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LIFE EXTENSION SIMULATION OF AGED RPV MATERIAL USING PROBABILISTIC FRACTURE MECHANICS ANALYSIS ON MASSIVELY PARALLEL COMPUTER

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Abstract

This paper describes a probabilistic fracture mechanics (PFM) computer program based on the parallel Monte Carlo (MC) algorithm. In the stratified MC algorithm, a sampling space of probabilistic variables such as fracture toughness value, the depth and aspect ratio of an initial semi-elliptical surface crack is divided into a number of small cells. Fatigue crack growth simulations and failure judgements of those samples are performed cell by cell in parallel. The developed PFM program is implemented on a massively parallel computer composed of 512 processors, combined with a function of dynamic workload balancing. As an example, some life extension simulations of aged RPV material are performed, taking analysis conditions of normal and upset operations of PWRs. The results show that cumulative breakage probabilities of the analyzed model are of an order of 10^{-7} (1/crack) and that parallel performance always exceeds 90%. It is also demonstrated that degradation of fracture toughness values due to neutron irradiation and probabilistic variation of fracture toughness values significantly influence failure probabilities.

1. Introduction

The studies on the efficient utilization and life extension of operating nuclear power plants have become increasingly important since the ages of the first-generation nuclear power plants are approaching their design lives. It is easy to imagine that a practical life of the plant might be usually longer than its design life by considering conservatism embedded in design practices. In order to predict a remaining life of the plant, it is necessary to select those critical components that strongly influence the plant life, and to evaluate their remaining lives by considering aging effects of materials and other factors (Akiyama, et al. 1991, Yagawa, et al. 1992).

However, when evaluating the reliability of nuclear structural components, some problems are quite formidable because of the lack of information regarding a past operating history, material property change and uncertainty in damage models. Accordingly, if structural integrity and safety are evaluated by the deterministic fracture mechanics approach, it is expected that the results obtained are too conservative to perform a rational evaluation of plant life and to make judgement of life extension because of the accumulation of conservatisms of all related factors.

Probabilistic Fracture Mechanics (PFM) has become an important tool (Bloom, 1984, Yagawa, 1988). The PFM approach is regarded as an appropriate method to rationally evaluate plant life since a number of uncertainties such as sizes and distributions of cracks, degradation of material strength due to aging effects, accuracy and frequency of pre- and in-service inspections are considered in this analysis.

As for PFM analyses, some problems related to computational accuracy and efficiency still remain to be solved or improved. Firstly, compared with deterministic evaluation approaches, a fracture mechanics model used in the PFM analysis, which is mostly based on the Linear Elastic Fracture Mechanics (LEFM), is rather simple, and not sufficient to evaluate nuclear structural components made of highly ductile materials. Secondly, when the Monte Carlo algorithm is used to evaluate the structural reliability of nuclear components with extremely low failure probabilities, a large number of samples have to be taken in order to achieve high computational accuracy. This will result in a very high computational cost. Several improved methods have been proposed so far, but not always satisfactory owing to their intrinsic features (Becker, Pedersen, 1974, Harris, et al. 1984, Ye et al., 1991). For a better evaluation of practical problems, it is necessary to increase the number of probabilistic variables. The latter requirement also arises new questions on computational efficiency. Thus, an efficient calculation of PFM problems with extremely low failure probabilities and with a large number of probabilistic variables is one of the most critical issues.

Owing to a dramatic progress of computer technology, the recent introduction of parallel computers has generated new challenges in the computational mechanics. The parallel computers may increase a computational speed of the time-consuming PFM analyses.

In this study, a new PFM computer program based on the parallel Monte Carlo algorithm is developed, and implemented on a massively parallel computer composed of up to 512 processors. Through some evaluations of failure probabilities of an aged RPV material, the parallel efficiency of the present PFM program is investigated, and finally some life extension simulations of the aged RPV material are performed.

2. Outline of PFM Analysis

Figure 1 illustrates the flow of a usual PFM analysis (Harris et al., 1984). Firstly, some probabilistic variables are selected according to an analysis model employed. In general, probabilistic variables to be considered include initial crack sizes (crack depth and crack aspect ratio), accuracy and frequency of non-destructive tests, material properties, cycles and amplitudes of applied loads.

