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A STRUCTURAL RELIABILITY-BASED NON-DESTRUCTIVE INSPECTION GUIDELINE

W.-F. Wu[1] and J.-J. Syau[2]

[1]Department of Mechanical Engineering, National Taiwan University, Taipei, Taiwan, R.O.C.
[2]Materials Research Laboratories, Industrial Technology Research Institute, Hsinchu, Taiwan, R.O.C.

ABSTRACT

To assure the safety and reliability of mechanical components and structures, in-service non-destructive inspection (NDI) is frequently employed in the nuclear industry. In the present paper, the effect of NDI on the structural reliability is studied within the content of probabilistic fracture mechanics. Special attention is given to the influence of inspection interval on the fatigue crack size distribution and the failure probability of the structure. Using the proposed analytical method, an optimal inspection guideline can be made which results in the least cost structural maintenance within a minimally required reliability assurance.

1. INTRODUCTION

Fatigue reliability has been studied extensively by many researchers. Among them, Yang and his co-workers have studied the fatigue reliability of aeronautical structural components under scheduled periodic inspection maintenance [4,5]. Madsen has studied the influence of loading, fatigue crack growth, and inspection on the failure probability of an ocean structure [2]. Itagagi and Yamamoto have applied Bayesian analysis to study the structural reliability and inspection for ships [1].

In a recent study on the reliability analysis of nuclear pressure vessels, fatigue crack growth was considered the major reason to cause the potential disruptive vessel failure [3]. Since NDT and NDE are frequently applied in nuclear industry to improve the structural reliability, therefore, in the present paper, a fatigue reliability-based non-destructive inspection guideline is studied. Many concepts proposed by Yang and Madsen respectively are adopted in the present analysis.

2. ELEMENTARY CONCEPTS

According to the philosophy of retirement for cause (RFC) used in aeronautical industry, the initial fatigue quality (IFQ) can be represented by the statistical distribution of the time for a structural component to initiate a reference crack size a_0 [5]. This distribution is referred to as time to crack initiation (TTCI). In general, it is assumed to follow a log-normal distribution with parameters μ_T and σ_T.

For a component first put in service, *i.e.* at time $t = 0$, the initial crack size $a(0)$ can be assumed to have a log-normal distribution with parameters μ_1 and σ_1 as follows

$$f_{a(0)}(x) = \frac{\log e}{\sqrt{2\pi}x\sigma_1} \exp\left\{-\frac{1}{2}\left[\frac{\log x - \mu_1}{\sigma_1}\right]^2\right\} \tag{1}$$

The above distribution consists of two groups, namely 'non-defective population' and 'defective population' respectively. The former indicates those cracks smaller than a_0 and the latter represents those larger than a_0. They are denoted respectively as

$$f_{a(0)}(x) = \begin{cases} f_{a(0)}(x;1), & x < a_0 \\ f_{a(0)}(x;2), & x \geq a_0 \end{cases} \tag{2}$$

For the non-defective population, the iucubation period is simply the TTCI which is log-normally distributed as mentioned previously. For defective population as well as those incubated cracks, the fatigue crack size grows following a crack growth equation such as Paris law

$$\frac{da}{dt} = ZQa^b \tag{3}$$

in which Q and b are material constants, and a random parameter Z is added to randomize the crack growth equation. In the present analysis, a log-normal distribution with parameters μ_Z and σ_Z is assumed for the random variable Z.

In considering the non-destructive evaluation (NDE) as well as its capacity in detecting an existing crack, the probability of detection (POD) is frequently used. In the present analysis, the following log odds function is assumed for the POD curve

$$\mathrm{POD}(a) = \frac{\exp(\alpha + \beta \ln a)}{1 + \exp(\alpha + \beta \ln a)}, \qquad 0 < a < \infty \tag{4}$$

in which POD(a) is the probability of detection for a crack size a, and α and β are constants representing respectively the bandwidth and central location of the POD curve.

3. FORMULATION

In the present analysis, a component is assumed to consist of only one critical location and the failure of such a critical location will imply the failure of the entire component. Furthermore, when a crack is detected in such a critical location, the component will be replaced or repaired. In the following derivation, the in-service inspection maintenance is assumed to be periodic and has an identical interval τ. This assumption, however, can be revoked as will be seen later in the discussion of optimal inspection interval.

