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# NUMERICAL IMPLEMENTATION OF SIMPLIFIED METHODS OF INELASTIC ANALYSIS OF STRUCTURES SUBJECTED TO SHORT PERIOD LOADS

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## ABSTRACT

For structures that exhibit inelastic deformations when subjected to cyclic loadings of short period, the stress can be decomposed into an elastic stress vector which varies with time following the variation of the loading, and a self-equilibrating residual stress vector constant inside the cycle. In the present paper the steady-state values of this residual stress vector are found by iterations. Thus the long term deflections in a structure can be estimated.

## 1 INTRODUCTION

In nuclear and aerospace industry, many structures operate under stress and temperature conditions that result in accumulation of creep and plastic strains throughout their lives.

In many cases the loading on the structure is applied periodically. It has been rightly recognized that simplified methods of analysis, which are not based on very time consuming and often numerically unstabe time stepping inelastic finite element procedures, provide a better alternative. These methods are based on the assumption that after some initial cycles, the structure finally develops a cyclic stationary stress state, in which the stress repeats itself with a period equal to the cyclic loading period. Moreover, if the cycle time is short, the stress can be decomposed into an elastic part and a residual part constant with time inside the cycle. The search therefore is for this residual part. It has been proved(1) that for values of loading less than a certain ratio of the elastoplastic shakedown loading, creep effects dominate and only these need to be considered. It has also been discussed (2) that one can go beyond this ratio.

The present work shows that this residual stress part can be found, by including plasticity effects which can be easily accomodated together with creep effects in a single analysis and establish therefore, at the same time, whether adaptation of the structure occurs.

## 2 NUMERICAL PROCEDURE

The structure is discretized into finite elements. The elements are assumed to be interconnected at a discrete number of nodal points situated on their boundaries. Bold letters indicate vectors and matrices.

When one cycle has been completed the changes in residual strains in terms of the changes of the nodal displacements $\Delta\mathbf{r}_r$ are given by:

$$\Delta\boldsymbol{\varepsilon}_r = \mathbf{B}\Delta\mathbf{r}_r \qquad (1)$$

On the other hand, the changes in strains can be decomposed into three terms : elastic, creep and plastic strains

$$\Delta\boldsymbol{\varepsilon}_r = \Delta\boldsymbol{\varepsilon}_r^{el} + \Delta\boldsymbol{\varepsilon}_r^{cr} + \Delta\boldsymbol{\varepsilon}_r^{pl} \qquad (2)$$

$\Delta\boldsymbol{\varepsilon}_r^{el}$ can be expressed in terms of the changes $\Delta\boldsymbol{\rho}$ of the residual stresses through the matrix of elastic constants **D**. Equation (2) therefore becomes:

$$\Delta\boldsymbol{\rho} = \mathbf{D}(\Delta\boldsymbol{\varepsilon}_r - \Delta\boldsymbol{\varepsilon}_r^{cr} - \Delta\boldsymbol{\varepsilon}_r^{pl}) \qquad (3)$$

Since $\Delta\boldsymbol{\rho}$ is in equilibrium with zero loads and $\Delta\boldsymbol{\varepsilon}_r$ is compatible, from PVW we finally get:

$$\left(\int_V (\mathbf{B}^T\mathbf{D}\mathbf{B})dV\right)\Delta\mathbf{r}_r = \int_V \mathbf{B}^T\mathbf{D}\Delta\boldsymbol{\varepsilon}_r^{cr}dV + \int_V \mathbf{B}^T\mathbf{D}\Delta\boldsymbol{\varepsilon}_r^{pl}dV \qquad (4)$$

The left-hand side of (4) is the linear stiffness matrix **K** of the structure, whereas the right-hand side are nodal loads that are due to creep and plastic effects correspondingly.

