Progresses on the Computation of Added Masses for Fluid Structure Interaction

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Abstract

The problem of coupled vibrations of fluids and structures is analyzed, in the case of irrotational incompressible fluid fields the effect is modelled as an added mass matrix. The Modified Boundary Elements technique is used; a particular case (cylindrical reservoirs with sloshing) and the general case are examined.

1. Introduction

In a previous paper /1/ the authors had presented a method dealing with the computation of generalized added masses in a fluid for the particular case of axysymmetric cylinders. Basically the method makes reference to the theory of modified boundary elements (i.e. a variant of the classical method introduced by Brebbia et al /2/) and chooses a particular solution, (fitting the peculiar boundary conditions of the problem under consideration) for the potential of the fluid velocities. A method was also presented to deal with free surface effects in cylindrical reservoirs (sloshing). It is the aim of this paper to generalize the results obtained in ref./1/ and to introduce a novel element to deal with more complicated geometries.

2. The theory of modified boundary elements

The method is applicable to all the physical problems governed by differential equations and finds a classical application in the field of fluid-structure interaction. Leaving to a particular paper /3/ the general theory, it is sufficient to say that the field equation is assumed to be given by (where $\phi$ is the fluid potential).

\[
\nabla^2 \phi = 0 \quad \text{in domain} \quad \Gamma
\]

while the boundary conditions are specified in terms of displacement $\mathbf{S}$ (similar treatment is given to other types of boundary conditions) then the work lost on the boundary, i.e.:

\[
\mathbf{W} = \int_{\partial \Omega} \left( S - \mathbf{w} \cdot \nabla \phi / \nabla \phi \cdot \mathbf{n} \right) \rho \mathbf{w} \cdot \mathbf{n} \, d\mathbf{y}
\]

where $\mathbf{w}$ is the pulsation, $\rho$ density and $\mathbf{n}$ the outwards normal to the surface. If the following positions are made:
(3a) \[ \sum_m = \sum_m \beta_m (x_i) \]
(3b) \[ \varnothing = \sum_k \alpha_k \cdot \varnothing (x_i) \]
where \( \alpha_k \) is the coefficient of epansion and \( \varnothing_k \) is any function satisfying the field equation (1).

The minimization of the work lost and the application of the virtual works principle leads to the definition of a symmetric mass matrix as given by:

\[
[M] = [q]^T [p]^{-1} [q]
\]

where:

\[ q_m = \int_{m} \beta_m \varnothing_k dV \]

\[ p_{ij} = \int_{m} \left( \frac{\partial \varnothing_j}{\partial x_i} + \frac{\partial \varnothing_i}{\partial x_j} \right) dV \]

3. Axysymmetric cylindrical case

3.1 Theory

In the case of axysymmetric cylinders the following expression for \( \varnothing \) has been used:

\[ \varnothing_r = \cos \Theta \cos (\lambda j \cdot z + \beta j) + (A j J_1 (\lambda j r) + B j K_1 (\lambda j r)) \]

where \((r, \Theta, z)\) is a set of cylindrical coordinates and \( \lambda j \) is generally chosen to fit the axial boundary conditions (no flow or constant pressure). The application of this formula leads to a very simple formulation for the case of lateral coupling of the fluid the structure, in good agreement with experimental results /4/ despite its simplicity. However, the dual solution to (6) can be used as well:

\[ \varnothing_r = \cos \Theta \cosh (\lambda j \cdot z + \beta j) \left( C j J_1 (\lambda j \cdot r) + D j Y_1 (\lambda j \cdot r) \right) \]

where the \( \lambda j \) coefficients are found in a similar way to the \( \lambda j \) coefficients for (6) correlation. If it is assumed that the cylinder has radial boundaries on the radii \( r = a, b \), this leads to a transcendent equation of the type:

\[ J_1 (\lambda j \cdot r) + D j Y_1 (\lambda j \cdot r) = 0 \quad \text{for} \quad r = a, b \]

where the prime denotes differentiation. Consequently a series of \( \lambda j, \lambda j \) values are found as a function of the particular geometry. The complete solution of the problem (using both \( \varnothing_r \) and \( \varnothing_\theta \) solutions) leads to the formulation of both a mass matrix and a stiffness matrix if the free surface equation (g gravity) on \( z = l \).

\[ \gamma_{+} p_{+} \varnothing / \omega \cdot \left( \varnothing / \partial z \right)_{z=L} = \gamma_{+} \varnothing / \omega \quad \text{at} \quad z=L \]

is used, as detailed in ref./1/.