At the end of the first interval τ, the distribution of crack size consists of non-defective and defective portions. The defective portion $f_{a(\tau)}(x;2)$ is what we are mostly concerned about. It, in turn, consists of (i) the originally defective population denoted now by $f'_{a(\tau)}(x;2)$ and (ii) some of the originally non-defective population denoted now by $f''_{a(\tau)}(x;2)$. Based on the concepts introduced in the preceding section, one can derive that

$$f'_{a(\tau)}(x;2) = \int_0^\infty f_{a(0)}[Y_1(x;\tau,z);2] J_1(x;\tau,z) f_Z(z)\, dz, \qquad x \geq a_0 \tag{5}$$

in which

$$Y_1(x;\tau,z) = \frac{x}{(1 + \tau c Q z x^c)^{\frac{1}{c}}} \tag{6}$$

$$J_1(x;\tau,z) = \frac{1}{(1 + \tau c Q z x^c)^{\frac{1}{c}+1}} \tag{7}$$

and $c = b - 1$.

$$f''_{a(\tau)}(x;2) = q(0)\int_0^{\tau} f_Z[W(x;\tau-t)]I(x;\tau-t)f_T(t)\,dt \\ = q(0)g(x), \qquad x \geq a_0 \tag{8}$$

in which

$$W(x;\tau-t) = [cQ(\tau-t)]^{-1}({a_0}^{-c} - x^{-c}) \tag{9}$$

$$I(x;\tau-t) = [Q(\tau-t)]^{-1}x^{-b} \tag{10}$$

and $q(0) = \int_0^{a_0} f_{a(0)}(x)dx$.

After the defective crack distribution is obtained, the failure probability before inspection denoted by $F(1)$, failure probability after inspection denoted by $\bar{F}(1)$, and the probability for a crack to remain less than a_0, *i.e.* non-defective population, denoted by $q(1)$ can all be calculated by the following formulae at the end of the first interval.

$$F(1) = \int_{a_c}^{\infty} f_{a(\tau)}(x;2)\,dx \tag{11}$$

$$\bar{F}(1) = \int_{a_c}^{\infty} f_{a(\tau)}(x;2)\,[1 - \mathrm{POD}(x)]\;dx \tag{12}$$

$$q(1) = \int_{a_0}^{\infty} f_{a(\tau)}(x;2)\mathrm{POD}(x)dx + q(0)\left[1 - \int_{a_0}^{\infty} g(x)\,dx\right] \tag{13}$$

in which a_c is the critical crack size the structural component can sustain. The quantity inside the square bracket of Eq. (13) indicates the remaining portion of the original non-defective population.

Proceeding with a similar procedure for the second, third, and nth service interval, the following general expressions can be derived for the defective crack distribution at the nth inspection:

$$f''_{a(n\tau)}(x;2) = q(n-1)g(x) \tag{14}$$

$$f'_{a(n\tau)}(x;2) = \int_0^{\infty}\left\{\prod_{m=1}^{n-1}(1 - \mathrm{POD}[A_1(x;m\tau,z)])\right\} f_{a(0)}[Y_1(x;n\tau,z);2]J_1(x;n\tau,z) \\ \times f_Z(z)\,dz + \sum_{i=0}^{n-2} q(i)\int_0^{\infty}\left\{\prod_{m=0}^{n-i-2}(1 - \mathrm{POD}[A_2(x;m\tau,z)])\right\} \\ \times g\left\{Y_2[x;(n-i-1)\tau,z]\right\} J_2[x;(n-i-1)\tau,z]f_Z(z)\,dz, \tag{15}$$

in which

$$Y_1(x;n\tau,z) = \frac{x}{(1+n\tau cQzx^c)^{\frac{1}{c}}}\quad, \qquad Y_1 \geq a_0 \tag{16}$$

$$J_1(x;n\tau,z) = \frac{1}{(1+n\tau cQzx^c)^{\frac{1}{c}+1}} \tag{17}$$

$$A_1(Y_1; m\tau, z) = \frac{Y_1}{(1 - m\tau c Q z Y_1^c)^{\frac{1}{c}}} \tag{18}$$

$$Y_2[x; (n-i-1)\tau, z] = \frac{x}{[1 + (n-i-1)\tau c Q z x^c]^{\frac{1}{c}}} \quad , \qquad Y_2 \geq a_0 \tag{19}$$

$$J_2[x; (n-i-1)\tau, z] = \frac{1}{[1 + (n-i-1)\tau c Q z x^c]^{\frac{1}{c}+1}} \tag{20}$$

$$A_2(Y_2; m\tau, z) = \frac{Y_2}{(1 - m\tau c Q z Y_2^c)^{\frac{1}{c}}} \tag{21}$$

The failure probability before inspection, failure probability after inspection, and the probability for a crack to remain less than a_0 can then be calculated as

$$F(n) = \int_{a_c}^{\infty} f_{a(n\tau)}(x; 2)\, dx \tag{22}$$

$$\bar{F}(n) = \int_{a_c}^{\infty} f_{a(n\tau)}(x; 2)\, [1 - \mathrm{POD}(x)]\, dx \tag{23}$$

$$q(n) = \int_{a_0}^{\infty} f_{a(n\tau)}(x; 2) \mathrm{POD}(x)\, dx + q(n-1) \left[1 - \int_{a_0}^{\infty} g(x)\, dx\right] \tag{24}$$

