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SLIDING BEHAVIORS OF ELASTIC CYLINDRICAL TANKS UNDER SEISMIC LOADING

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1. Introduction

There is a paper that reports on the occurrence of sliding in several oil tanks on Alaskan earthquake of 1964(1). This incident appears to be in need of further investigation for the following reasons:

First, in usual seismic designing of cylindrical tanks ("tanks"), sliding is considered to occur when the lateral inertial force exceeds the static friction force. When the tank in question can be taken as a rigid body, this rule is known to hold true. If the tank is capable of undergoing a considerable amount of elastic deformation, however, its applicability has not been proved.

Second, although several studies have been done on the critical conditions for static sliding(2), the present author is unaware of like ones made on the dynamic sliding, except for the pioneering work of Sogabe(3), in which they have empirically indicated possibility of sliding to occur under the force of sloshing.

Third, this author has shown earlier on that tanks, if not anchored properly, will start rocking, inducing uplifting of the baseplate, even at a relatively small seismic acceleration of 10 gal or so(4).

The present study has been conducted with these observations for the background. Namely, based on a notion that elastic deformation given rise to by rocking oscillation should be incorporated as an important factor in any set of critical conditions for the onset of sliding, a series of shaking table experiments were performed for rigid steel block to represent the rigid tanks ("rigid model") and a model tank having a same sort of plate thickness-to-diameter ratio as industrial tanks to represent the elastic cylindrical tanks ("elastic model").

2. Conditions for Onset of Sliding

The condition for the onset of sliding usually adopted in seismic design is that, from the Coulomb's theorem, the lateral inertial force by seismic loading to exceed the static friction force, with an assumption that the structure concerned is a rigid body and that the force is quasi-static. Namely:

$$F_s \geq \mu_s m \cdot (g - \alpha_v), \tag{1}$$

where μ_s is the static friction coefficient.

Now, let us model an unanchored cylindrical tank undergoing rocking deformation under a lateral seismic force as shown in Fig.1. Since the grounded area can be approximated with a circle of radius $(a - l_b/2)$, the vertical force F_v that acts on the baseplate will be:

$$F_v = p_b \pi \cdot (a - l_b/2)^2 + 2\int_b^{\pi} \tilde{q}_r \cdot a \cdot d\theta, \tag{2}$$

where p_b is the mean liquid pressure working on the baseplate.

From this equation and Eq.1, we get Eq.3 below, provided that the vertical force vary little while the tank is rocking:

$$F_s = \mu_s \{p_b \pi \cdot (a - l_b/2)^2 + 2 \int_b^{\pi} \tilde{q}_r \cdot a \cdot d\theta\}. \tag{3}$$

That is to say, inasmuch as tank's mass remains unchangeable, the reaction force from the foundation that corresponds to that portion of the liquid weight which is acting on the uplifted parts of the baseplate is now concentrated on the tilt-compressed parts. Thus, the reaction force is no longer uniform: it is distributed as the second term on the righthand side dictates.

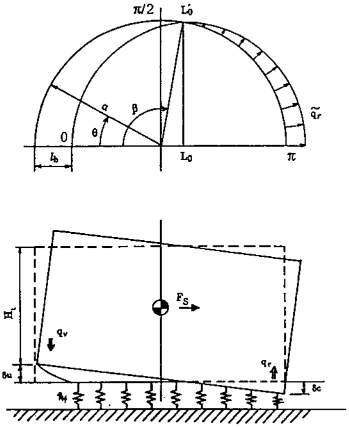


Fig.1 Rokcing Model

3. Sliding Test for Rigid Tanks

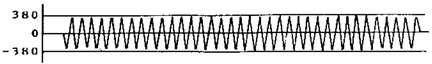
3. 1 Model and Experimental Method

The model for the rigid tanks was a sufficiently rigid steel block of 1,000 x 500 x 418 mm, weighing 3.15 kN; it was placed on a plywood plate, which was fixed to the shaking table. From a static pulling test, the static friction coefficient between this model and the plate was μ_s = 0.30.

