The Influence of Fluid Flow Characteristics on Stochastic Behaviour of Cylindrical Shell

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Abstract
In the paper the method of numerical solution of vibrational response of cylindrical shell to random exciting forces is applied to explore the influence of parallel fluid flow characteristics on the stochastic behaviour of considered shell. A physical model of cylindrical shell supported at both ends is used to demonstrate numerically the mentioned influence.

1. Introduction
Vibrations induced by a fluid flow are sometimes considered to be a secondary parameter in design, but only until a failure has been occurred. Failures resulting from the vibrations of fluidodynamic origin have not been avoided even in nuclear reactors, though extraordinary attention has been given to their safe operation. The problem under discussion has an interdisciplinary character and lies on the interface of several scientific fields. This paper applies the method of numerical solution of structural random response to explore the influence of parallel fluid flow parameters on vibrational behaviour of a cylindrical shell excited by mentioned flow.

2. Theoretical Background
A cylindrical shell under consideration is replaced by an r-degree-of-freedom system. An i-th element of the cylindrical shell is regarded as a continuum. The frequency modal properties of the cylindrical shell are known. To simplify the problem, we assume further: (1) there is no coupling between individual modes of vibration, (2) no coupling between the flow excitation and cylindrical shell response is introduced, (3) the flow random excitation, expressed in terms of the fluctuating pressure \( p(t) \) of the fluid flow where it is in a contact with the cylindrical shell, is stationary and ergodic.

Using modal analysis and Fourier transform to solve the equations of motion, we obtain an expression for the vector \( \mathbf{S}_\nu(\omega) \) of dimension \( r \) of the power spectral density of cylindrical shell displacements. In matrix symbo-
where $\tilde{W}$ is the matrix of dimension $(r \times r^2)$ of modal vectors with elements $w_\alpha^{(i)} w_\beta^{(k)}$ of the associated conservative system; $\tilde{H}(\omega)$ is the diagonal matrix of dimension $(r^2 \times r^2)$ of generalized spectral compliances with elements $H_\alpha^\omega H_\beta^\omega$; $\tilde{\tau}(\omega)$ is the vector of dimension $r^2$ of generalized spectral loadings given by

$$\tilde{\tau}(\omega) = \tilde{W} \hat{\rho}(\omega)$$

where $\tilde{W}$ is the matrix of dimension $(r^2 \times r^2)$ of modal vectors with elements $w_\alpha^{(i)} w_\beta^{(k)}$; $\hat{\rho}(\omega)$ is the vector of dimension $r^2$ with elements $\hat{p}_{ik}(\omega)$ given by

$$\hat{p}_{ik}(\omega) = \lim_{T \to \infty} \frac{1}{T} \int_0^T \int_{t_1}^t p_i(t) p_k(t) \exp(\omega(t_1-t)) \, dt_1 \, dt$$

where $T$ is the half of the realization time of random process. Introducing the coherence function $G_{p_i p_k}(i,k,\omega)$ of the exciting pressure field, eq. (3) can be rewritten

$$\hat{p}_{ik}(\omega) = S_i S_k \left( S_{p_i}(i,\omega) S_{p_k}(k,\omega) \right)^{1/2} G_{p_i p_k}(i,k,\omega)$$

where $S_i$, $S_k$ are the contact surfaces of the $i$-th, resp. $k$-th element of the cylindrical shell; $S_{p_i}(i,\omega)$, $S_{p_k}(k,\omega)$ are the power spectral density of fluctuating pressure at the places of $i$-th, resp. $k$-th element.

If an analytical expression is used for the vector $\tilde{\tau}(\omega)$ its elements are given [1]

$$\tau_{ik}(\omega) = \int_S w_\alpha(A_i) w_\beta(A_\beta) S_{p_i p_k}(A_i, A_\beta, \omega) \, dA_i \, dA_\beta$$

where $A_i$, $A_\beta$ are the places on the contact surface $S$ of the cylindrical shell; $w_\alpha(A_i)$, $w_\beta(A_\beta)$ are the coefficients of the distribution of the displacement amplitudes of the $\alpha$-th resp. $\beta$-th mode of vibration at $A_i$, resp. $A_\beta$; $S_{p_i p_k}(A_i, A_\beta, \omega)$ is the cross (spatial) power spectral density of the pressure field at the places $A_i$ and $A_\beta$ on the contact surface of the cylindrical shell. If $G_{p_i p_k}$ is used, instead of $S_{p_i p_k}$, the generalized spectral loadings $L_{\alpha \beta}(\omega)$ related to a unit surface can be expressed as

$$L_{\alpha \beta}(\omega) = S^2 \int_S w_\alpha(A_i) w_\beta(A_\beta) G_{p_i p_k}(A_i, A_\beta, \omega) \, dA_i \, dA_\beta.$$
spectral loadings in axial and in circumferential directions, and a total
generalized spectral loadings. To express the coherence function of a homo-
genous turbulent flow a Bessel function of the first kind of zero order,
dependent on fluid flow parameters, was used in agreement with e.g. Lesueur,
Milan and Payen [3].

