot{\varepsilon}_{\theta\zeta}\}^T,\quad \{\dot{\varepsilon}^{vp}\}=\{\dot{\varepsilon}_\xi^{vp},\dot{\varepsilon}_\theta^{vp},\dot{\varepsilon}_{\xi\theta}^{vp},\dot{\varepsilon}_{\xi\zeta}^{vp},\dot{\varepsilon}_{\theta\zeta}^{vp}\}^T,\quad \{\dot{\varepsilon}^t\}=\{\alpha\dot{T}_e,\alpha\dot{T}_e,0,0,0\}^T,$$

$$[D]=\frac{E}{1-\nu^2}\times\begin{bmatrix}1&\nu&0&0&0\\ \nu&1&0&0&0\\ 0&0&1-\nu&0&0\\ 0&0&0&1-\nu&0\\ 0&0&0&0&1-\nu\end{bmatrix},\quad \{\dot{\varepsilon}^{vp}\}=\frac{2}{\sqrt{3}}\gamma_o(T)<\Psi\left(\frac{\bar{\sigma}-\sigma^*(T)}{\sigma^*(T)}\right)>\times\frac{1}{\bar{\sigma}}\begin{bmatrix}1&-1/2&0&0&0\\ -1/2&1&0&0&0\\ 0&0&3/2&0&0\\ 0&0&0&3/2&0\\ 0&0&0&0&3/2\end{bmatrix}\{\sigma\} \quad (17)$$

where $\{\ \}^T$ represents the transposed matrix, and E , ν and α are Young's modulus, Poisson's ratio and the thermal expansion coefficient, respectively. T_e is the temperature rise from the original temperature $\bar{T}_0$ to the present temperature T, namely,

$$T_e(\xi,\theta,\zeta,t)=T(\xi,\theta,\zeta,t)-\bar{T}_0 \tag{18}$$

Solving eq.(16) for stresses, the stresses are;

$$\{\dot{\sigma}\}=[D]\{\dot{\varepsilon}\}-\{\dot{\sigma}^{vp}\}-\{\dot{\sigma}^{t}\} \tag{19}$$

where

$$\{\dot{\sigma}^{vp}\}=\{\dot{\sigma}_{\xi}^{vp},\dot{\sigma}_{\theta}^{vp},\dot{\sigma}_{\xi\theta}^{vp},\dot{\sigma}_{\xi\zeta}^{vp},\dot{\sigma}_{\theta\zeta}^{vp}\}^T=[D]\{\dot{\varepsilon}^{vp}\},\qquad \{\dot{\sigma}^{t}\}=\{\dot{\sigma}^{t},\dot{\sigma}^{t},0,0,0\}^T=[D]\{\dot{\varepsilon}^{t}\}=\frac{\alpha E}{1-\nu}\{\dot{T}_e,\dot{T}_e,0,0,0\}^T \tag{20}$$

By the use of eq.(19) with the eq.(5), the resultant forces $\{N\}$ and the resultant moments $\{M\}$ per unit length may be obtained as follows;

$$\left.\begin{aligned}
\dot{N}_{\xi}&=\frac{Eh}{1-\nu^2}[\{1+\frac{h^2}{12a^2}\omega_{\xi}(\omega_{\xi}-\omega_{\theta})\}\dot{\varepsilon}_{\xi m}+\nu\dot{\varepsilon}_{\theta m}-\frac{h^2}{12a}(\omega_{\xi}-\omega_{\theta})\dot{\kappa}_{\xi}]-\dot{N}_{\xi}^{vp}-\dot{N}_{\xi}^{t}\\
\dot{M}_{\xi}&=\frac{Eh^3}{12(1-\nu^2)}[\{1+\frac{3}{20}\frac{h^2}{a^2}\omega_{\xi}(\omega_{\xi}-\omega_{\theta})\}\dot{\kappa}_{\xi}+\nu\dot{\kappa}_{\theta}-\frac{1}{a}(\omega_{\xi}-\omega_{\theta})\dot{\varepsilon}_{\xi m}]-\dot{M}_{\xi}^{vp}-\dot{M}_{\xi}^{t}\\
\dot{Q}_{\xi}&=\frac{5}{6}\frac{Eh}{1+\nu}\dot{\varepsilon}_{\xi\zeta m}\{1+\frac{h^2}{12a^2}\omega_{\xi}(\omega_{\xi}-\omega_{\theta})\}-\dot{Q}_{\xi}^{vp}
\end{aligned}\right\} \tag{21}$$