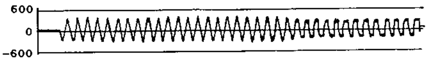
The lateral acceleration was steadily increased at a rate of 3 gal/s ("acceleration sweeping") with sinusoidal waves until sliding occurred. In some experiments, moreover, vertical oscillation was imposed in the same frequency as the lateral, though at a given constant acceleration. This was to see the effect of biaxial combined oscillation.

3. 2 Shaking Experiments

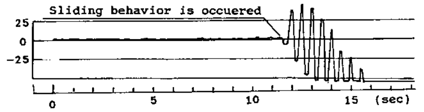
The results are summarized in Figs.2 and 3. Fig.2, which presents a set of time-resolved diagrams to identify the onset of sliding in a lateral acceleration sweep test conducted at 2 Hz, in particular, shows sliding took place for an acceleration of 323 gal.



(a) Acc. of Shaking Table



(b) Acc. of Rigid Model



(c) Disp. of Rigid Model

Fig.2 Response of Rigid Model

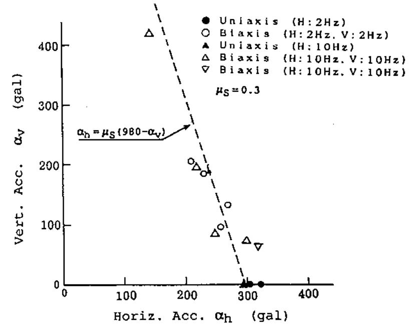


Fig.3 Critical Acc. of Sliding

The cases of biaxial shaking are seen in Fig.3. Here the dotted line denotes the slide-inducing lateral accelerations, α_{hs}, calculated in Eq.1. Agreement between the theoretical and the experimental is quite satisfactory.

Thus, we conclude that the critical condition for sliding of rigid tanks should be given by Eq.4, which is an Eq.1 modified so as to take oscillation frequency, phase difference between the lateral and the vertical waves, and the added mass effect of the liquid contained:

$$(m_t + m_s)\cdot\alpha_{hs} > \mu_s\{(m_t + m_l)\cdot g - (m_t + m_v)\cdot\alpha_v\cdot\cos\theta_s\}, \qquad (4)$$

where θ_s is the phase difference between the two oscillations, m_t and m_l, the masses respectively of the tank and the liquid, and m_s and m_v, the added mass of the liquid reckoned respectively in the lateral and the vertical directions.

4. Sliding Test for Elastic Tanks

4. 1 Model and Experimental Method

The model tank is depicted in Fig.4: it is a cylindrical steel tank of 2,034 (dia.) x 1,547 (h) mm with a 1.0 mm thick shell and a 0.5 mm thick baseplate. It will be noted that the wall thickness-to-diameter ratio falls in the range generally adopted for industrial large tanks of like design. Weighing ca. 1.4 kN dry, the tank was placed on ALC plate, which was fixed to the shaking table. From a static pulling test, the static friction coefficient between this model and the ALC plate was μ_s = 0.48.

The tank was filled with water to a chosen level: either the "low level", which was 840 mm (H_l/a = 0.83), or the "high level", which was 1,350 mm (H_l/a = 1.33).
The oscillation conditions in the latter experiments were as follows:

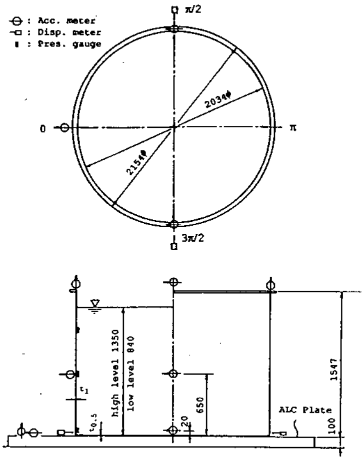


Fig.4 Elastic Tank Model

Lateral acceleration sweep rate:3 gal/s;
Vertical acceleration:0 or 200 gal for the low level;
and 0, 100, or 200 gal for the high level.

