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SIMPLE ANALYTIC SOLUTIONS FOR THE COUPLED RESPONSES OF FLUID-RECTANGULAR CONTAINER SYSTEMS

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ABSTRACT

An analytic method for analyzing the fluid-structure interaction problem in a flexible rectangular container under horizontal and vertical earthquake excitations is presented. The equations of motion of the coupled fluid-structure system are solved by applying Galerkin method. Computation results for the coupled response of fluid-rectangular container systems by the present method are discussed.

1. INTRODUCTION

Nuclear spent fuel assemblies or canisters are usually stored under the water inside rectangular reinforced concrete container structures. Since these reinforced concrete structures are not designed to behave rigidly under earthquake excitations, the fluid-structure interaction effects need to be investigated carefully prior to their design.

The dynamic behavior of earthquake excited liquid storage tanks has been an active area of theoretical researches and engineering investigations [1]. However, most of these works have been concerned with the analysis of cylindrical tanks, and little has been done on the study of fluid-structure interaction effects in flexible rectangular tanks. Haroun [2] presented a method for calculating both the loads and the associated moments for a rectangular tank, but the method did not account for dynamic fluid-structure interaction effects. Recently, Park, Koh and Kim [3] proposed a numerical procedure using a coupled boundary element-finite element method for the analysis of dynamic fluid-structure interaction in a flexible rectangular container.

This paper presents an analytic method for evaluating the effects of fluid-structure interaction in a rectangular container structure subjected to horizontal and vertical earthquake excitations. The motion of fluid is assumed to be governed by Laplace equation. Series expressions for the velocity potential and appropriate expressions of vibration mode shape for a wall structure are used to satisfy in a weak sense the governing equation of motion for the coupled fluid-structure system by Galerkin method. The behavior of wall structure can be described by either a 2-dimensional cantilever beam model or a 3-dimensional plate model, but this paper presents only an analysis procedure using the cantilever beam model. The present method is applied to the analysis of a fluid container structure which is typically used in nuclear spent fuel storage facilities. Numerical results show that the present method is very convenient for investigating the dynamic characteristics of the coupled system.

2. ANALYSIS OF COUPLED RESPONSE TO HORIZONTAL EXCITATION

A two-dimensional model of a rectangular fluid container is shown in Fig. 1 with cross-sectional dimensions of $2L_x$, H_s and t_s. The container is partially filled with liquid to a depth H_l, and is fixed to the ground with a rigid base. A Cartesian coordinate system (x,z) is used with the origin at the center of base surface.

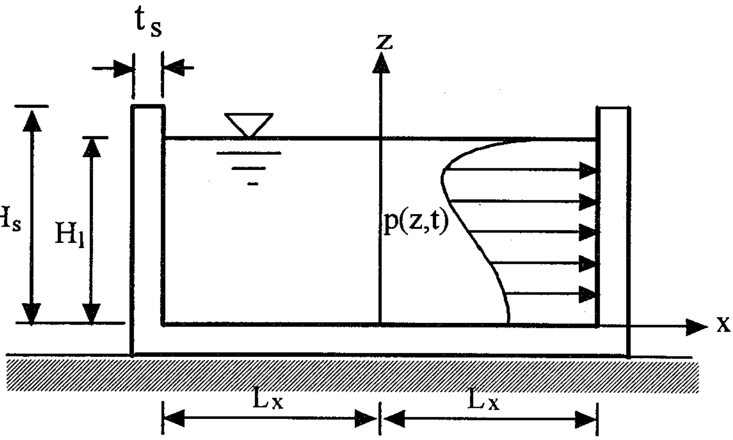

Fig. 1. Geometry and Dimensions of a rectangular container with rigid base fixed to the ground

Under a horizontal earthquake acceleration, $\ddot{G}_x(t)$, the governing equation of motion of the coupled fluid-wall structure system is expressed as

$$EI\frac{\partial^4 w(z,t)}{\partial z^4} + t_s\rho_s\frac{\partial^2 w(z,t)}{\partial t^2} = -t_s\rho_s\frac{\partial^2 G_x(t)}{\partial t^2} + p(z,t) \tag{1}$$

where $p(z,t)$ is the hydrodynamic pressure acting on the wall and ρ_s the mass density of structure. The wall is modeled as a cantilever beam fixed at the lower boundary, thus $w(z,t)$ denotes a deflection of the cantilever beam in the x-direction. The deflection can be expressed as

$$\begin{aligned} w(z,t) &= \psi_1^z(z) f_1(t) \\ &= [\cosh(a_1 z) - \cos(a_1 z) - \sigma_1(\sinh(a_1 z) - \sin(a_1 z))] f_1(t) \end{aligned} \tag{2}$$

where $\psi_1^z(z)$ is the fundamental vibration mode shape of a cantilever beam, and constants a_1, σ_1 are defined in [4]. For a more accurate solution, a sufficient number of vibration mode shapes can be used.