The vector of the mean square displacements $\mathbf{\tilde{y}}$ of dimension $r$ can be
obtained from the power spectral density of the displacements through inte-
gration over all the angular frequencies $\omega$,
\[
\mathbf{\tilde{y}} = (2\pi)^{-r} \int_{0}^{\infty} S_y(\omega) \, d\omega.
\]  

(7)

3. Numerical Results

To study the influence of fluid flow characteristics on dynamic beha-
viour of cylindrical shell we shall use a PWR core barrel physical model
[4]. It is a cylindrical shell of dimensions: $t/a = 0.046$, $a/l = 0.231$
($a$ - middle surface radius, $t$ - thickness, $l$ - length) with continuous
support at both ends. The shell will be divided into $\bar{n} = 30$ identical parts
along its length and into $\bar{m} = 80$ identical parts along its circumference,
so we get a symmetrical division of $\bar{m} \bar{n}$ elements and thus a system of
$\bar{r} = 2400$ freedom degrees. This number of freedom degrees is sufficient to
get reasonable results of the shell spectral compliances [5]. The frequency
spectrum of the cylindrical shell was fixed by a semi-experimental way in
both air and water. The values of natural frequencies and of generalized
damping ratios were considered for 24 selected vibration modes including
the influence of the liquid and of limited surroundings [4, 6].

Generalized spectral loadings were computed for the various fluid flow
parameters such as: mean flow velocity $v$, turbulent velocity $u$, pressure
wave correlation length $\lambda$. Some results of computations are plotted in
figs. 1 to 6. The courses of total generalized spectral loadings for some
joined factors at different correlation lengths are shown in figs. 1 to 4,
while fig. 5 presents the spectral loading courses at different turbulent
velocities due to their dependence on exciting frequencies. An illustration
of the obtained values of the circumferential generalized spectral cross
loading is displayed in fig. 6. Comparing the plotted cross loadings with
the joined loading $L_{22}$ it is clear that the cross terms are substantially
smaller and become very quickly damped with the increasing frequency. As
for the correlation length we can say with its growth even the spectral
loading values are rising and their maxima are shifted to the region of
higher exciting frequencies.

Some results of computations of the power spectral density of radial
displacements are plotted in figs. 7 to 10. In figs. 7 and 8 the courses
are concerned with a certain place on the cylindrical shell surface marked
as $l = 1121$. While evaluating the results we find out that: (1) the response
due to the cross factors(lower curve) is negligible in comparison with the
response induced by the joined factors (upper curve), (2) the turbulent velocity dependence on the frequency of turbulent eddies (rising - fig. 7, sinking - fig. 8) has a significant effect on the magnitude and on the character of the response. As an example of the computed values of the psd of displacements at each element of the cylindrical shell, figs. 9 and 10 present the courses of the mentioned quantity at two different places. The courses differ not only in the magnitude levels, but also in the resonance peaks.

The graphical expressions of the results of computations of the vectors of the mean square displacements at two different, both axial and circumferential, correlation lengths are presented in figs. 11 and 12. It is obvious that ms displacements of the cylindrical shell are significantly dependent on a fluid flow parameter.

4. Conclusions

From the results presented above and concerned with the influence of fluid flow parameters on the stochastic behaviour of cylindrical shell excited by a parallel fluid flow, we can conclude:

(a) The turbulent velocity and its dependence on exciting frequencies affects the magnitude as well as the course of spectral loadings, and consequently, both the magnitude and the character of the response of the cylindrical shell.

(b) The mean flow velocity affects the magnitude and the courses of the spectral loadings, also the magnitude of the response, though the character of the cylindrical shell response remains untouched by a change of this velocity.

(c) The values of the maxima of generalized spectral loadings in axial and in circumferential directions are independent of the flow parameters. With growing pressure wave correlations lengths, however, they shift into the region of higher exciting frequencies.

(d) The courses of psd of displacements, with regards to the magnitude and number of resonance peaks, are dependent on the location of considered place on the cylindrical shell surface.

(e) The distributions of the ms displacements are symmetrical to the plane dividing the axis of cylindrical shell in half, non harmonic, with several waves around the circumference which have sufficiently steep tops.

(f) A symmetry in the ms displacement distributions can be employed to a substantial cut in the computing time.

(g) The knowledge of the course and distribution of the ms displacements can serve as a basis for the determination of ms stress distributions in the cylindrical shell.
References


Fig. 5. Total generalized spectral dimensionless loadings at parameters $u, v$.

Fig. 6. Circumferential generalized spectral dimensionless cross loadings at parameter $\lambda$.

Fig. 7. Power spectral displacement density due to joined and cross factors at parameters $u, v$.

Fig. 8. Power spectral displacement density due to joined and cross factors at parameters $u, v$.

Fig. 9. Power spectral displacement density at the place $x, \varphi$.

Fig. 10. Power spectral displacement density at the place $x, \varphi$.

Fig. 11. Mean square displacement distribution at parameters $u^2, v^2$.

Fig. 12. Mean square displacement distribution at parameters $u^2, v^2$.