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APPLICATION OF A FEW ORTHOGONAL POLYNOMIALS TO THE ASSESSMENT OF THE FRACTURE FAILURE PROBABILITY OF A SPHERICAL TANK

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ABSTRACT

This paper presents some methods to assess the fracture failure probability of a spherical tank. These methods convert the assessment of the fracture failure probability into the calculation of the moment of cracks and a one-dimensional integral. In the paper, we first derive series' formulae to calculation the moments of cracks on the occasion of the crack fatigue growth and the moments of crack opening displacements according to JWES-2805 code. We then use the first n moments of crack opening displacements and a few orthogonal polynomials to compose the probability density function of the crack opening displacement. Lastly, the fracture faiture probability is obstained according to the interference theory. A example proves that these methods are simpler, quicker,and more accurate. At the same time,these methods avoid the disadvantage of Edgeworth's series method.

NOTATION

a:effective size of a surface crack; a_o:initial effective size of a surface crack;C,m: material constants;e:strain;E:Young's modulus;E[•]:expected,or mean,value; $F_x(\cdot)$: probability distribution function of variable x;$f_x(\cdot)$: probability density function of variable x;K_h:coefficient of stress concentration caused by alternate edge;K_t:coefficient of stress concentration;K_w: coefficient of stress concentration caused by angular deformation;N:number of load repetition; P_f: fracture failure probability; P[•]: probability of event[•];Y:geometry factor of a crack; δ:crack opening displacement; δ_c:critical crack opening displacement; ΔK:stress intensity range; $\Delta\sigma$:stress range during cyclic loading; σ_m:membrane stress

1 INTRODUCTION

In general, the calculation of the fracture failure probability is to calculate a multi-dimensional probability integral.Because it is very difficult to accurately calculate this integral,some approximate methods are used to assess the fracture failure probability,such as Direct Monte Carlo method, Crude Monte Carlo method,JC method,and so on[1,2].Monte Carlo method is a simulative method. It often takes a lot of time;JC method is a method in which the failure probability must be obtained by iteration,and it often brings bigger error.Reference [3] utilized Edgeworth's series,which is very complicated,to assess the fracture failure probability of a spherical tank. This paper will present some other methods to assess the fracture failure probability of a spherical tank by a few orthogonal polynomials.These methods convert the assessment of the fracture failure probability into the calculation of the moments of the crack opening displacement and a one-dimensional integral. A example will be used to demonstrate that these methods are simpler, quicker, and more accurate.

2 INTRODUCTION TO THE METHODS

In order to utilize the moments of the crack opening displacememt to approximately express the probability density function of the crack opening displacement, we first derive the series' formulae to calculate the moments of the crack opening displacement.If we use Paris law $da/dN=C(\Delta K)^m$, and we take m=4 in the law , crack size a can be determined as follows

$$a=a_o/(1-c\pi^2Y^4K_t^4\Delta\sigma^4Na_o) \tag{1}$$

where,

$$K_t=q+K_h+K_w$$

$$q=\begin{cases}1.5, & \text{taking the height of a welding seam into account}\\ 1.0, & \text{not taking the height of a welding seam into account}\end{cases}$$

we may see from eqn(1) that although a is a function of a_o, C, K_t, Y, $\Delta\sigma$, and N, $C\pi^2Y^4K_t^4\Delta\sigma^4Na_o$ comes out in one product.The product can be temporarily considered as one variable x, such that

$$a=\frac{a_o}{1-x} \qquad 0< x <1 \tag{2}$$

similarly,

$$a^j=\frac{a_o^j}{(1-x)^j} \qquad 0< x <1\ ,\ j=1,2,\ldots,n \tag{3}$$

Expressing eqn (3) into a finite series, it follows

$$a^j=\frac{a_o^j}{(j-1)!}\sum_{i=0}^{M}\frac{(j+i-1)!}{i!}x^i$$

$$=\frac{1}{(j-1)!}\sum_{i=0}^{M}\frac{(j+i-1)!}{i!}\pi^{2i}Y^{4i}N^{i}C^{i}K_t^{4i}\Delta\sigma^{4i}a_o^{j+i} \quad (4)$$