The motion of fluid inside a container is assumed to be governed by Laplace equation

$$\nabla^2\Phi = 0 \tag{3}$$

where ∇^2 is a Laplace operator and Φ the velocity potential. Boundary condition at $z = H_l$ is, assuming a negligible sloshing effect [1],

$$\frac{\partial \Phi}{\partial t} = 0 \qquad at\, z = H_l \tag{4}$$

and, at $z = 0$,

$$\frac{\partial \Phi}{\partial z} = 0 \quad at\, z = 0 \tag{5}$$

The velocity of a fluid particle contacting with the wall is the sum of horizontal excitation velocity and relative velocity of the wall in the x-direction, thus

$$\frac{\partial \Phi}{\partial x} = \frac{\partial w}{\partial t} + \frac{\partial G_x(t)}{\partial t} \quad at\, x = -L_x \text{ and } L_x \tag{6}$$

Hydrodynamic pressure at a point is defined as

$$p(z,t) = -\rho_l \frac{\partial \Phi}{\partial t} \tag{7}$$

where ρ_l is the mass density of fluid and can be obtained as following from Eqn. (6) by using both Eqn. (2) and a series solution satisfying Eqn. (3), (4) and (5).

$$p(z,t) = -\sum_{j=0}^{\infty} \frac{2\rho_l \tanh(k_j L_x)}{k_j H_l} \cos(k_j z)[\frac{\sin(k_j H_l)}{k_j}\frac{\partial^2 G_x(t)}{\partial t^2} + \frac{\partial^2 f_1(t)}{\partial t^2}\int_0^{H_l} \psi_1^z(z)\cos(k_j z)dz] \tag{8}$$

Galerkin method is now applied to satisfy the governing equation of motion, Eqn. (1), in a weak sense.

$$\int_0^{H_s} [EI\frac{\partial^4 w}{\partial z^4} + t_s\rho_s\frac{\partial^2 w}{\partial t^2} + t_s\rho_s\frac{\partial^2 w}{\partial t^2} - p(z,t)]\psi_1^z(z)dz = 0 \tag{9}$$

By substituting Eqn. (2) and Eqn. (8) into Eqn. (9), the generalized equation for a time function $f_1(t)$ is obtained.

$$(M^* + M^A)\frac{\partial^2 f_1(t)}{\partial t^2} + K^* f_1(t) = -L^* \frac{\partial^2 G_x(t)}{\partial t^2} \tag{10}$$

where

$$M^* = t_s\rho_s\int_0^{H_s} (\psi_1^z(z))^2 dz \tag{11}$$

$$M^A = \sum_{j=0}^{\infty} \frac{2\rho_l \tanh(k_j L_x)}{k_j H_l}[\int_0^{H_l} \psi_1^z(z)\cos(k_j z)dz]^2 \tag{12}$$

$$K^* = EIa_1^4 \int_0^{H_s} (\psi_1^z(z))^2 dz \tag{13}$$

$$L^* = t_s \rho_s \int_0^{H_s} \psi_1^z(z)dz + \sum_{j=0}^{\infty} \frac{2\rho_l \sin(k_j H_l)\tanh(k_j L_x)}{k_j^2 H_l} \int_0^{H_l} \psi_1^z(z)\cos(k_j z)dz \tag{14}$$

Eqn. (10) can be solved for a time function, $f_1(t)$, using Duhamel's integral method. When structural damping effects are significant, a modal damping term can be added to Eqn. (10).

3. ANALYSIS OF COUPLED RESPONSE TO VERTICAL EXCITATION

The governig equation of the coupled system under a vertical earthquake acceleration, $\ddot{G}_z(t)$, and the deflection form are the same as those for a horizontal excitation case. Boundary conditions for the fluid motion are different from the horizontal case in the followings.

$$\frac{\partial \Phi}{\partial z} = \dot{G}_z(t) \quad at\, z = 0 \tag{15}$$

$$\frac{\partial \Phi}{\partial x} = \frac{\partial w}{\partial t} \quad at\, x = L_x,\ and\ \frac{\partial \Phi}{\partial x} = -\frac{\partial w}{\partial t} \quad at\, x = -L_x \tag{16}$$

Unlike the horizontal acceleration case, the velocity potential is obtained as a sum of two parts: a potential related to wall vibration and the other with rigid wall in order to satisfy the boundary conditions above. By using the same procedure as in the horizontal case, the general equation for the vertical earthquake excitation can be obtained in a form similar to Eqn. (10) [4].

4. NUMERICAL RESULTS AND DISCUSSIONS

The present method is applied to a fluid container structure shown in Fig. 1 with the following dimensions and material properties: $t_S = 1.2$ m, $L_x = 9.8$ m, $H_s = 12.3$ m, $\rho_s = 2300\ \mathrm{kg/m^3}$, $E = 20.776 \times 10^9\ \mathrm{N/m^2}$, $\rho_l = 1000\ \mathrm{kg/m^3}$. The N-S component of the 1940 El Centro earthquake record with the peak acceleration 0.2g is used as input ground motion for both horizontal and vertical response analyses. The present analyses consider no structural damping for convenience.