Next, crack growth simulations are performed. Fracture mechanics models employed in the present PFM program are based on the linear elastic fracture mechanics (i.e. Newman-Raju solutions (Newman and Raju, 1984), the nonlinear fracture mechanics (i.e. fully plastic solutions (Yagawa et al., 1993) or their combination. Besides, creep crack growth can be simulated based on the nonlinear fracture mechanics (Yagawa et al., 1989).

During the crack growth simulation, pre- and in-service inspections are considered, and failure judgements of leakage and breakage are performed. Failure probabilities are calculated as functions of operation time.

3. Stratified Sampling Monte Carlo Algorithm

When the Monte Carlo algorithm is used to evaluate the structural reliability of nuclear components with very low failure probabilities, a large number of samples have to be taken in order to achieve high computational accuracy. This will lead to a very high computational cost. Several methods have been proposed so for. The Stratified sampling Monte Carlo (SMC) algorithm is one of such attempts. Figure 2 illustrates a typical stratification of an initial crack sampling space composed of a non-dimensional depth a/t and aspect ratio a/c of a semi-elliptical surface crack, where t is a plate thickness, a is a crack length and c is a half of crack length. This sampling space is divided into a number of mutually exclusive small subspaces, named cells. A pre-determined number of samples are then taken from those cells. Within each cell, an individual sampling is carried out according to a postulated initial crack distribution. The failure probability up to time t is calculated by the follow equation (Harris et al., 1984) :

$$P_f(T \leq t) = \sum_{j}^{m} \frac{N_f(t)}{N_j} P_j \qquad (1)$$

where m is the total number of cells, Nj is the number of samples taken from the j-th cell, Nf (t) is the number of failed samples in the j-th cell up to time t, Pj is the probability that an initial crack exists in the j-th cell.

As shown in Fig.2, the crack samples located in the upper portions of the sampling space seem obviously more likely to fail than the crack samples in its lower portions. A region of uncertainty may exist between failure and non-failure regions. Since the samples taken from these non-failure regions would never lead to failure, a considerable computational improvement can be obtained by ignoring those cells in the sampling plan.

However, since the classification of these regions cannot be known before calculation. Many cells and samples would be selected from the whole sampling space to assure computational accuracy. Computational time would increase in accordance with the number of cells and samples. To overcome this problem, the following parallel processing algorithm based on the SMC method is proposed here.

4. Parallel Processing Technique

Parallel processing is a general name for those computational methods in which multiple processors operate simultaneously for increasing a calculation speed. The following measures are often utilized to evaluate the efficiency of parallel processing techniques composed of n processors :

$$\text{Parallel performance } P_n(\%) = T_1 / (T_n \times n) \times 100 \qquad (2a)$$

$$\text{Speedup } S_n = T_1 / T_n \qquad (2b)$$

where T is a computational time and the subscript denotes the number of processors employed.

Ideally, the processing speedup of using n processors is expected to be n times as much as one processor, and parallel performance reaches 100%. However, in reality, the efficiency of a parallel processing system is inevitably reduced due to the following overheads :

1) data transfer,
2) idling of processor during waiting input data.

To overcome the problems, the following features have to be achieved

1) larger granularity of parallel tasks,
2) well balanced workload distribution.

In order to obtain high parallel performance, it is important to keep workloads balanced well among processors. In usual parallel processing approaches, a physical problem is mapped "statically" on a multiple processor network before starting the calculation with the consideration of workload balance among processors. However, such pre-processing is in general troublesome in the PFM analysis, since computation times for cells are usually different from each other.

In the present study, the parallel processing algorithm with a function of dynamic workload balancing is developed. As shown in Fig.3, the computer system employed includes a parent processor and a number of children processors, which are called network processors. The parent processor controls and renews the calculation data in each cell, while the children processors perform the PFM analyses. The roles of the parent processor and the children processors are described as follows.

Firstly, the parent processor reads a whole calculation data, and divides the sampling space into many independent small cells. The data of each cell required to the calculation of a fracture mechanics model, such as the cell number, the cell size, the number of samples and an initial value of generating random numbers, are sent to the network whenever confirming that any idling processor exists in the network. At the same time, the parent processor receives the analysis results, i.e. the numbers of breakage and leakage samples, calculated by the children processors. After completing the calculation of all cells, the cumulative breakage and leakage probabilities are evaluated as functions of operation years. Each child processor continuously monitors the state of the execution of the PFM analysis and the cell data flowing in the network. If any child processor is idling, it receives the cell data from the network. After the calculation, the results are sent back to the network.

