SEISMIC RISK ANALYSIS OF NON-LINEAR MDOF STRUCTURES

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ABSTRACT
Seismic risk of a non-linear multi-degree-of-freedom (MDOF) structure was evaluated using a spectral attenuation equation and a spectral reduction factor. Simulated ground motions matching the spectral attenuation equation were generated. Non-linear response analyses were performed to obtain the spectral reduction factor for shear failure of a reactor building model. The probability of shear failure of the structure at a site was evaluated by the response spectral acceleration at the structural first natural period and the spectral reduction factor.

1. INTRODUCTION
Conventional seismic hazard analysis calculates the exceedance probability of peak ground acceleration (PGA) or response spectral values. This method, however, cannot easily take into account the non-linear behavior of structures. Sewell (1988) studied seismic risk of SDOF non-linear structures using a spectral attenuation equation and a spectral reduction factor. Inoue and Cornell (1991) studied seismic risk of non-linear structures using a factor that accounts for the contribution of higher modes. Kanda (1985) proposed a second moment seismic safety margin index for inelastic structures using an approximate linear relationship between the input peak ground acceleration (PGA) and the equivalent elastic response. In this paper, these methods are combined and the seismic risk of a prototype reactor building model was evaluated.

2. ATTENUATION EQUATION AND SIMULATED GROUND MOTIONS
Kawashima and Aizawa (1984) proposed the following attenuation equation of earthquake acceleration response spectra based on 394 horizontal components of strong motions recorded in Japan.

\[ S_a(T, M, \Delta, G) = a(T, G) \times 10^t(T, G) \times (\Delta + 30)^c \]  

where \( T, M, \Delta, G \) are the natural period, the magnitude, the epicentral distance and the ground condition (i.e. hard, medium, and soft ground) respectively. The coefficients \( a \) and \( d \) are given for each natural period and ground condition. The coefficient \( c \) is constant so that spectral shape does not change with respect to the distance. Standard deviation of \( \ln S_a \) is also given for each natural period and ground condition. They also proposed an attenuation equation of PGA in a similar form.
Simulated ground motions matching the proposed acceleration response spectra were generated by the sinusoidal wave superimposition method with random phase angles. Jennings type envelope function was used to simulate the time history. Ten ground motions were generated for each magnitude, i.e., M=5.5, 6.5, 7.5, 8.5 and the ground condition of the hard ground was assumed whose natural period is less than 0.2sec. Spectral ratios (the ratio between the value of Eq. 1 and that of the attenuation equation of PGA) of target spectra for the distance of 10km are shown in Fig. 1. As magnitude increases, the spectral ratio at a long period range increases.

3. STRUCTURAL MODEL
The structural model used for this study is shown in Fig. 2. Dynamic characteristics are shown in Table 1 and the load-deformation relationship of members is shown in Table 2. The model was determined based on a BWR Mark II standard type reactor building model (Osaki and Watabe, 1987) with a little modification. The first natural period is 0.3 sec. An origin-oriented hysteretic rule and a degrading tri-linear hysteretic rule are employed for the shear deformation and for the flexural deformation respectively. The skeleton curve is tri-linear for both rules. The skeleton curve for shear deformation is shown in Fig. 3. The model has sway and rocking springs at the bottom. Mean ultimate limit state of each member is assumed to be defined by the ultimate shear strain $\gamma_u=5.0\times 10^{-3}$ (6.0 times $\gamma_y$, i.e. $\mu=6.0$).