4. NUMERICAL EXAMPLES

In the present analysis, the reference crack size a_0 is assumed to be 0.084 mm and the critical crack size a_c is 9.400 mm. The initial crack size is assumed to have a median value 0.047 mm and coefficient of variation 79.38%. According to Eq. (1), it indicates that $\mu_1 = \log(0.047) = -1.328$ and $\sigma_1 = \sqrt{\ln(1 + 0.794^2)}/\ln 10 = 0.304$. For the originally non-defective population, the TTCI distribution has median value 4,000 hr and coefficient of variation 95.630% which indicates that $\mu_T = 3.602$ and $\sigma_T = 0.350$. The fatigue crack growth equation is assumed to have parameters $b = 1.046$ and $Q = 6.660 \times 10^{-4}$. The random variable Z has log-normal distribution with $\mu_Z = 0$ and $\sigma_Z = 0.128$. Parameters in the POD curve are chosen to be $\alpha = 0.660$ and $\beta = 3.960$, which results in a curve shown in Fig. 1. For a certain ultrasonic inspection device, experimentally obtained data is also shown in the same figure for comparison.

A typical result for the failure probability of the component is shown in Fig. 2 in which the solid curve represents the situation that no inspection is applied to the component and the dashed curves indicate that periodic inspection is applied to the component. The result shows that if no inspection is applied, the failure probability increases monotonically with time, and periodic inspection does decrease the failure probability from time to time. Under the condition of no inspection at all, the defective crack size distribution at different times are shown in Fig. 3. It is interesting to note that at the early stage, *i.e.* at $t = 1,800$ hr twin peaks are found in the figure which may indicate that equal weights are found for those contributed from defective population and those from non-defective population. Since only defective population is shown in the figure, areas under the curves are not one.

In considering periodic inspection, Figure 4 shows a typical result for the crack size distribution just before an inspection is employed. Curves with similar shape are found for the crack size distribution after the inspection. Since inspection acts as a filter which filter out larger crack sizes in a probabilistic way, it is interesting to find the probability distribution of detected crack size. Figure 5 shows three of such curves which have different shape as compared to those shown in Figs. 1 and 4.

It is obvious that more frequent inspection results in more reliable of a component or structure but, of course, at larger amount of cost. How to make a compromise between the expense and a certain reliability measure becomes an important issue. In the present study, two analyses have been performed. The first analysis used a try and error method to find the optimal non-periodic inspection intervals based on a prescribed failure probability. For example, if the failure probability is set to be less than 10^{-4}, the calculation results in a curve which has similar shape as those shown in Fig. 2 but with five crests, all below the level of 10^{-4}. The second analysis followed a procedure used by Yang and Trapp [4] who have proposed an inspection optimization scheme for aircraft structures based on reliability analysis. The analysis, however, considers periodic inspection only. One of the results in the present study is shown in Fig. 6 in which C_r represents the nondimensional total cost and r indicates a relative ratio between the cost of each inspection and the loss of property resulting from structural failure. The dotted curve at the lower left side labeled by $P(N)$ indicates that failure probability (at a specified time) decreases as the number of inspection increases. The dashed curve represents the least cost curve for different r's without considering the requirement of failure probability. In the present study, if we set failure probability to remain under 10^{-4}, then Fig. 6 tells us that the optimal periodic inspection number (in $9,000\,\mathrm{hr}$) is solely governed by $P(N)$ curve and is found to be five. This result agrees with what we have found previously using the first method.

5. CONCLUSION

Based on the philosophy of retirement for cause introduced in aeronautical industry, the present paper studied the effect of in-service inspection on the improvement of the deteriorated structural reliability owing to fatigue crack growth. Either periodic or non-periodic optimal inspection guidelines can be made through the analytical study. Although many techniques used in the present analysis were developed in the aircraft industry, it is believed that they can also be employed in nuclear industry. For example, in the reliability study of mechanical components or structures such as pressure vessels and piping [3].