Owing to this kind of data transfer mechanism, a whole workload of the PFM analysis is well distributed dynamically among children processors. Besides, it can be estimated that the overhand due to data transfer is negligibly small because only a small amount of data, i.e. cell number, cell size, the number of samples, and the number of failure samples are transferred among processors.

The present parallel PFM algorithm is implemented on a massively parallel computer, which is a kind of Multiple Instruction-stream Multiple Data-stream (MIMD) type computer composed of one host processor and 512 network processors (i.e. children processors). Its peak computation speed is 4.2 GFLOPS (Giga FLOating point Operations Per Seconds). The host computer is one of popular engineering workstations.

5. Analysis Problem

To investigate computational efficiency and accuracy of the present PFM computer program based on the parallel SMC algorithm, it is applied to evaluate failure probabilities of a beltline portion in RPV, which is set up referring one of the round-robin problem given by the LE (Life Evaluation)-PFM subcommittee in the Japan Welding Engineering Society (Yagawa et al., 1992).

5.1 Analysis Model

A plate with a semi-elliptical surface crack as shown in Fig.4 is analyzed. Its thickness and width are taken to be $t = 0.2$ m and $2b = 12.6$ m, considering the beltline portion of PWR pressure vessels. The plate is assumed to be subjected to various magnitudes of remote uniform tensile and bending stresses. For the purpose of simplicity, we ignore curvature effects of actual pressure vessels in evaluating a three dimensional stress intensity factor from Newman-Raju's solutions (Newman and Raju, 1984) ($c/a > 0.5$ and $a/t < 0.9$), where a is the crack depth and c is the half of the crack length.

Cumulative failure probabilities of one existing crack, whose unit is 1/crack, are calculated as functions of operation years.

5.2 Cyclic Loads

Nineteen kinds of cyclic tensile and bending stresses given in Table 1 are chosen from the design loading conditions of Level A (normal operation condition) and Level B (upset condition) which occur in the beltline portion of PWR pressure vessels listed in the Marshall report (Marshall, 1982). The nineteen loads are applied as listed in the table.

5.3 Failure Criteria

The failure modes considered here are leakage and breakage, whose criteria are simply defined as follows :

Breakage criterion : $K_{max} \geq K_{Ic}$ or $b/c \leq 1$ (3a)

Leakage criterion : $a / t \geq 0.8$ (3b)

where K_{max} is taken to be the larger one of the K values at either the surface or the deepest points of the semi-elliptical surface crack. For simplicity, breakage phenomenon after leakage is not considered. The critical value of a/t = 0.8 in the above leakage criterion is conservatively selected by considering the crack extension from the back-side surface.

5.4 Fatigue Crack Growth

For fatigue crack growth, the Paris' law is used, whose coefficients are taken from the fatigue crack growth rate of nuclear pressure vessel steels in water given in the ASME Code Section XI, Appendix A (ASME, 1973) :

$$da/dN \text{ (m/cycle)} = 1.738 \times 10^{-13} (\Delta K)^{5.95} \quad (\text{for } \Delta K < 13.2 \text{ MPa}\sqrt{m}) \tag{4a}$$

$$da/dN \text{ (m/cycle)} = 5.325 \times 10^{-9} (\Delta K)^{1.95} \quad (\text{for } \Delta K \geq 13.2 \text{ MPa}\sqrt{m}) \tag{4b}$$

5.5 Distribution of Initial Crack Shapes

An initial crack depth (a) and a crack aspect ratio (c/a) are assumed to be probabilistic variables. Their probabilistic density functions are given as follow (Harris et al., 1984) :

$$P(a) = \frac{\exp(-a/\mu)}{\mu\,(1 - \exp(t/\mu))} \tag{5a}$$

$$P(c/a) = \frac{\alpha}{c/a\,\gamma\,\sqrt{2\pi}} \exp\frac{-\ln^2(c/a/\beta)}{2\gamma^2} \tag{5b}$$

where $\alpha = 1.035$,
$\beta = 1.336$,
$\gamma = 0.5382$,
$\mu = 6.248 \times 10^{-3}$ m.