3.2 ADAM Code /5/ ADAM code uses the whole procedure as detailed in 3.1; the input data are the geometry, the characteristics of the fluid and the nodalization, the output gives:

a) the mass matrix
b) the stiffness matrix at the free surface (if the sloshing boundary conditions are specified)
c) the eigenvalue solution for case b) if the external reservoir walls are rigid.

Comparisons have been performed for various geometries as shown in Table A in the case of rigid reservoirs:

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Unit</th>
<th>Ri</th>
<th>Re</th>
<th>H</th>
<th>Frequencies</th>
<th>Comparisons</th>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>1st</td>
<td>2nd</td>
</tr>
<tr>
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<td>0.3</td>
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</tr>
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<td>48.0</td>
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<td>0.384</td>
<td>1.45</td>
</tr>
</tbody>
</table>

Th = theory                  Exp = Experimental

It should be noted that the hypotheses on the free surface (sloshing or constant pressure) are not influent on the vibratory behaviour of the reservoir wall, as already discussed in Ref. /1/.

4. General case

Most obviously the theory is applicable in an independent way from the particular choice of the function $\phi$ with the only limitation that it should satisfy (eq.1). The general case of a domain of fluid can be then solved in different ways depending from the particular type of the geometry under consideration. It might be considered the generalization of correlations 6,7 under the form:

(6) $\phi_j = \cos n \theta \cos (\lambda_j n Z + \beta j n) + (A_{nj} \cos (\lambda_j n Z + \beta j n) + B_{nj} \sin (\lambda_j n Z + \beta j n))$

and the same correlation could be used for (7'). However a much simpler and more elegant correlation can be employed referring to a particular representation of the solution to Laplace equation under the form.

(10) $\phi_{ij} = A_i / r_{ij}$

where $A_i$ represents the potential of a charge located on the $i$th node of coordinates $X_i$, $J$ denotes a point inside the domain of coordinates $X_j$ and $r_{ij}$ is the distance between the $i$th node and the $j$th point. As pointed out in the general theory of the Laplace equations the calculation of the nodal point should be on the boundary of the domain. Consequently the $\phi$ functions can be represented as a series of potentials of point charges located on the external boundary of the domain (nodes); in the same time the following functions are considered:

(11a) $R_{ij}^0 = 1 / r_{ij}$
\[(1b, d) \quad R_{ij}^{1k} = \Box R_{ij}^0 / \Box X_k = (X_k^i - X_k^j) / r_{ij}^3 \]

If then \(n_k^n\) are the principal cosines of the perpendicular to the surface of the domain on point \(j\), the displacement normal to the surface is:

\[(12) \quad \sum_j^n = \frac{1}{\Omega} \int \nabla \cdot \varphi d \Omega - \frac{1}{\Omega} \sum_i \sum_k A_l n_k^i R_{ij}^{1k} \]

and consequently the relevant coefficients are given by:

\[(5a) \quad \varphi_{mk} = \sum_j \sum_{\Omega} R_{ij}^0 S_j \]

\[(5b) \quad \varphi_{li} = \sum_j \sum_{\Omega} R_{ij}^0 S_j (R_{ij}^0 n_k^i R_{ij}^{1k} + R_{ij}^0 n_k^j R_{ij}^{1k}) \]

where \(S_j\) is the integration coefficient and the subscript \(j\) denotes the values of the function on point \(j\). This formulation leads to formulae particularly simple and compact and does not depend from the particular geometry under consideration and it can be applied whichever is the number of nodes under consideration. On the contrary a particular type of programming is opportune (dynamic memory allocation) which is included in MBE program.

5. Conclusions

In the particular case of irrotational, incompressible fluids, the effect of the fluid can be analyzed as a "added masses" matrix. Under this aspect the theory of modified boundary elements is a simple practical way of computing the added masses. Two cases have been considered:

a) the special case of cylindrical reservoirs with gravity effects (sloshing) and possible coupling between the vibrations of the fluid and of the reservoir walls, where the particular shape functions in (6), (7) are used (ADAM Code).

b) the general case of fluid fields with nodal influence functions (elementary solutions of Laplace equations) as shown in (11) (MBE Code).

References

/1/ Lazzeri L, Cecconl S, Scala M. "Generalized added masses computation for fluid structure interaction" paper B6/7, SMIRF Conf., Chicago 1983


/6/ Housner GW "Dynamic analysis of fluids on containers subjected to accelerations" TID-7024, 1976.


Basic sloshing geometry

Fig. 1

Sloshing in tank

Fig. 2

Sloshing in annular tank

Fig. 3