and so on. In eqs.(21) $(\ )^{vp}$ and $(\ )^{t}$ denote the apparent internal forces due to visco-plasticity and the internal forces due to temperature rise T_e, respectively, and they are given by:

$$\left.\begin{aligned}
\{\dot{N}_{\xi}^{vp},\dot{N}_{\xi\theta}^{vp},\dot{Q}_{\xi}^{vp},\dot{M}_{\xi}^{vp},\dot{M}_{\xi\theta}^{vp},\dot{N}_{\xi}^{t},\dot{M}_{\xi}^{t}\}&=\int_{-h/2}^{h/2}\{\dot{\sigma}_{\xi}^{vp},\dot{\sigma}_{\xi\theta}^{vp},\dot{\sigma}_{\xi\zeta}^{vp},\dot{\sigma}_{\xi}^{vp}\zeta,\dot{\sigma}_{\xi\theta}^{vp}\zeta,\dot{\sigma}^{t},\dot{\sigma}^{t}\zeta\}L_{\theta}d\zeta\\
\{\dot{N}_{\theta}^{vp},\dot{N}_{\theta\xi}^{vp},\dot{Q}_{\theta}^{vp},\dot{M}_{\theta}^{vp},\dot{M}_{\theta\xi}^{vp},\dot{N}_{\theta}^{t},\dot{M}_{\theta}^{t}\}&=\int_{-h/2}^{h/2}\{\dot{\sigma}_{\theta}^{vp},\dot{\sigma}_{\xi\theta}^{vp},\dot{\sigma}_{\theta\zeta}^{vp},\dot{\sigma}_{\theta}^{vp}\zeta,\dot{\sigma}_{\xi\theta}^{vp}\zeta,\dot{\sigma}^{t},\dot{\sigma}^{t}\zeta\}L_{\xi}d\zeta
\end{aligned}\right\} \tag{22}$$

A complete set of field equations for 56 independent variables: $\dot{U}_{\xi},\dot{U}_{\theta},\dot{W},\dot{\Phi}_{\xi},\dot{\Phi}_{\theta},\dot{\Phi}_{n},\dot{\kappa}_{\xi},\dot{\kappa}_{\theta},\dot{\kappa}_{\xi\theta},\dot{\kappa}_{\theta\xi},\{\dot{\varepsilon}_m\},\{\dot{\varepsilon}^{vp}\},\{\dot{\varepsilon}^{t}\},\{\dot{\sigma}\},\{\dot{\sigma}^{vp}\},\{\dot{\sigma}^{t}\},\{\dot{N}\},\{\dot{M}\},\{\dot{N}^{vp}\},\{\dot{M}^{vp}\},\dot{N}_{\xi}^{t},\dot{N}_{\theta}^{t},\dot{M}_{\xi}^{t},\dot{M}_{\theta}^{t}$ is now given by 56 equations: (8)-(11),(17),(19)-(22).

3 NON-DIMENSIONAL EQUATIONS

In order to analyze the problem of the shells under arbitrary unsymmetrical loads, the distributed loads, the ambient fluid temperature of the shell, the heat generation and the 53 independent variables, except for $\{\dot{\varepsilon}^{vp}\}$ and $\dot{\varepsilon}^{t}$, are expanded into Fourier series (Takezono et al., 1990, 1991).

Substituting these into the above fundamental equations, the equations among the Fourier coefficients (denoted by small letters) relating to the variables are obtained. From the heat conduction equations, the simultaneous differential equations for the coefficients $t_o^{(n)}$, $t_1^{(n)}$, $t_2^{(n)}$ can be obtained. Similarly from the thermal deformation equations, appropriately eliminating the variables, the simultaneous differential equations for the displacement rates and the rotation rates can be derived (Takezono et al., 1991).

By the use of eqs.(20) and (22), the rates of internal forces related to visco-plasticity and the temperature rise become the following;