Cracks of c/a < 0.5 are neglected to avoid exceeding the validity range of Newman-Raju's solutions. Crack shapes are assumed to keep the semi-elliptical shape during the crack growth.

5.6 Fracture Toughness

5.6.1 Time variation of fracture toughness value due to neutron irradiation

Based on the reduction of the upper shelf Charpy absorbed energy due to neutron irradiation (Tipping et al., 1987) and some experimental data by the LE (Life Evaluation) subcommittee (JWES, 1990), the time variation of K_{Ic} at 300°C due to neutron irradiation is formulated as follows :

$$\begin{aligned} 300\,^{\circ}\mathrm{C}:\quad K_{Ic}(t) &= 135.0\ (\mathrm{MPa}\sqrt{\mathrm{m}}) && (\text{for } F \leq 0.361) \\ &= 3.29 + 118.71 \times F^{-0.102} && (\text{for } F > 0.361) \end{aligned} \tag{6}$$

where F is the neutron fluence (10^{19} n/cm^2). The neutron fluence after a 40 year operation is assumed to be 3 x 10^{19} n/cm^2 according to that of the beltline portion of the PWR pressure vessels in the Marshall report (Marshall, 1982). According to Eq. 6, the KIc value reduces by about 19 % due to neutron irradiation during a 40 year operation.

In reality, since the neutron fluence varies along the thickness direction, the neutron fluence F is formulated as a function of the distance d(mm) from the inside surface of plate assumming F0 to be the neutron fluence at the inside surface as (NRC, 1988) :

$$F = F0 \times \exp(-0.00945 \times d) \tag{7}$$

According to Eq. 7, the neutron fluence at the outside surface is about 15 % of that of the inside surface.

5.6.2 Statistical distribution of KIc

In practice, fracture toughness values are statistically distributed. It is often assumed that the fracture toughness value is of a normal distribution, and that its standard deviation is proportional to a mean value (Cheverton and Ball, 1984).

6. Results and Discussions

6.1 Parallel Efficiency

At first, the parallel performance and the speedup of the present parallel PFM algorithm are investigated by varying the number of processors employed. The total number of cells is set to be 960, being equal to the product of 12 cells for a/t, 5 cells for a/c and 16 cells for KIc. The number of samples per cell is chosen to be 160. The sampling conditions are proved to assure sufficient solution accuracy.

Table 2 shows the parallel performance and the speedup of the present PFM algorithm measured on the massively parallel computer. It can be seen from the table that parallel performance always exceeds 90%, and that speedup reaches 479.7 when using 512 processors. It is clearly demonstrated from these results that the total workload is well distributed among processors during the calculation. As a result, an extremely high computation speed of 3.9 GFLOPS is achieved in the present PFM analysis, while the peak speed of the massively parallel computer employed here is 4.2 GFLOPS. A better reliable evaluation is expected by using this parallel PFM algorithm when the number of probabilistic variables increase more.

6.2 Aging Effects vs. Failure Probabilities

In order to investigate the aging effects of the fracture toughness KIc on breakage probabilities, the following two cases are assumed.

1) According to Eq. 6, the KIc value decreases about 20% due to neutron irradiation during a 40-year operation. The neutron fluence is assumed to be uniformly distributed along the thickness direction.
2) According to Eqs. 6 and 7, the plate is non-uniformly irradiated along the thickness direction.

The breakage probabilities during a 40-year operation are given in Fig. 5. The figure clearly shows that the breakage probability strongly depends on the magnitude of neutron irradiation, and that the consideration of non-uniform irradiation along the thickness direction results in less breakage probability.

6.3 Probabilistic Fracture Toughness vs. Failure Probabilities

Here the effect of a probabilistic distibution of KIc is investigated assuming a normal distribution as described in Sec. 5.6. Figure 6 shows breakage probabilities after a 40-year operation plotted against the standard deviation of KIc distribution. In accordance with the increase of the standard deviation of the KIc, the breakage probability increases. The breakage probability after a 40-year operation is by one order larger in the case of the standard deviation of 20% of the mean KIc value than in the case of no probabilistic distribution in KIc.