Since the cycle time is short, it is reasonable to assume(1), that there is no time for any stress redistribution and the residual stress remains constant with time inside the cycle. This leads to the following numerical procedure:

The linear elastic cyclic loading is solved first and the elastic stresses are calculated at each Gauss point. Then assuming an initial constant in time distribution of residual stresses $^{o}\boldsymbol{\rho}$, normally zero, we add them to the elastic stresses, which are functions of time, at the Gauss points and the creep effects are calculated by performing numerical time integration over a complete cycle. The plasticity effects are also calculated, as will be explained below, and therefore the right-hand side of (4) is now assembled for all elements. Solving (4), we get an increment of $\Delta\mathbf{r}_r$ and from (1) and (3) an increment $\Delta\boldsymbol{\rho}$. The increment is added to $^{o}\boldsymbol{\rho}$ and a new residual stress distribution arises. The process is repeated till $\Delta\boldsymbol{\rho}$ is almost negligible, and the stationary residual stress state is found.

## 2.1 Plastic effects

A simple and effective way to implement plasticity effects is proposed. A perfectly plastic material has been assumed and a von Mises type of yield criteria has been used with $\sigma^y$ uniaxial yield stress. Analogous approach would also apply for any other yield criteria. Since the value of the residual stress does not change inside the cycle, after the completion of the cycle we can add to the values of the residual stresses obtained, the maximum or minimum values of the elastic stress, and calculate for the total stress at each Gauss point, the equivalent stress $\bar{\sigma}^{max}$ and $\bar{\sigma}^{min}$. After we have checked for all the Gauss points of the structure, if we find $\bar{\sigma}^{max} > \bar{\sigma}^{min} > \sigma^y$, the maximum values of elastic stresses are taken to be the most critical, otherwise if $\bar{\sigma}^{min} > \bar{\sigma}^{max} > \sigma^y$, the minimum values of elastic stresses are the most critical. We further proceed with the most critical values of the elastic stresses $\boldsymbol{\sigma}_*^{el} = \boldsymbol{\sigma}_{max}^{el}$ or

$\sigma^{el}_{min}$, and perform a radial return operation (Fig.1).
Let us suppose that the initial state of stress after the completion of a cycle is $\rho^{in}$. The final state of stress ,had the material responded elastically, is $\sigma^A = \sigma^{el}_* + \rho^{in}$. A radial return with ratio r is performed so that the stress is brought back on the yield surface. If we denote by $\rho^{fin}$ the true final, after the correction, residual stresses, the following equation must hold:

$$(\sigma^{el}_* + \rho^{in}) - r(\sigma^{el}_* + \rho^{in}) = \sigma^{el}_* + \rho^{fin}$$

from which

$$\rho^{fin} = \rho^{in} - r(\sigma^{el}_* + \rho^{in}) \tag{5}$$

The ratio r is determined from the condition that the final state of stress $\sigma^{el}_* + \rho^{fin}$ lies on the yield surface.Therefore:

$$f\left\{(1 - r)(\sigma^{el}_* + \rho^{in})\right\} = 0 \tag{6}$$

The unbalanced force $r(\sigma^{el}_* + \rho^{in})$ is redistributed as equivalent nodal forces in the whole structure.

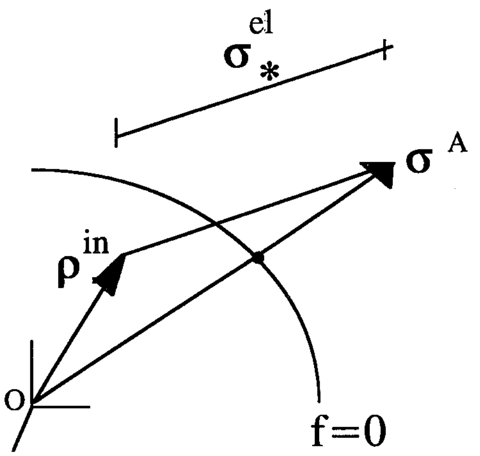



Fig.1. Radial return rule for a perfectly plastic material.

## 3 NUMERICAL APPLICATIONS

In the applications that follow, creep effects are calculated using the multiaxial, for each case, equivalent to uniaxial Norton's creep law , $\dot{\varepsilon}=K\sigma^n$.

### 3.1 Thick cylinder

A thick cylinder under internal pressure(3) is considered. The finite element discretization and load variation is shown in Fig.2. The values of material constants used are $K=0.636 \times 10^{-10}$ (SI units), creep index $n=3$, yield stress $\sigma^y=24$ dN/mm$^2$. Plane strain conditions are assumed and incompressibility of the material, $\mu=0.4999$ is also assumed. 8-node isoparametric elements with 2x2 Gauss point integration for each of them was used.