Time history of base shear acting on a wall due to horizontal excitation when the fluid level is 11.2 m is shown in Fig. 2. The result by the present method agrees very closely with the one by the coupled BEM-FEM method [3]. Fig. 3 shows maximum hydrodynamic pressure distributions along the wall height. Due to the fluid-structure interaction effects, hydrodynamic pressure in the flexible container is greatly amplified and its distribution shape becomes similar to the one in a tall cylindrical steel tank. The difference between solutions by using a fundamental mode only and multiple modes is less than 10 % at their maximum points. This indicates that the structural reponse is dominated by the fundamental mode, and the present method using a fundamental mode only can be used as a simple procedure for design purpose. Maximum pressure distribution due to vertical excitation is plotted in Fig. 4, where amplification of the pressure is shown to be also significant.

Fig. 5 and Fig. 6 show the dependency of maximum base shear on the fluid level due to horizontal excitation and the variation of natural frequency of the coupled system with corresponding fluid level, respectively. Base shear amplifies when the natural frequency of the coupled system drops from that of dry structure as the fluid level rises. The peaks and troughs in Fig. 5 are related to undamped spectral acceleration of the input ground motion.

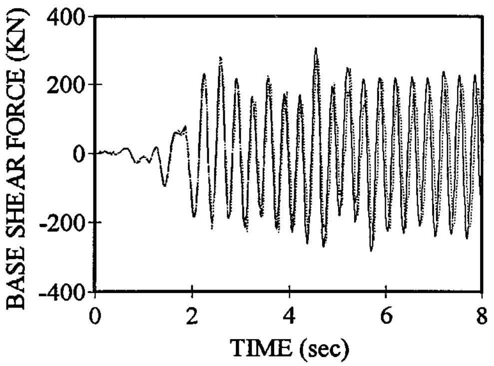

Fig. 2. Time history of base shear due to horizontal excitation

—— Present method(single mode)

········ Coupled BEM-FEM

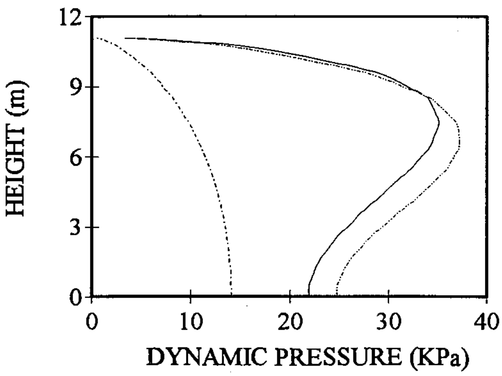

Fig. 3. Maximum hydrodynamic pressure distribution along the wall height due to horizontal excitation

-·-·-· Rigid wall

—— Present method(single mode)

········ Present method(multiple modes)

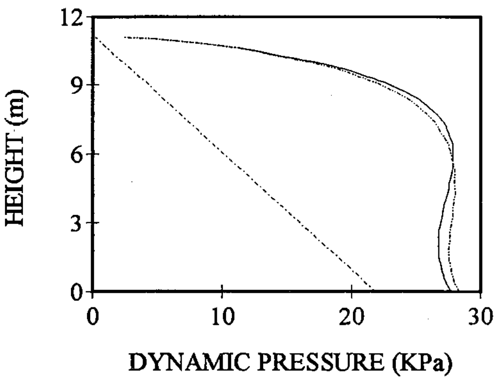

Fig. 4. Maximum hydrodynamic pressure distribution along the wall height due to vertical excitation

-·-·-· Rigid wall

—— Present method(single mode)

········ Present method(multiple modes)

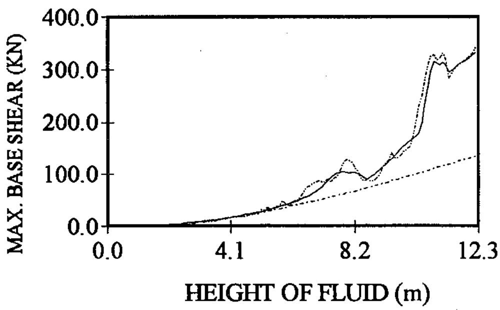

Fig. 5. Dependency of maximum base shear on the fluid level due to horizontal excitation

------ Rigid wall

—— Present method(single mode)

-------- Present method(multiple modes)

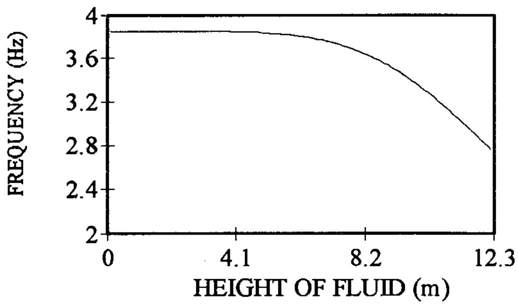

Fig. 6. Variation of natural frequency for horizontal motion with the fluid level

—— Present method(single mode)

5. CONCLUSIONS

An analytic method for the analysis of fluid-structure interaction problems in a flexible rectangular container has been presented. Example results show that the present method is very reliable and convenient for investigating the coupled response characteristics. The method can be easily used in the response spectrum analysis for aseismic design of rectangular container structures.

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