6.4 Life Extension Simulations

Simulations of life extension are carried out based on the following PFM analyses. At first we evaluate a breakage probability after a design life, say, 40 years, by using design values of material properties and operating conditions. The calculated value is regarded as a critical value of the breakage probability at a plant operating limit. On the other hand, some data collected after several years of operation are usually different from the design data. Then, the evaluation of failure probabilities using this data would lead to values which are different from the above simulation results based on the design data. Such difference would give us a certain rational foundation for life extension. In other words, the difference between the design data and the measured data influences the degree of life extension significantly in the present simulation.

Design scenario
The KIc value is given by Eq. 6, in which the total neutron fluence is assumed to be 3 x 10^{19} n/cm^2 during a 40-year operation. We suppose that at the designing stage, an operation life is determined to be 40 years under the operating conditions shown in Table 1. The breakage probability after a 40-year operation corresponding to this scenario is regarded as a criterion for life extension judgement.

Scenario 1 It is assumed that the measured value of the total neutron fluence is half of the design value, i.e. 1.5 x 10^{19} n/cm^2 .

Scenario 2 A probabilistic distribution of KIc with a standard deviation of 15% of its mean value is assumed.

Figure 7 shows the time variations of the cumulative breakage probabilities obtained in the three scenarios. It can be seen from the figure that the critical breakage probability is about 8.0x10^{-7}. The comparison between the design scenario and scenario 1 shows that the reduction of neutron fluence from 3.0 x 10^{19} to 1.5 x 10^{19} n/cm^2 results in great difference in breakage probability, and suggests the possibility of plant life extension. The comparison between the design scenario and scenario 2 shows that the probabilistic distribution of the KIc value results in the increase of breakage probability, and suggests the possibility of plant life shortening.

7. Conclusions

In the present study, a new method for the probabilistic fracture mechanics analysis based on the parallel Stratified Monte Carlo algorithm is developed. The developed PFM computer program is implemented on a massively parallel computer composed of 512 processors. The parallel performance of the present program is shown to exceed 90% in practical PFM analyses.

The effects of neutron irradiation on the cumulative breakage probabilities are investigated using the program. It is found that the cumulative breakage probability increases by about one order due to the 20% decrease of KIc caused by the neutron irradiation during a 40-year operation.

The effects of the probabilistic distribution of fracture toughness on the cumulative breakage probability are also investigated. It is clarified that the cumulative breakage probability increases due to the probabilisitc distribution of KIc value.

Finally, focusing on the difference between the design data and the measured data on neutron fluence, some simulations for life extension are performed. These simulations demonstrate the effectiveness of the PFM analyses to the life extension evaluation.

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Table 1 Loading conditions

Load No.		Design transient	Applied stress (MPa)				Cycles for 40 years
			Tension		Bending		
			$\sigma_{t,max}$	$\sigma_{t,min}$	$\sigma_{b,max}$	$\sigma_{b,min}$	
Level A	1/2	Heatup/cooldown	160.9	0.0	8.0	0.0	200
	3	Unit loading/unloading	159.2	160.6	11.3	5.6	500
	4/5	Plant loading/unloading	160.7	158.0	11.3	8.3	13,200
	6/7	Step load change of 10%	164.4	157.6	5.1	5.6	2,000
	8	Steam dump	167.5	150.5	4.7	6.4	200
	9a	Steady state fluctuations(A)	160.9	158.8	8.0	5.8	$1.5\text{x}10^5$
	9b	Steady state fluctuations(B)	161.3	160.5	8.0	7.9	$3\text{x}10^6$
	10	Feedwater cycling	154.9	162.9	16.4	1.2	2,000
	11/12	Loop out of service	167.6	160.4	6.2	6.1	80
Level B	15	Loss of load	187.7	131.7	5.7	- 7.6	80
	16	Loss of power	176.0	132.1	4.5	9.9	40
	17	Partial loss of flow	164.7	138.2	6.0	4.7	80
	18a	Reactor trip A	160.9	142.7	8.0	5.6	230
	18b	Reactor trip B	160.9	114.5	8.0	17.1	160
	18c	Reactor trip C	160.9	114.5	8.0	11.0	10
	19	Inadvertent depressurization	160.9	9.3	8.0	52.6	20
	20	Inadvertent startup	167.5	142.1	10.3	8.3	10
	21	Control rod drop	160.9	131.0	8.0	9.8	80
	22	Inadvertent safety injection	164.0	143.7	13.1	4.6	60