4. 2 Shaking Test

4. 2. 1 Acceleration sweep experiments

Fig.5 presents a typical set of time-resolved diagrams to show the onset of sliding in lateral shaking for the low level. We note:

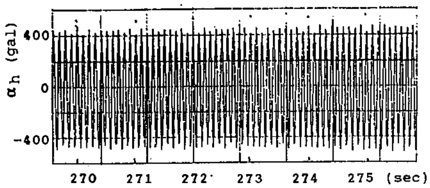
- at 274 sec. of sampling time (75 sec. in the real time), when the acceleration had attained 400 gal (Fig. 5-a), the acceleration at the shell top started to increase rapidly (b);
- this was accompanied by rapid increase in the vertical displacement at the shell bottom (c), indicating the onset of rocking oscillation and uplifting of the tank at one end;
- at the same time, the lateral displacement of the tank took a sudden change (d and e);
- here, particular attention should be paid to the observation that, since the diagram (d) gives the translational component of the tank's movement, while (e), the rotational component, (d) and (e) prove together that the occurrences of the translational sliding and the rotational sliding were simultaneous; and
- concurrent with (d) and (e), furthermore, marked reduction in the lateral force F_s occurred (f), indicating the friction to have changed from static to dynamic.

The case of the high level, biaxial shaking (100 gal in the vertical direction) is shown in Fig.6 with a similar set of diagrams. Features to be observed here are:

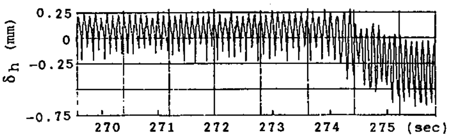
- the sudden increase in the acceleration at shell top, indicating the onset of rocking motion, was for a lateral acceleration of only ca. 170 gal (a and b);
- but there was no translational sliding taking place (c), even though the rotational sliding did occur (d); and
- the waveform of the lateral force was essentially the same as that of the shell top (e and b), demonstrating that there was no reduction in the lateral force F_s on occurrence of sliding.

That is to say, when the liquid level is high enough, rocking can be initiated, accompanied by rotational sliding, at an acceleration which may be markedly lower than one half of the acceleration needed to induce translational sliding. This observation should be taken as strongly suggesting that rocking is closely related to rotation.

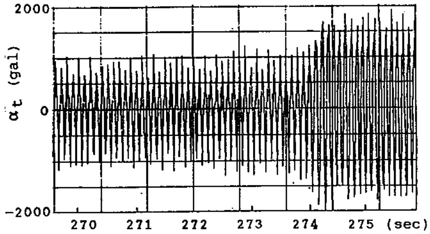
On the other hand, however, there was in the present experiments a concentration of electric cables, amounting to ca. 400 N in the total weight, on the shell plate located at right angles to the shaking axis, and the waveform of tangential acceleration deter-



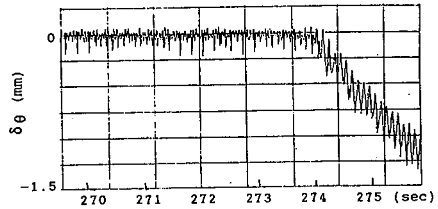
(a) Acc. of Shaking Table



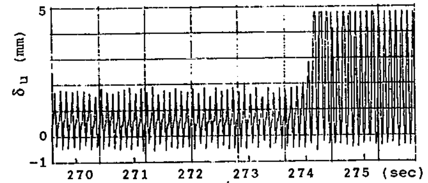
(d) Trans. Disp. of Tank



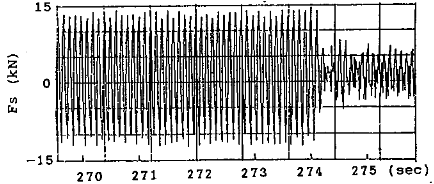
(b) Acc. of Shell Top



(e) Rotat. Disp. of Tank

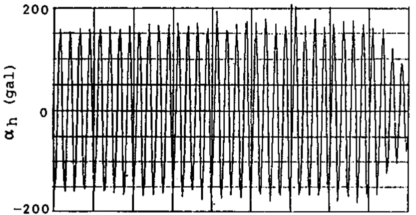


(c) Uplift Disp. of Shell Bottom

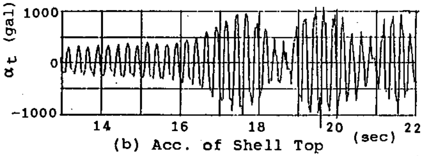


(f) Lateral Force of Tank

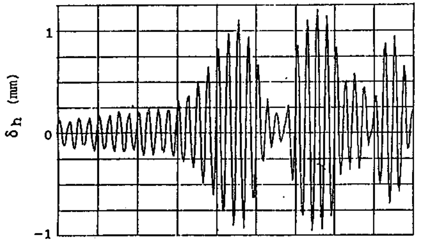
Fig.5 Response of Elastic Tank (Lower Level)