4. SPECTRAL REDUCTION FACTOR
PGA which induces an incipient shear crack ($Q/Q_o$) of at least one member, $a_o$, was obtained for the 40 simulated motions. The inelastic responses were calculated for PGA, $a_o$ of 2, 3, and 5 times $a_o$. Let $S_{a_1}$ and $S_{a_{c1}}$ denote the response spectral acceleration at the first natural period of the structure corresponding to PGA of $a$ and $a_o$ respectively. Then the ratio $a/a_o$ is equal to $S_{a_1}/S_{a_{c1}}$. Peak inelastic shear responses are transformed into equivalent elastic shear response, $Q^*$, which is defined to have the same strain energy with the inelastic response. The largest values of $Q^*/Q_o$'s among members, max[$Q^*/Q_o$], which occurs at member 5 or 6 for most cases, are plotted versus $S_{a_1}/S_{a_{c1}}$ in Fig. 4. Although the variation of max[$Q^*/Q_o$] increases with $S_{a_1}/S_{a_{c1}}$ in a range of $S_{a_1}/S_{a_{c1}}=1$ to 3, it is almost unchanged at around $S_{a_1}/S_{a_{c1}}=3$ and 5.

The value of max[$Q^*/Q_o$] corresponding to $\gamma_u$ is 8.8 for all members according to the definition. Let $F$ be the $S_{a_1}/S_{a_{c1}}$ value at which max[$Q^*/Q_o$]=8.8 is reached. $F$ is called a spectral reduction factor for the shear failure. As the relationship between max[$Q^*/Q_o$] and $S_{a_1}/S_{a_{c1}}$ is almost linear, $F$ was evaluated by linear interpolation or extrapolation, and shown in Fig. 5. The mean value of $F$ is about 5 and decreases with the magnitude. The variation of $F$ is small (COV=0.07) and changes little with respect to magnitude. Bazzurro and Cornell (1992) obtained $F$ for a jacket-type offshore platform using 15 ground motion records and the COV of $F$ for $\mu=6.0$ is 0.24. One of reasons for smaller COV values in our study seems that simulated ground motions with smaller spectral variation than observed ground motions are used for a structural model with a fair distribution of stiffness and strength.

4. SEISMIC RISK ANALYSIS OF MDOF STRUCTURES
The relationship between magnitude and $S_{a_{c1}}$ is shown in Fig. 6. The mean value of $S_{a_{c1}}$ is about 600gal and $S_{a_{c1}}$ values increase with the magnitude. The variation of $S_{a_{c1}}$ is due to contribution of higher modes to the response. As the first mode
dominates the response in this model, the variation of $S_{a1}$ is generally small.

Given a event of magnitude $m$ and distance $\delta$, the probability that any member of the model structure experiences shear failure is given as follows:

$$P \left[ S_{a1}/S_{ac1} > F \right| m, \delta] = P \left[ S_{a1}/(S_{ac1}F) > 1 \right| m, \delta]$$

(2)

Then the annual rate of shear failure of the structure is given by:

$$\lambda = \int_0^\infty \int_0^\infty P \left[ S_{a1}/(S_{ac1}F) > 1 \right| m, \delta] f_{m, \delta} (m, \delta) \, dm \, d\delta$$

(3)

By expressing the means of $\ln S_{a1}$ and $\ln F$ as a function of the magnitude as shown in Fig. 5 and 6, and assuming $S_{a1}$ and $F$ are lognormally distributed, Eq. 3 can easily be evaluated. The standard deviation of $\ln S_{a1}$ obtained by Kawashima et al. is 0.56. As the standard deviations of $\ln S_{a1}$ and $\ln F$ obtained in this study are small compared with that of $\ln S_{a1}$ of recorded motions, the standard deviation of $\ln S_{a1}/(\ln S_{a1} \ln F)$ was assumed to be equal to that of $\ln S_{a1}(=0.56)$.

5. NUMERICAL EXAMPLE

Seismic hazard of the model structure was evaluated for a site in Japan. Earthquake data of 300 years (1686-1985) were used to evaluate the value of $b$ in Gutenberg-Richter occurrence model and the annual occurrence rate of events $v$ having magnitude greater than 5.0. Earthquake data of 1307 years were used to evaluate the maximum magnitude, $m_\infty$. Seismic source parameters are shown in Fig. 7. The site is located at 34.62'N and 138.15'E. Program EQRISK (McGuire, 1976) was used for this study. The annual risk of exceedance of acceleration response spectral value at T=0.3 sec is shown in Fig. 8. This is a hazard curve for SDOF structure of the natural period = 0.3 sec, i.e. $S_{a1}$. The annual probability that shear strain of any member of the model structure exceeds $5.0 \times 10^{-3}$ is $0.356 \times 10^{-4}$ (corresponding reliability index $\beta = 3.97$). The variation of structural resistance is not considered herein.