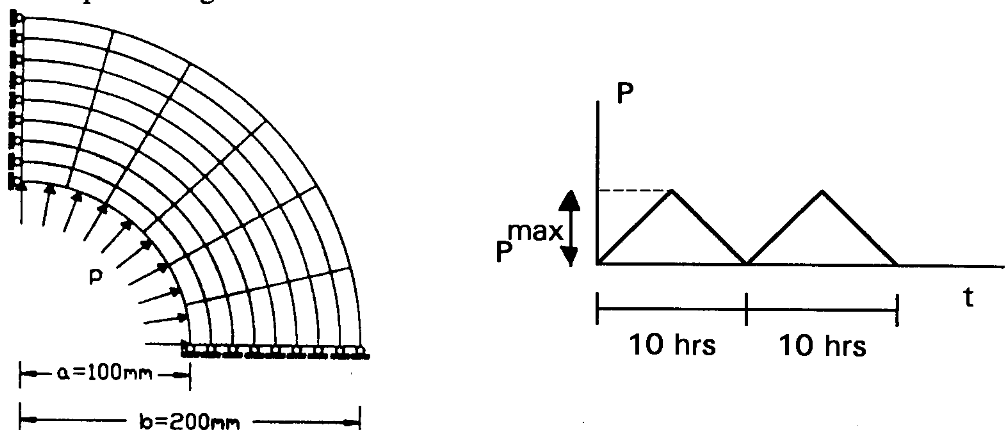



Fig.2 Finite element discretization and load variation with time.

In order to test the computer program, the load applied on the structure was kept constant in time. A pure creep behaviour was considered first and a very good agreement with analytical results(4) was achieved.

An elastoplastic behaviour for the material was considered secondly and results for a pressure of 15 dN/mm$^2$ are shown in Fig.3. A very good agreement with analytical results were obtained. The shakedown elastoplastic pressure(5) comes out to be 19.2 dN/mm$^2$

The case of the cyclic loding (Fig.2) with $p^{max}$ = 15dN/mm$^2$ was considered and results are shown in Fig.3. The curves corresponding to the cyclic creep and to the cyclic creep-plastic behaviours almost coincide and this indicates that creep effects are the dominant effects, for this level of loading. For load levels greater than this value, convergence within the specified tolerance becomes quite tedious and definitely above 16 dN/mm$^2$ stresses near the region where loading is applied always exceed the yield surface indicating that this is, approximately, the level of shakedown in the presence of creep.

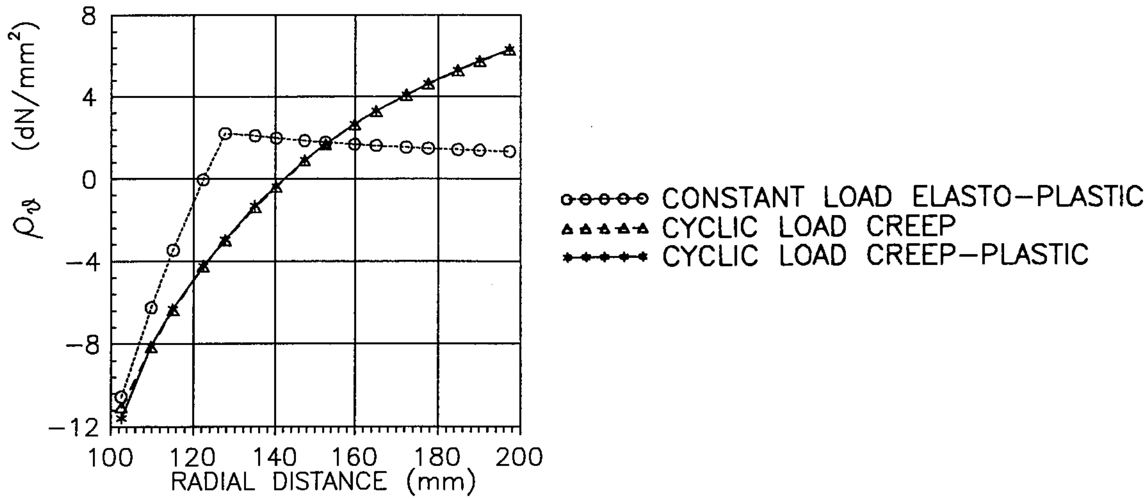
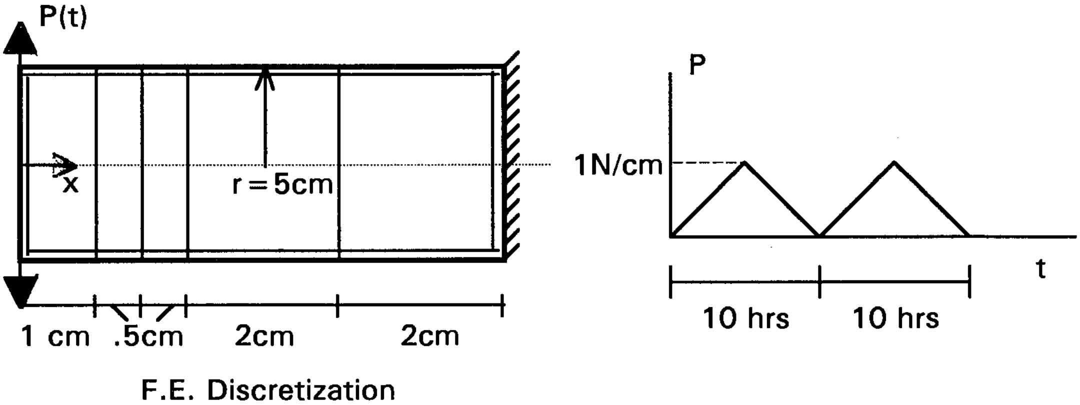