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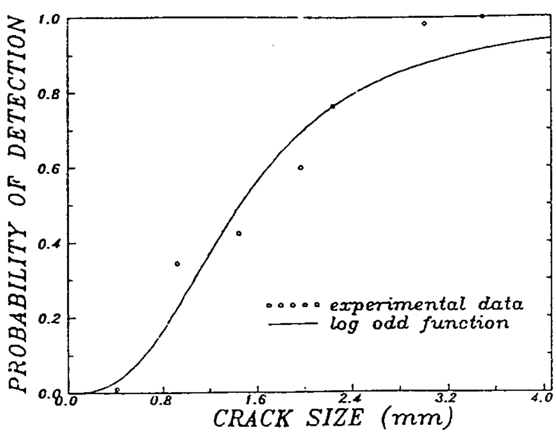

Fig. 1 A POD curve

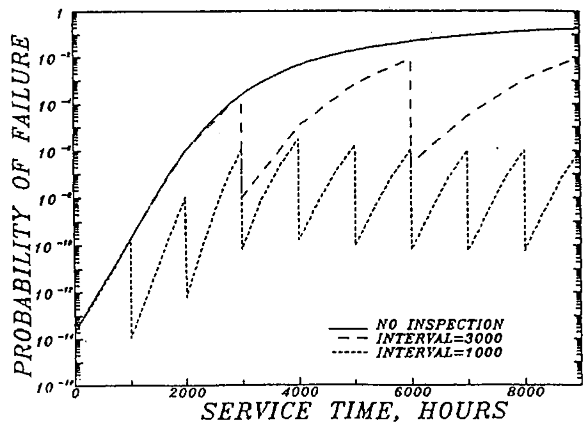

Fig. 2 Failure probability under periodic inspection

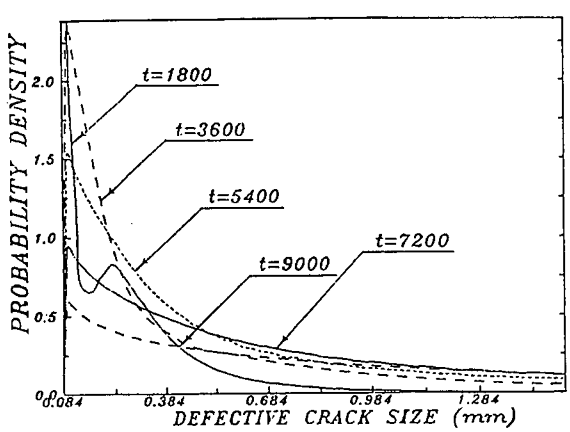

Fig. 3 Distribution of defective crack size (without inspection)

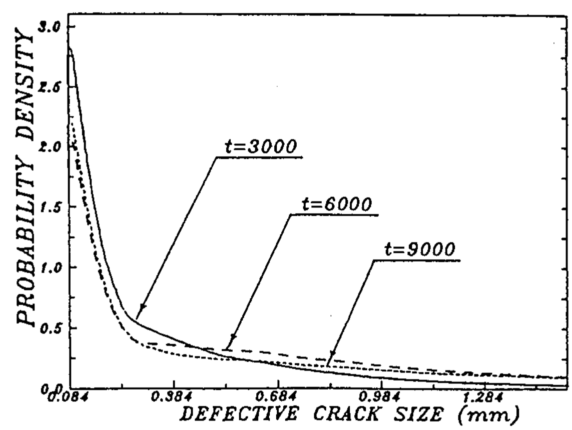

Fig. 4 Distribution of defective crack size (with inspection)

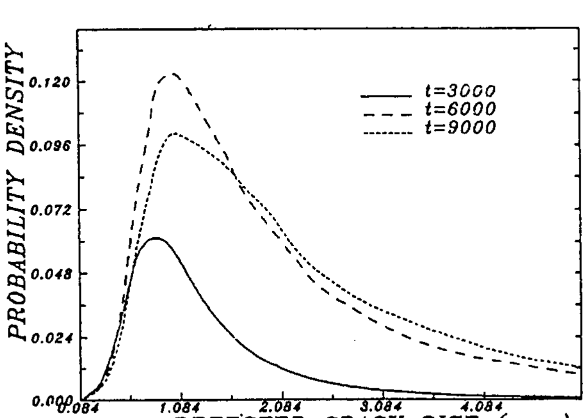

Fig. 5 Distribution of detected crack size

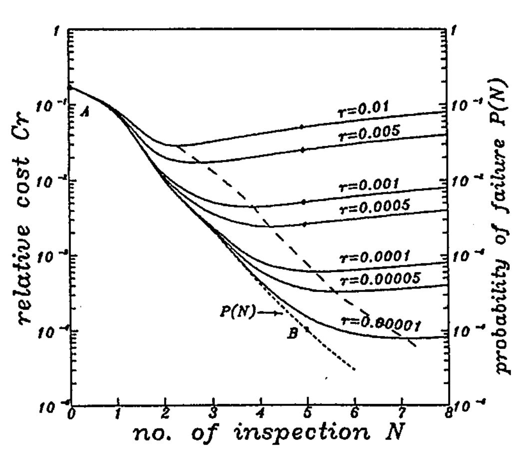

Fig. 6 The least cost number of inspection