Table 2 Parallel performance, speedup vs. no. of processors

No. of processors (n)	1	8	32	64	128	512
Performance (%)	-	99.8	99.6	99.6	99.0	93.7
Speedup	-	8.0	31.9	63.7	126.7	479.7

No. of cells = 5 for a/c x 12 for a/t x 16 for K_{Ic} = 960
No. of samples = 160 samples / cell x 960 cells = 153,600

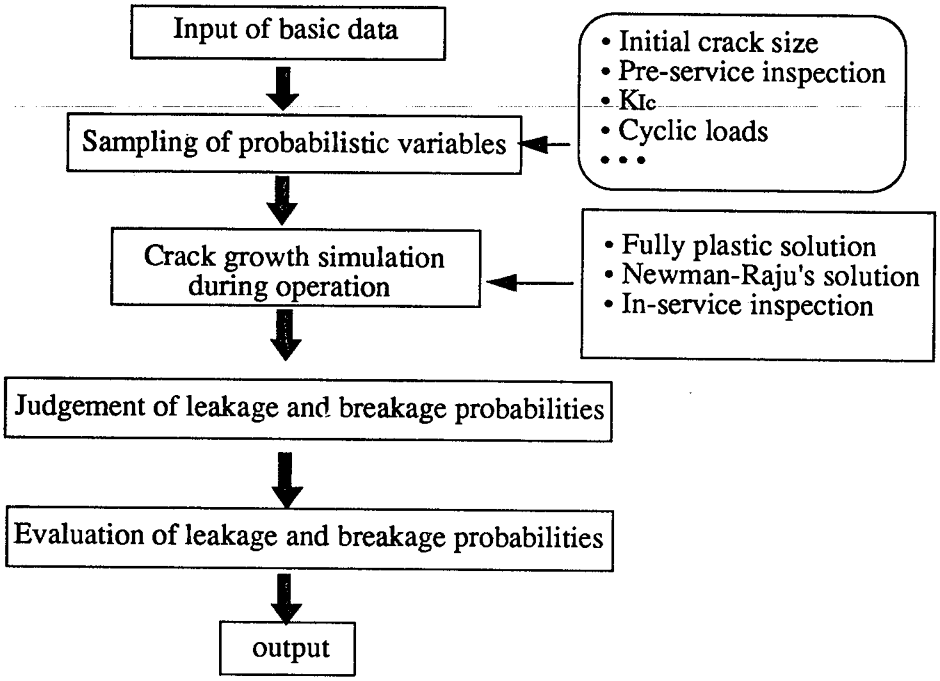


Fig.1 Flow of PFM analysis

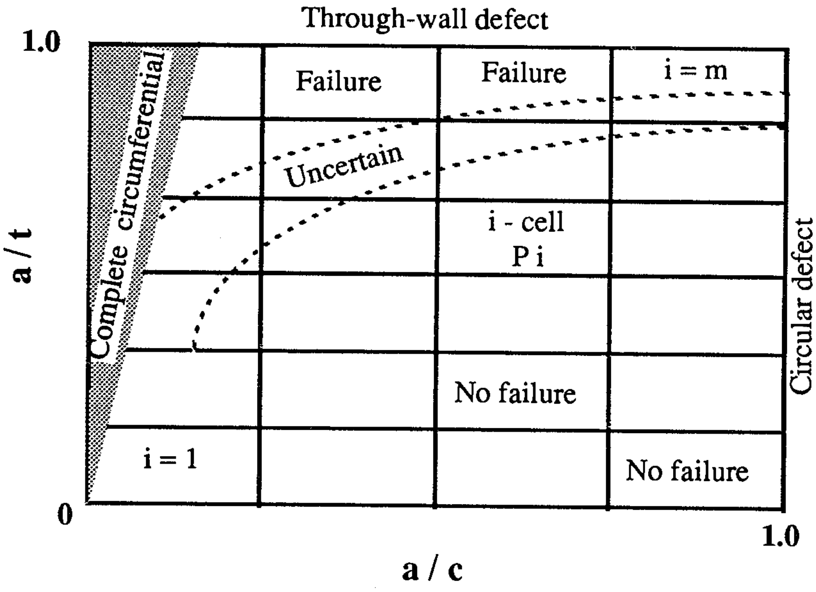


Fig. 2 Stratification of sampling space

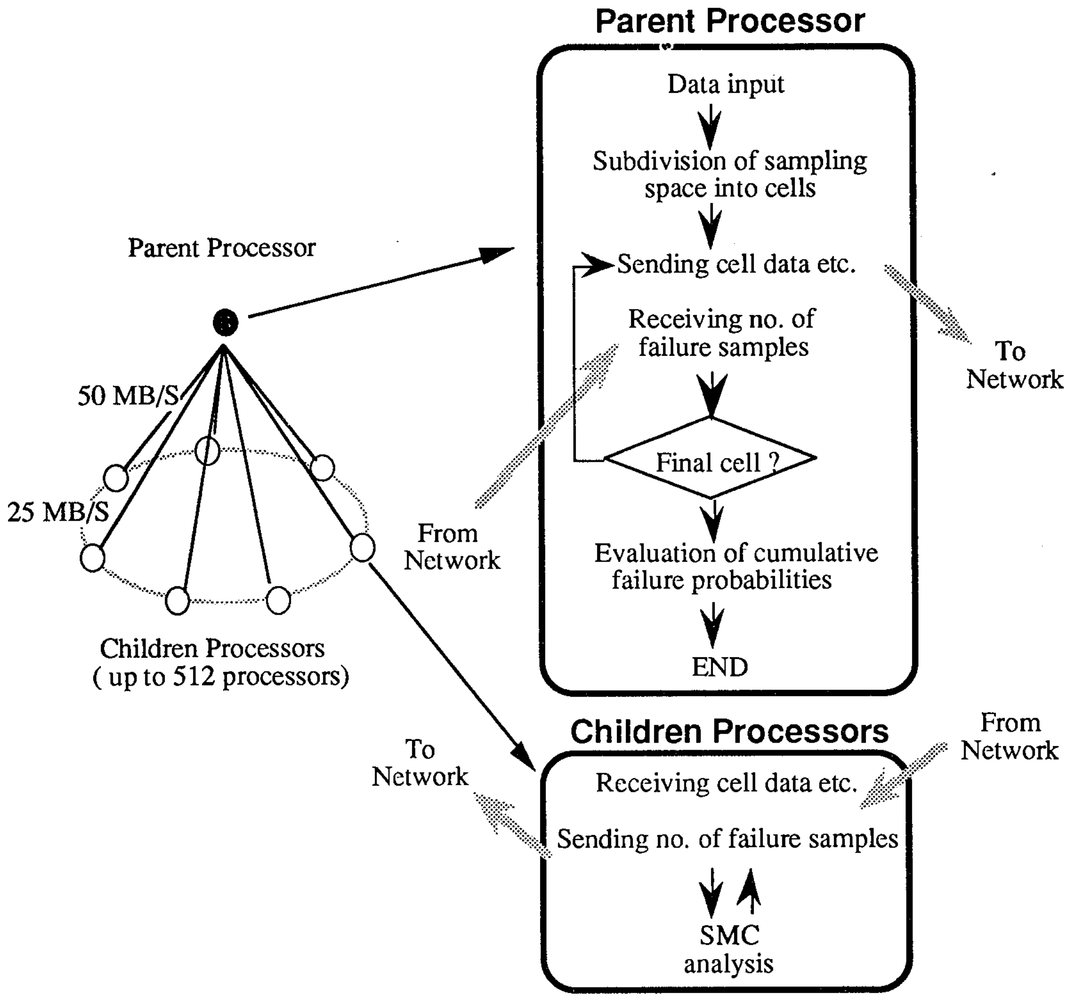
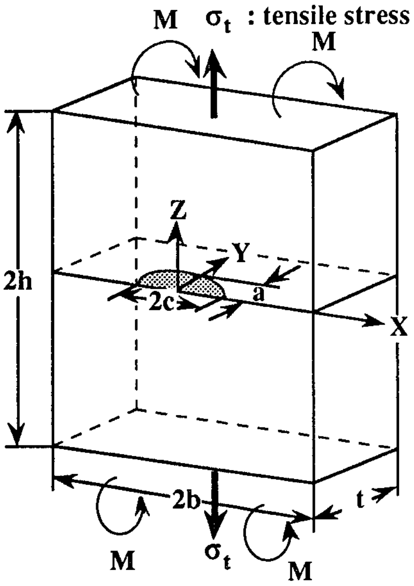


