ON SOME TYPES OF DEPENDENCE IN PSA OF NUCLEAR POWER PLANTS
AND THEIR QUANTIFICATION

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Abstract

Some types of dependence between component failures considered in the probabilistic safety assessment of nuclear power plants are discussed, namely conditional stochastic and SoK (state of knowledge) dependences, as well as common cause models. A couple of ways to quantify them are proposed, mainly in terms of fragility curves.

1 Introduction

The numerous studies on the PSA (probabilistic safety assessment) of NPPs (nuclear power plants) that have been performed over the last decade have revealed the importance of treating correctly the dependencies between the component failure events and/or between the uncertain quantities describing the stochastic behaviour of the components in a NPP [1,2,3]. According to [4], the various kinds of dependence that have been identified frequently cause a conceptual confusion as to what exactly they represent. The CCP (common cause failure) type of dependence cannot cover all types of dependence arising in the PSA of NPPs.

A clear distinction is made by Apostolakis in [4] between the stochastic and state-of-knowledge (SoK) dependences. We have also considered it in [5] and we come back with some extensions in the next section of this paper. The concept of fragility has been introduced and rather widely used in the PSA of NPPs [6,7,8,9]. When formulated in the so-called 'double lognormal format', it allows to treat dependences in terms of variabilities due to both randomness and uncertainty. We propose, in Section 3, a method to evaluate failure fractions associated to accident sequences in terms of fragility models which differs from the ones of [9,10] in what regards the way the compound variabilities are derived. Finally, we try to establish some connections between the previous dependence models and concepts as coherent / incoherent failure, degrees of redundancy (a.o.) introduced by Dörre [11].

2 Stochastic and SoK Dependences

In a rather general approach to the evaluation of risk and
reliability assessment for various structural or technological systems of [12], Apostolakis & Kaplan consider functions of the form

\[ Q = q(q_1, q_2, \ldots, q_N) \quad (1) \]

in which \( Q \) may denote the unavailability of a system, while \( q_i \) (\( 1 \leq i \leq N \)) may be component unavailabilities / failure rates. In the context of PSAs for NPPs, \( Q \) may be the occurrence frequency of a particular accident sequence, in which case the \( q_i \)'s would represent the frequencies of the (component) event failures comprising the sequence. The function \( q \) is derived from the structure of the system or sequence, e.g. from a fault tree. However, it could be noticed that such a quite general functional model as described by Eq. (1) would hardly give account for possible dependences (of causal nature) between the successive stages of an accident sequence (1*2*...*N). Instead, failure rates would have to be associated to partial accident sequences in a recursive way:

\[ q_{1,2,\ldots,k}^* = g_k(q_1, q_2, \ldots, q_{k-1}) = g_k(q_{1,2,\ldots,k-1}^*, q_k) \quad (2) \]

The system unavailability corresponding to this sequence would then be given by

\[ Q = q_{1,2,\ldots,N}^* \quad (3) \]

In order to define the stochastic independence/dependence between component failures of a system \( S \), indicator variables are used in [4]. If \( A \) is a component of \( S \), then \( X_A = 1 \) if \( A \) is down and it is \( = 0 \) otherwise. The unavailability of \( A \) is defined as the fraction of times (cases) in which component \( A \) is down, that is,

\[ q_A = \text{fr}(X_A = 1) \quad (4) \]

If system \( S \) fails when two components \( A, B \) fail then \( X_S = X_AX_B \) and if \( A \) and \( B \) fail independently then

\[ \text{fr}(X_S = 1|q_A, q_B) = q_A q_B \quad (5) \]

Eq. (5) characterizes conditional stochastic independence. To define the conditional stochastic dependence, unavailabilities of a number \( j \) (\( 0 \leq j \leq N \)) have to be considered:

\[ q_j = \text{the fraction of times in which exactly } j \text{ components fail} \]

In terms of these parameters \( q_j \), the system unavailability is given by

\[ \text{fr}(X_S = 1|q^0, q^1, q^2) = q^2 \quad (6) \]

Eq. (6) applies to redundant systems consisting of nominally identical components. Let us remark that our notations differ from the ones of [4]: subscript \( i \) identifies a component while
superscript \( j \) equals the number of failing components. Let us also notice that Eq.(6) applies to parallel systems only. For a parallel system consisting of \( N \) components, Eq.6 simply extends to

\[
\text{fr}(X_S = 1 | q^0, q^1, \ldots, q^N) = q^N, \quad \sum_{j=0}^{N} q^j = 1. \tag{7}
\]