$$\left.\begin{aligned}
&\sigma_o h\sum_{n=0}^{\infty}\{\dot{n}_{\xi}^{vp(n)},\dot{m}_{\xi}^{vp(n)}\}\cos n\theta=\frac{E}{1-\nu^2}\int_{-h/2}^{h/2}\{1,\frac{a}{h^2}\zeta\}(\dot{\varepsilon}_{\xi}^{vp}+\nu\dot{\varepsilon}_{\theta}^{vp})L_{\theta}d\zeta,\quad h\{\dot{n}_{\xi}^{t(n)},\dot{m}_{\xi}^{t(n)}\}=\frac{1}{1-\nu}\int_{-h/2}^{h/2}\{1,\frac{a\zeta}{h^2}\}\dot{t}_e L_{\theta}d\zeta\\
&\sigma_o h\sum_{n=1}^{\infty}\{\dot{n}_{\xi\theta}^{vp(n)},\dot{m}_{\xi\theta}^{vp(n)}\}\sin n\theta=\frac{E}{1+\nu}\int_{-h/2}^{h/2}\{1,\frac{a}{h^2}\zeta\}\dot{\varepsilon}_{\xi\theta}^{vp}L_{\theta}d\zeta
\end{aligned}\right\} \tag{23}$$

and so on. σ_0 is a reference stress, and the visco-plastic strain rates on the right-hand sides of eqs.(23) can be related to the stresses by eqs.(17). The integrations are carried out numerically by the use of Simpson's 1/3 rule.

4 NUMERICAL METHOD

A finite difference method is employed for the solutions of above two second simultaneous differential equations. The usual central difference formulas are used for every mesh point except the discontinuity points and the boundary points of the shell. For the discontinuity points and the boundary points forward and backward difference equations are employed. The derivatives with respect to time in the difference formulas for heat conduction equations are treated with the Crank-Nicolson method. The solutions at any time are obtained by a summation of the incremental values due to the time increment.

5 NUMERICAL EXAMPLE

As a numerical example, the simply supported cylindrical shell made of mild steel subjected to locally distributed axisymmetric thermal loading due to fluid is analyzed (Fig.2). $\overline{T}_o$ and Θ_i are both 0°C, and the both ends are assumed to be adiabatic. h_i and h_o on inner and outer surfaces of the shell are 0.02, 100 1/m, respectively, and the value of σ_0 in eqs.(23) has been selected as $\sigma_0=1$.

The material constants employed in the calculations are as follows (Perzyna, 1966),

$$\left.\begin{array}{l} E = 189.5\ \text{GPa},\quad \rho_o = 7.78\ \text{g/cm}^3,\quad \nu = 0.3,\quad \lambda_o = 50.1\ \text{W/(m}\cdot\text{K)},\quad \alpha = 11.7\times10^{-6}\ \text{1/K},\quad c = 0.460\ \text{kJ/(kg}\cdot\text{K)} \\ \sigma^*(T) = 207\exp\{0.45(288/T-1)\}\ \text{MPa},\quad \gamma_o(T) = 30.12[1+2.6\{(220-T)/273\}^2]\ \text{1/s},\quad \Psi(F) = [(\bar{\sigma}-\sigma^*(T))/\sigma^*(T)]^5 \end{array}\right\} \tag{24}$$