Fig.3. Steady-state residual circumferential stress distribution $\rho_\theta$ .

3.2 Thin cylinder

A thin cylinder, loaded by an axisymmetric edge cyclic loading (Fig.4) was considered next. 24 conical frustra elements were used for discretization, 20 of them from x=0 to x=1cm.




$E=10^7$ N/cm$^2$ $\nu$=0.3 K=0.636x10$^{-10}$ (SI units) n=3.0 $N^y$=30N/cm

Fig.4. Cylindrical shell, properties, and variation of the cyclic loading with time.

The stress vector consists of an axial force and a bending moment acting longitudinally as well as an axial force (hoop force) and a bending moment acting transversely. The yield surface[6], was used with $N^y$ being the equivalent quantity to $\sigma^y$ in this case.
In the left part of Fig.5 the elastic axial hoop force distribution, whereas in tge right part the elastic bending moment distribution along the length of the cylinder for the maximum value of loading are plotted. As expected the effects of the edge loading are localised and they only extend in a small region of the cylinder which belongs to the "long cylinder" category.

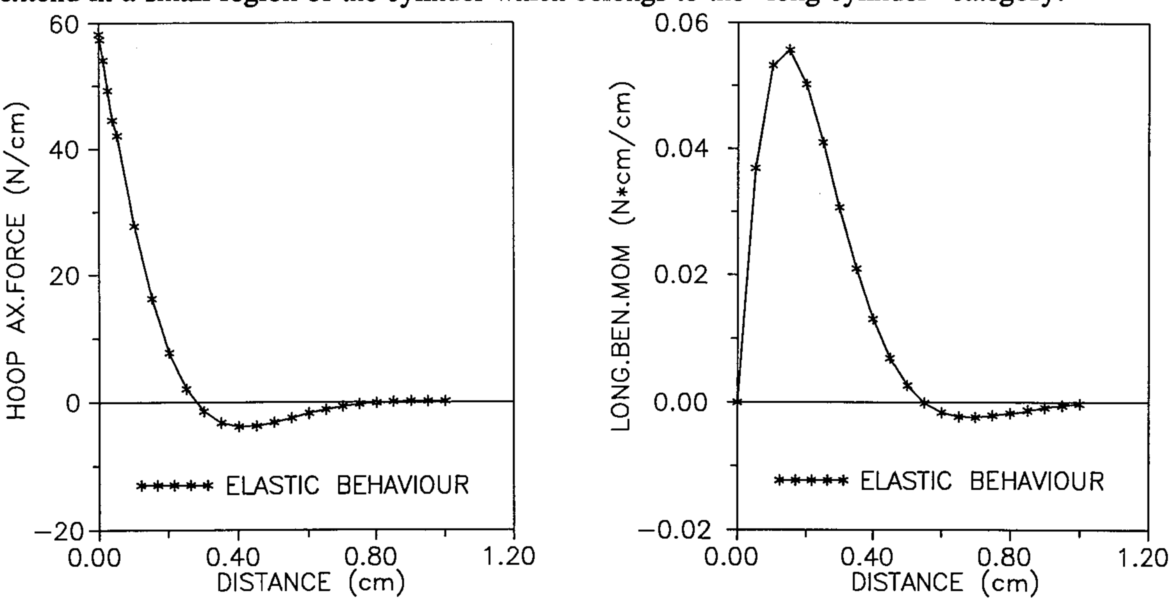



Fig.5. Distribution of elastic hoop axial force and longitudinal bending moment for the maximum value of loading