The failure frequency of such a system with independent component failures can also be expressed in terms of frequencies \( q_i \) by an extension of Eq. (5):

\[
\text{fr}(X_S = 1 | q_1, q_2, \ldots, q_N) = q_1 q_2 \ldots q_N. \tag{8}
\]

Passing now to series systems, Eqs. (7) and (8) turn to

\[
\text{fr}(X_S = 1 | q^0, q^1, \ldots, q^N) = \sum_{j=1}^{N} q^j = 1 - q^0, \tag{9}
\]

\[
\text{fr}(X_S = 1 | q_1, q_2, \ldots, q_N) = \sum q_i - \sum q_{ij} + \sum q_{ijk} + \ldots + (-1)^{n-1} q^N \tag{10}
\]

respectively. Of course, the frequencies with 2, 3 or more subscripts in the r.h.s. of Eq.(10) correspond to simultaneous failures of several components; for instance,

\[
q_{ijk} = \text{fr}(X_i X_j X_k = 1), \tag{11}
\]

but all these multiple frequencies are = 0 in the case when the component failures are mutually exclusive. The connection between the failure fractions for individual components (subscripted \( q \)'s) and fractions of numbers of failures (superscripted \( q \)'s) is given by

\[
q^k = \sum_{\substack{1 \leq i_1 < \ldots < i_k \leq N \\text{for } 1 \neq i_m}} \text{fr}(X_{i_1} X_{i_2} \ldots X_{i_k} = 1 | X_1 = 0 \text{ for } 1 \neq i_m). \tag{12}
\]

The frequency parameters \( q_i \) and \( q^k \) are further involved in the evaluation of the unconditional unavailabilities of subsystems consisting of \( k \) components, but joint pdfs are needed for this purpose:

\[
\pi(q_{i_1}, q_{i_2}, \ldots, q_{i_k}), \pi(q^0, q^1, \ldots, q^k). \tag{13}
\]

Now, for the model of stochastic independence, the unconditional unavailability (or failure probability) of component \( i \) and of the system \( S \) are given by

\[
p_i = \int \ldots \int q_i \pi(q_{i_1}, \ldots, q_{i_k}) dq_{i_1} \ldots dq_{i_k}, \tag{14}
\]
\[ Q_S = \int \cdots \int q_{i_1} q_{i_2} \cdots q_{i_k} \pi(q_{i_1}, \ldots, q_{i_k}) dq_{i_1} \cdots dq_{i_k}. \]  

(15)

For the model of stochastic dependence, the same unavailabilities are given by

\[ p_i = \int \cdots \int (q^1 + q^2 + \cdots + q^k) \pi(q^0, q^1, \ldots, q^k) dq^0 \cdots dq^k, \]  

(16)

\[ Q_S = \int \cdots \int q^k \pi(q^0, q^1, \ldots, q^k) dq^0 \cdots dq^k. \]  

(17)

The integrals in Eqs. (14) thru (17) are \( k \)-fold, for \( 1 \leq k \leq N \); \( k \) is the number of potentially failing components in a subsystem or in an accident sequence.

At the \( \Sigma \Pi \) level, the complete dependence between two nominally identical components \( A, B \) is characterized in \([4]\) by

\[ \pi(q_A, q_B) = \pi(q_A) \delta(q_A - q_B), \]  

(18)

where \( \delta \) is Dirac's delta-function. With the pdf of Eq. (18), the unconditional unavailabilities \( p_i \), \( Q_S \) become

\[ p_i = \int q_A \pi(q_A) dq_A \quad (i = A, B), \quad Q_S = \int q_A^2 \pi(q_A) dq_A \]  

(19)

respectively. This model may be extended for any number \( k (k \geq 2) \) of components among which only \( m \) \((m < k)\) are nominally distinct. That is, there are \( k_j \) components identical to \( A_j \) \((j = 1, \ldots, m)\) with the common pdf \( \pi(q_j) \) and \( k_1 + \cdots + k_m = k \). Then Eq. (15) will turn to

\[ Q_S = \int \cdots \int q_1^{k_1} q_2^{k_2} \cdots q_m^{k_m} \pi(q_1, q_2, \ldots, q_m) dq_1 dq_2 \cdots dq_m \]  