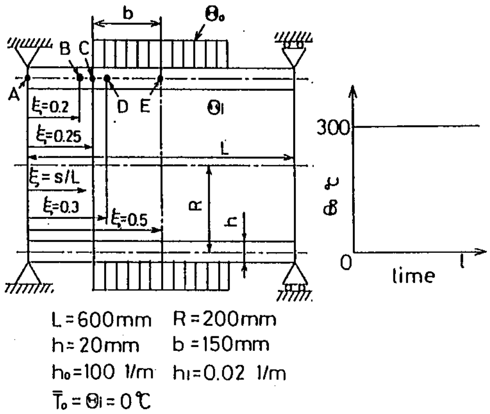

Fig.2 Numerical example

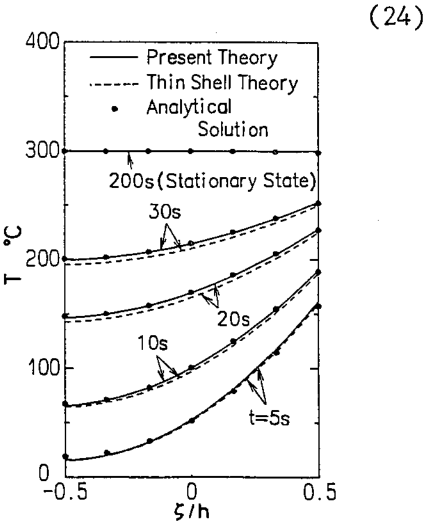

Fig.3 Temperature distribution through thickness at point E

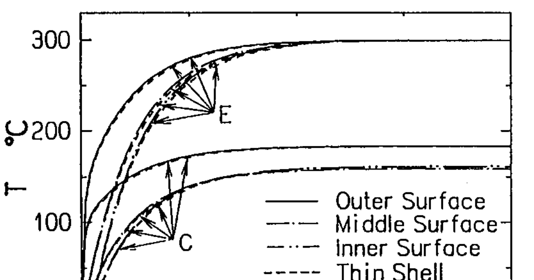

Fig.4 Variations of T with time

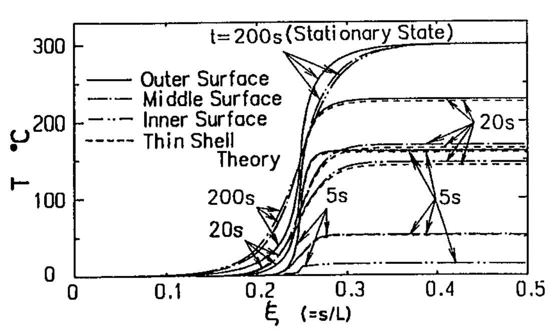

Fig.5 Meridional distributions of T

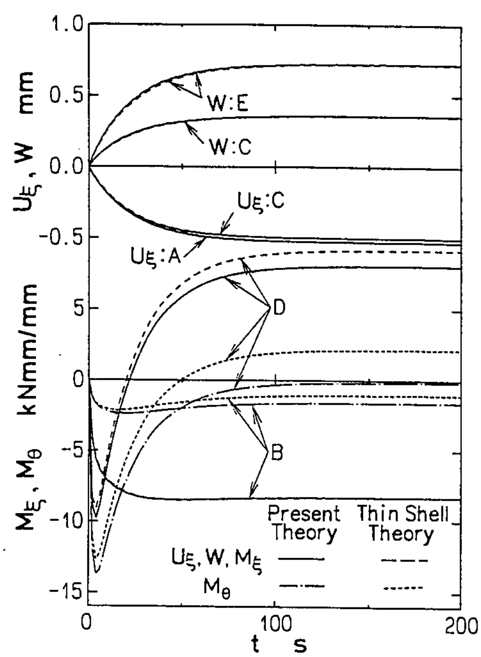

Fig.6 Variations of U_ξ , W , M_ξ and M_θ with time

The number of mesh point is 101, the division number through the thickness is chosen to be 19, and the increment of time Δt is determined as 0.1 s.

Some of the essential features of the solutions are shown in Figs.3-9. The results from the thin shell theory (Sanders,1959), which neglects the effect of shear deformations, are plotted by broken and dotted lines. From the computations, the following has been found:

(1) In Fig.3 the present solutions are in very good agreement with the analytical solutions by Ishida and Takeuchi (1984).

(2) The differences between solutions from the present theory and those from the thin shell theory are small in the displacements and resultant stresses, but are significant in the resultant moments.

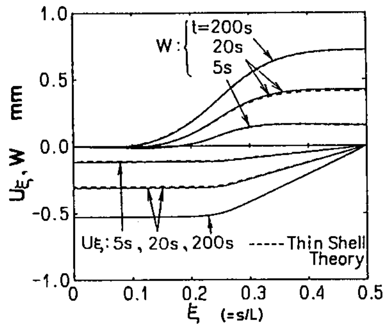

Fig.7 Meridional distributions of U_ξ and W

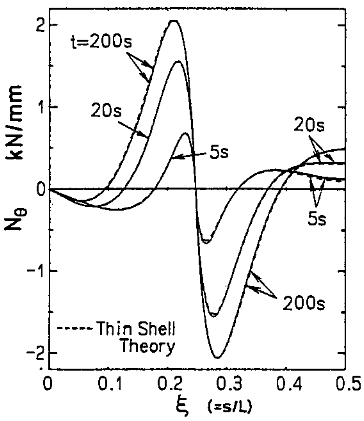

Fig.8 Meridional distributions of N_θ

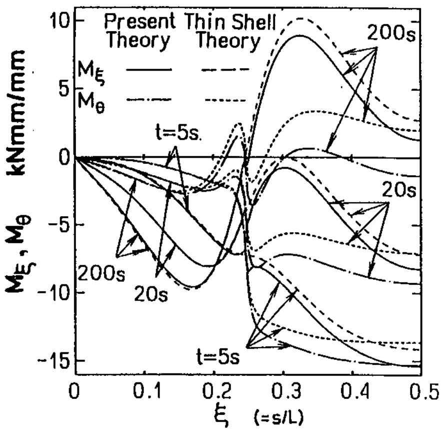

Fig.9 Meridional distributions of M_ξ and M_θ

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