Fig. 3 Schematic view of the parallel PFM algorithm implemented on a massively parallel computer



M: bending moment

$\sigma_t = 3M / bt^2$: bending stress

Fig. 4 Plate with a semi-elliptical surface crack subjected to tensile and bending stresses

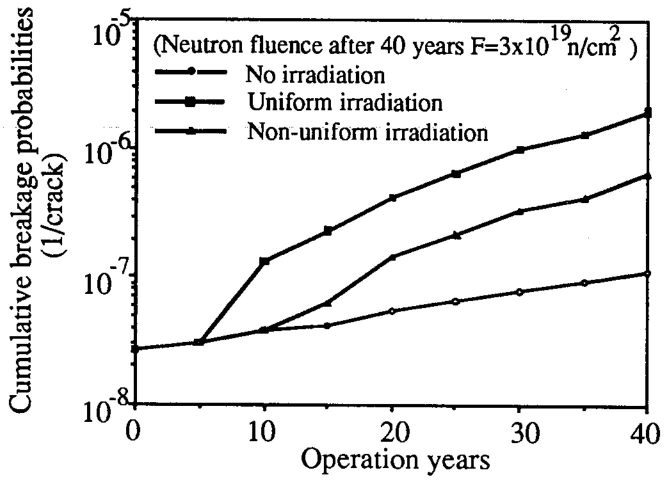


Fig. 5 Time variations of cumulative breakage probabilities for three different aging scenarios

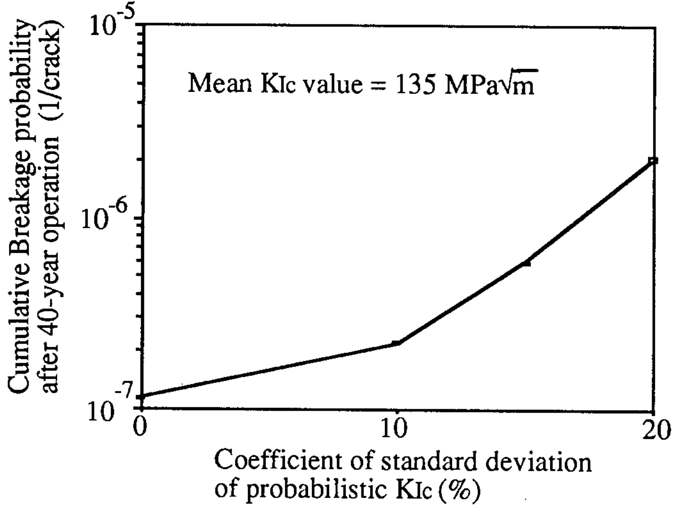


Fig. 6 Cumulative breakage probabilities after a 40-year operation vs. standand deviation of KIc value

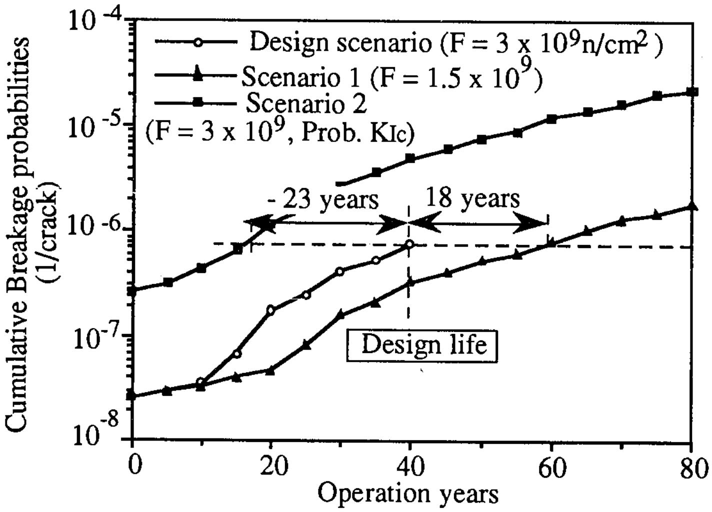


Fig. 7 Time variations of cumulative breakage probabilities for three operation scenarios