Assuming that C, K_h, K_w, $\Delta\sigma$, and a_o are random variables and they are independent of each other, one can easily determine the first n moments of the crack a from eqn (4) as follows

$$E[a^j]=\frac{1}{(j-1)!}\sum_{i=0}^{M}\frac{(j+i-1)!}{i!}\pi^{2i}Y^{4i}N^{i}E[C^i]E[K_t^{4i}]E[\Delta\sigma^{4i}]E[a_o^{j+i}]$$

$$j=1,2,\ldots,n \quad (5)$$

If JWES-2805 code δ=3.5ea is adopted, one has

$$E[\delta^j] = 3.5^jE[(ea)^j] = (3.5/E)^jE[\sigma_m^j]E[(K_ta)^j]$$

$$j=1,2,\ldots,n \quad (6)$$

where,

$$E[(K_ta)^j]=\frac{1}{(j-1)!}\sum_{i=0}^{M}\frac{(j+i-1)!}{i!}\pi^{2i}Y^{4i}N^{i}E[C^i]$$

$$E[K_t^{4i+j}]\ E[\Delta\sigma^{4i}]\ E[a_o^{j+i}]$$

By the use of the first n moments of δ one can compose a series to approach the probability density function of δ by a few orthogonal polynomials. And the fracture failure probability can be obtained according to the interference theory.

2.1 Laguerre's orthogonal polynomial

Express the probability density function of the crack opening displacement as follows

$$f_\delta(\delta) = \sum_{i=0}^{n} A_iL_i(\lambda\delta)\lambda e^{-\lambda\delta} \quad (7)$$

where, λ can be determined by the mean value of δ, $L_i(\cdot)$ is the ith order Laguerre's orthogonal polynomial, whose progressive formulae are

$$L_O(\delta)=1,\ L_1(\delta)=1-\delta,\ L_{i+1}(\delta)=(2i+1-\delta)L_i(\delta)-i^2L_{i-1}(\delta)$$

By the use of the orthogonality of Laguerre's orthogonal polynomial, one has

$$A^i = \sum_{j=0}^{i}(-1)^j\frac{\lambda^j}{(i-j)!j!^2}E[\delta^j]$$

2.2 General Laguerre's orthogonal polynomial

Express the probability density function of the crack opening displacement as follows

$$f_\delta(\delta) = \sum_{i=0}^{n} A_iL_i^{(\alpha)}(\beta\delta)\frac{\beta^{\alpha+1}}{\Gamma(\alpha+1)}\delta^{\alpha}e^{-\beta\delta} \quad (8)$$

where, α and β can be determined by the first two moments of δ, $\Gamma(\cdot)$ is Gamma function, $L_i^{(\alpha)}(\cdot)$ is the ith order general Laguerre's orthogonal polynomial

$$L_0^{(\alpha)}(\delta)=1, \qquad L_1^{(\alpha)}(\delta)=1-\delta+\alpha$$

$$(i+1)L_{i+1}^{(\alpha)}(\delta)=(2i+\alpha+1-\delta)L_i^{(\alpha)}(\delta)-(i+\alpha)L_{i-1}^{(\alpha)}(\delta)$$

$$A_i=i!\,\Gamma(\alpha+1)\sum_{j=0}^{i}(-1)^j\frac{\beta^j}{\Gamma(i-j+1)\,\Gamma(j+1+\alpha)\,j!}E[\delta^j]$$