(20)

in which the integral is \( m \)-fold and \( \pi(q_1, q_2, \ldots, q_m) \) is a joint pdf. In the case of \( \Sigma \Pi \) independence between the distinct components, \( Q_S \) will be given by

\[ Q_S = \prod_{j=1}^{m} \left[ \int q_j^{k_j} \pi(q_j) dq_j \right]. \]  

(21)

3 Dependence in Fragility Models

As already mentioned in the Introduction, the notion of fragility has been rather widely accepted in studies of PSA for NPPs, but the ways it is presented sometimes differ from one source to another. According to Ravindra & Tiong [8], fragility means the seismic vulnerability of a structure or equipment as a probability of failure conditional on a ground motion parameter (e.g. PGA). The component fragility is modeled in the form

\[ A = A_m e_R e_U \]  

(22)

where \( A \) is the ground motion (PGA) capacity, \( A_m \) is the median ground acceleration capacity and \( e_R, e_U \) are random variables.
with unit medians representing the inherent randomness about the median and the uncertainty in the median estimate, respectively. If it is assumed that both $e_R$ and $e_U$ are lognormally distributed with logarithmic standard deviations $\beta_R$ and $\beta_U$, then a fragility model may be formulated in terms of a pair of functions (as in [10]), namely a fragility curve $F(A)$ and a pdf $f(C)$, where $C$ is the median of $A$. But we shall associate such a model with each component $i$ ($1 \leq i \leq N$) of a system or an accident sequence by using subscripts:

$$F_i(A) = \Phi \left[ \frac{\ln(A/C_i)}{\beta_{Ri}} \right],$$

$$f_i(C) = \frac{1}{\sqrt{2\pi} \beta_{Ui} C} \exp \left[ -\frac{1}{2} \left( \frac{\ln(C/A_{Mi})}{\beta_{Ui}} \right)^2 \right]$$

where $C_i$ = median PGA capacity of component $i$ given by $F_i(C_i) = \frac{1}{2}$ with the median value $A_{Mi}$ and Log-N standard deviation $\beta_{Ui}$, while $\beta_{Ri}$ is the Log-N deviation of capacity $A$ for component $i$. The fragility model represented by Eqs. (23) and (24) is entirely determined by the 3-tuple of parameters $(A_{Mi}, \beta_{Ri}, \beta_{Ui})$.

The problem is now to formulate a procedure for deriving a compound fragility model for each possible accident sequence of a system $S$, modeled in terms of cutsets. Let $s = (1*2*...*i*...*k)$ be an accident sequence. For a component $i$ ($1 \leq i \leq k$) the composite variability is given (according to [6,7,8]) by $\beta_c = (\beta_R^2 + \beta_U^2)^{\frac{1}{2}}$ and the HCLPF (high confidence low probability of failure value) by $HCLPF = A_{Mi} \exp[-1.645(\beta_R + \beta_U)]$.

We propose a procedure to evaluate compound fragility curves associated to the successive stages of an accident sequence $s$ as follows. A common value is accepted for $\beta_{Ri}$ and it is assumed that the medians $A_{Mi}$ can be evaluated from statistical evidence and/or by engineering judgement. The compound variability $\beta_{C_i}^*$ and the compound HCLPF value for stage $i$ of sequence $s$ are recursively evaluated by

$$\beta_{C_i}^* = (\beta_{C_{i-1}}^* - \beta_{Ui}^2)^{\frac{1}{2}}, \quad HCLPF_i^* = A_{Mi} \exp[-1.645(\beta_{C_i}^* + \beta_{Ui})]$$

and a failure probability may be associated to a subsequence of $s$ by

$$P_f(1*2*...*i) = \left[ \frac{\ln(HCLPF_i^*/A_{Mi})}{\beta_{C_i}^*} \right] \text{ for } 1 \leq i \leq k.$$
ones due to Yamaguchi et al [10], but the latter are formulated for single components and not for sequences.

As regards the concepts and models developed by Dörre in [11], we limit ourselves to the remark that the generalized binomial distribution model appears to be applicable for modeling stochastic and SoK types of dependence as considered in [4,5]. It's also compatible with Kaplan's DPD Method.

Conclusions

Existing models for some types of dependence in the PSA of NPPs are discussed and extended. A procedure for building compound fragilities associated with accident sequences in proposed.

References