In the left part of Fig.6, the steady-state residual axial hoop force distribution for the cyclic loading of Fig.4 is plotted against the length of the cylinder, whereas in the right part, the corresponding distribution of the residual longitudinal bending moment is plotted. Two different cases like in the example with the thick cylinder were considered; one with only creep effects and another with both creep and plastic effects. It can be seen that for this level of loading, when plastic effects are being taken into account, they cause substantial difference to the stress distribution caused by creep only.

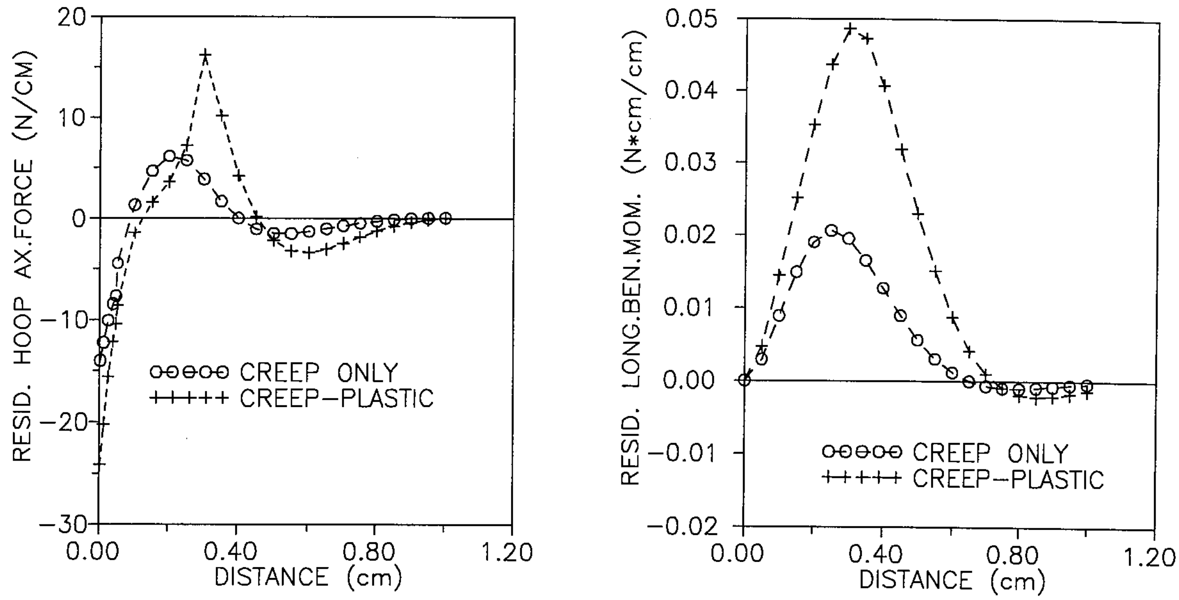



Fig.6. Distribution of steady-state residual axial force and longitudinal bending moment for cyclic loading.

## 4 CONCLUSIONS

In the search for a cyclic stationary stress state for a short period cyclic loading imposed on a structure that exhibits inelastic deformations, non-linear time stepping calculations can be avoided. In the present work it is shown that plasticity effects can easily be included together with creep effects and therefore establish, approximately, the shakedown load in the presence of creep. The whole procedure can be added to an existing linear finite element program.

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