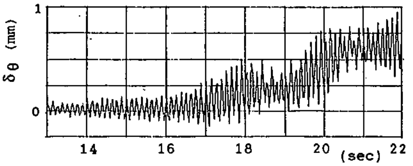
(a) Acc. of Shaking Table



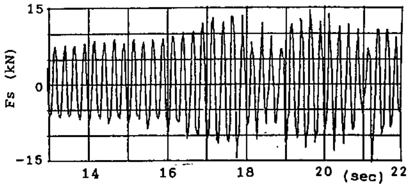
(b) Acc. of Shell Top



(c) Trans. Disp. of Tank



(d) Rotat. Disp. of Tank



(e) Lateral Force of Tank

Fig.6 Response of Elastic Tank (Higher Level) mined at the shell top suggested that, developing some torsional moment about the principal axis of the model tank, they may have contributed to the observed rotational motion. Since this effect remains indeterminable, though undoubtedly small enough, we shall refrain from discussing the tank's rotational sliding any further.

4. 2. 2 Lateral force at onset of sliding

To make fair comparison of different conditions for sliding in terms of the friction coefficient, let us define an equivalent friction coefficient μ_s^* as a ratio of the prevalent lateral force to that of the vertical force:

$$\mu_s^* = F_s/[(m_t + m_l)\cdot(g - \alpha_v)] \tag{5}$$

In Table 1, which compares the accelerations and lateral forces at the onset of sliding for two water levels, we note:

- for the low level, the equivalent friction coefficient μ_s^* of 0.46 to 0.50 agrees well with the static friction coefficient μ_s, which is 0.48;
- for the high level, on the other hand, $\mu_s^* = 0.38$ to 0.41 is some 15 to 25% lower than μ_s;
- the lateral force under biaxial oscillation is clearly lower than that under lateral oscillation alone; and
- the uplifting of shell bottom due to rocking is some ten times as large for the high level as for the low level.

Table Sliding Condition for Model Tank

Expermental No.		1	2	3	4	5	6
Liquid Level		840 mm			1350 mm		
Exciting Cond. Frequency Conbination		Uniaxis 10 Hz	Biaxis 10 Hz Inphase	Biaxis 10 Hz Outphase	Uniaxis 4 Hz	Biaxis 4 Hz Inphase	Biaxis 4 Hz Outphase
Exp.	Acc. αh gal	400	360	370	200	170	170
	Acc. αv gal	40	200	180	60	130	120
	Acc. αt gal	1350	1200	1200	1025	1000	900
	Uplift Disp. mm	1.8	1.6	1.8	over 20	over 20	over 20
	Sliding Mode	↑↻	↑↻	↓↻	↻	↻	↻
	Horiz. Force KN	12	11	11	17	15	15
	Equivalent μ^*	0.46	0.50	0.49	0.41	0.39	0.38
Cal.	Horiz. Force KN	11.9	10.6	10.7	17.2	16.3	15.0
	Equivalent μ^*	0.45	0.49	0.48	0.42	0.43	0.39

Note. static Friction Coefficent μ_s is 0.48
↑ show translational sliding, ↻ show rotational sliding

4. 2. 3 Critical condition for onset of sliding in elastic tanks

As described above, the empirical results indubitably prove that the rocking oscillation that allows a shell bottom to be lifted up must be incorporated in the critical conditions for onset of sliding. Here, the rocking model illustrated in Fig.1 and the Eq.2 show that the situation is now that of the liquid pressure acting on that part of the baseplate which has been lifted up counteracting the reaction force $\tilde{q}_r$ coming from that part of the foundation which supports the still grounded part of the baseplate. One result of this movement is that the distribution of friction force has become inhomogeneous.