2.3 Jacobi's orthogomal polynomial

Express the probability density function of the crack opening displacement as follows

$$f_\delta(\delta) = \sum_{i=0}^{n} A_i J_i^{(\alpha,\beta)}(\delta)\frac{(1-\delta)^\alpha\delta^\beta}{B(\alpha+1,\beta+1)} \tag{9}$$

where, α and β can be determined by the first two moments of δ, $B(\alpha+1,\beta+1)$ is Beta function

$$J_0^{(\alpha,\beta)}(\delta)=1, \quad J_1^{(\alpha,\beta)}(\delta)=(\alpha+\beta+2)\delta-(1+\beta)$$

$$2(i+1)(i+\alpha+\beta+1)(2i+\alpha+\beta)J_{i+1}^{(\alpha,\beta)}(\delta)$$
$$=(2i+\alpha+\beta+1)[(2i+\alpha+\beta)(2i+\alpha+\beta+2)(2\delta-1)+\alpha^2-\beta^2]J_i^{(\alpha,\beta)}(\delta)-$$
$$2(i+\alpha)(i+\beta)(2i+\alpha+\beta+2)J_{i-1}^{(\alpha,\beta)}(\delta)$$

$$A_i=D_i\sum_{j=0}^{i}\binom{i+\alpha}{j}\binom{i+\beta}{i-j}\sum_{k=0}^{i-j}(-1)^k\frac{(i-j)!}{(i-j-k)!\,k!}E[\delta^{i-j}]$$

$$\binom{i+\alpha}{j}=\frac{(i+\alpha)(i+\alpha-1)\cdots(i+\alpha-j+1)}{j!}$$

$$D_0=1, \quad D_{i+1}=\frac{(2i+\alpha+\beta+3)(i+\alpha+\beta+1)(i+1)}{(2i+\alpha+\beta+1)(i+\alpha+1)(i+\beta+1)}$$

On determining $f_\delta(\delta)$,one can estimate the failure probability P_f by the formula below

$$P_f=P[\delta_c<\delta]=\int_0^\infty f_\delta(\delta)F_{\delta c}(\delta)d\delta$$

3 EXAMPLE

In order to compare some approximate results with the exact result,we only select a_0 and δ_c as random variables.Their probability density functions and parameters are as follows

a_0: $f(x)=\lambda\exp\{-\lambda x\}$, $\lambda=0.65\times10^{-3}\ m^{-1}$

δ_c: $f(x)=\frac{\alpha}{\beta}(\frac{x}{\beta})^{\alpha-1}\exp\{-(\frac{x}{\beta})^\alpha\}$, $\alpha=1.91$, $\beta=\beta_0(1-r)^{N/1000}$

$\beta_0=0.138\times10^{-3}$ m, $r=0.05$

Others are constants. They are $E=2.06\times10^5$ MPa, $C=1.702\times10^{-12}$, $K_h=0.10$, $K_w=0.35$, $\Delta\sigma=86.2$ MPa, $\sigma_m=142.1$ MPa, $q=1.0$, $Y=1.0$

To some different number of the load repitition,the exact value of the fracture failure probability,the results calculated by Crude Monte Carlo method, by JC method, by Edgeworth's series method, and by a few orthogonal polynomials methods are shown in table 1.One can see from the table that the results calculated by the methods presented in the paper are better than that by some other approximate methods. The computation time taken by the presented methods is approximately nine times less than that taken by Crude Monte Carlo method.

Table 1. The fracture failure probability calculated by some methods (order:10^{-3})

Number of repetition	0	1000	2000	3000	4000	5000
exact value	3.729	4.254	4.858	5.553	6.354	7.278
Monte Carlo method	3.777	4.312	4.926	5.634	6.450	7.392
JC method	3.954	4.467	5.047	5.703	6.448	7.289
Edgeworth's series	3.737	4.344	5.028	5.802	6.682	7.688
Laguerre's polynomial	3.729	4.255	4.859	5.554	6.356	7.278
General Laguerre's Polynomial	3.729	4.255	4.859	5.554	6.356	7.279
Jacobi's Polynomial	3.729	4.255	4.859	5.554	6.356	7.279

4 CONCLUSION

This paper presented a few methods to assess the fracture failure probability of aspherical tank. These methods utilize a few orthogonal polynomials and the moments of the crack opening displacement to compose the probability density function of the crack opening displacement.So they convert the assessment of the fracture failure probability into the calculation of the moments of the crack opening displacement and a one-dimensional integral. These methods are similar to Edgeworth's series method,but their formulae are simpler.The example proves that these methods are quicker,and more accurate than some other approximate methods.

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