6. CONCLUSIONS

The seismic risk of the non-linear structure was evaluated using response spectral attenuation equation at the first natural period of the structure. The variation of spectral reduction factor $F$ was small for the simulated motions of the hard ground. The equation of damage evaluation can be easily substituted for the response spectral attenuation equation in the seismic hazard equation. The probability of structural damage of non-linear MDOF structures which takes into account the probabilistic distribution of earthquake magnitude and distance at a site can be evaluated by this method.

REFERENCES


Kanda, J. 1985. Probability-Based Seismic Margin Index for Inelastic Members of Reactor Buildings. 8th SMiRT, M1K2/5 : 353-359


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Fig. 1. Spectral Ratio of Target Spectra

Fig. 2. Structural Model

Fig. 3. Skeleton Curve for Shear Deformation

Table 1. Dynamic Characteristics of Model Structure

| Member No. | Mass (t) | Weight (x10^2 t/m^2/rad) | Moment of Inertia (x10^6 t) | GA (x10^6 t) | EI (x10^10 t m^2) | h (%)
|------------|---------|--------------------------|-----------------------------|--------------|-------------------|------
| 9          | 3520    | 0.7301                   | 44.1                        | 6.23         |                   |      
| 8          | 4050    | 0.8399                   | 57.3                        | 7.90         |                   |      
| 7          | 15200   | 3.162                    | 149.7                       | 11.12        |                   |      
| 6          | 29330   | 6.138                    | 167.3                       | 16.78        |                   |      
| 5          | 19530   | 4.067                    | 203.6                       | 21.19        | 5.0               |      
| 4          | 25210   | 13.28                    | 451.8                       | 85.85        |                   |      
| 3          | 46830   | 24.70                    | 484.5                       | 93.47        |                   |      
| 2          | 34880   | 18.39                    | 507.0                       | 101.12       |                   |      
| 1          | 31150   | 16.42                    | 507.0                       | 101.12       |                   |      
| F          | 124500  | 66.26                    |                             |              |                   |      

K_{ss} = 2.088 \times 10^{7} t/m
K_{s} = 3.613 \times 10^{7} t/m
K_{\theta} = 5.074 \times 10^{10} t m/\text{rad}
### Table 2. Load–Deformation Relationship of Members

<table>
<thead>
<tr>
<th>Member</th>
<th>$Q_c$ ($10^3$ t)</th>
<th>$Q_{\psi}$ ($10^3$ t)</th>
<th>$K_1/K_0$</th>
<th>$K_2/K_0$</th>
<th>$M_c$ ($10^5$ tm)</th>
<th>$M_{\psi}$ ($10^5$ tm)</th>
<th>$K_1/K_0$</th>
<th>$K_2/K_0$</th>
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<td>0.12</td>
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<td>5.39</td>
<td>0.134</td>
<td>0.001</td>
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<tr>
<td>8</td>
<td>15.9</td>
<td>23.9</td>
<td>0.25</td>
<td>0.12</td>
<td>3.62</td>
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<tr>
<td>7</td>
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<td>0.25</td>
<td>0.12</td>
<td>5.83</td>
<td>13.15</td>
<td>0.149</td>
<td>0.019</td>
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<tr>
<td>6</td>
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<td>35.47</td>
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<td>24.97</td>
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<td>0.12</td>
<td>30.91</td>
<td>58.16</td>
<td>0.393</td>
<td>0.057</td>
</tr>
</tbody>
</table>

**Fig. 4.** Values of max[$Q'/Q_c$] plotted versus $S_{a1}/S_{a01}$
Fig. 5. Spectral Reduction Factor $F$

Fig. 6. Values of $S_{\text{ac}1}$

Fig. 7. Seismic Source Parameters used in This Study

Fig. 8. Seismic Hazard Curve of SDOF(T=0.3sec) Structure