Since the lateral force working on the tank is transferred to the baseplate as a planar shearing force in the shell, on the other hand, it will take on a distribution that has its maximum in the region that crosses the shaking axis at right angles ($\theta = \pi/2$). Therefore, even though the friction force as a component of the resultant force remains larger than the lateral force as its component, there can be localities where this relation is reversed: it is there that local sliding can be initiated to lead eventually into total sliding.

We shall examine this possibility in terms of the lateral force and the friction force given rise to by the first term on the righthand side of Eq.3. Namely, rewriting Eq.3 in view of the tank's vertical motion and effects of the added mass, we obtain:

$$F_f = \mu_s(1 - l_b/2a)^2\cdot\{(m_t + m_l)g - (m_t + m_v)\alpha_v\cdot\cos\theta_s\}. \tag{6}$$

First, we calculate the vertical displacement δ_u and the radial uplift length l_b of the baseplate at the onset of sliding from the force q_v to resist uplifting. Computation carried out according to this author's method(4) gave a δ_u of ca. 1.8 mm and 20 mm and an l_b of ca. 71 mm and 154 mm for the low and the high levels, respectively. From this, the grounded area of baseplate, as approximated with a circle of radius $(a - l_b/2)$, is calculated to be 3.02 x10^6mm^2 for the low level and 2.77 x10^6mm^2 for the high, i.e., respectively 93% and 85% of the original baseplate area of 3.25 x10^6mm^2.

Finally, since $(m_t + m_s) = 4.47$ kN for the high level, Eq.6 yields F_f's as 16.8, 15.2 and 15.7 kN for experiments Nos. 4, 5, and 6, respectively.

Since these values agree quite well with the empirically determined lateral forces on onset of sliding given in Table 1, i.e., 17.0, 15.0, and 15.2 kN, we conclude that the Eq.6 adequately describes the critical condition for an elastic tank to initiate local sliding on rocking. Therefore, in the final form, the condition for onset of local sliding is given by Eqs. 4 and 6 as $F_s > F_f$. Namely,

$$(m_t + m_s)\cdot\alpha_{hs} > \mu_s(1 - l_b/2a)^2\cdot\{(m_t + m_l)g - (m_t + m_v)\alpha_v\cdot\cos\theta_s\}. \qquad (7)$$

4. 2. 4 Summary of elastic tank experiments

The observations gained in the experiments conducted for elastic model tanks may be summarized as follows:

(1) Total sliding

As long as the uplift of the shell bottom on rocking, hence the magnitude of l_b, remains small enough, the friction coefficient may be regarded as homogeneous over the baseplate. Then the tank can undergo a translational sliding as a whole if the equivalent friction coefficient μ_s^* defined in Eq.4 exceeds the static friction coefficient μ_s, i.e., $\mu_s^* > \mu_s$.

(2) Local sliding

As rocking is intensified, and when lb becomes so large as to reduce tank's grounded area to less than ca. 90 % of the original value, the distribution of friction force over the baseplate can no longer be regarded as homogeneous. It is then that local sliding can take place as a precursor to the total sliding. The critical condition for this can be deduced from Eqs.1, 3, and 6 as $F_s > F_f$.

5. Summary and Conclusions

Following observations have been obtained for the critical condition of the onset of sliding:

(1) Sliding of rigid tanks will occur when the lateral force given rise to by oscillation exceeds the static, or the Coulombic, friction force.

(2) If vertical oscillation is imposed on the lateral oscillation, the lateral force needed to induce sliding of a rigid tank will be reduced to the same level as the ratio of the vertical seismic acceleration to the gravitational acceleration.

(3) In the case of elastic tanks, on the other hand, the sliding motion comprises the total sliding and the local sliding, the latter being a precursor to the former.

(4) In (3) above, the total sliding is a translational motion in the direction of oscillation; this motion takes place when the lateral force due to shaking exceeds the static friction force.

(5) Whereas the local sliding is closely related to the rocking motion by which a part of the baseplate is lifted up.

Acknowledgement

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