Active Protection of Domains of Coupled Solid-Fluid Model by Means of Optimal Control Theory

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Abstract

The vibration analysis of solid structures immersed in a fluid is required in industrial problems as nuclear fuel bundles and exchanger tubes. The structure to be analized consists of a group of circular parallel cylinders,elastic rods immersed in an ideal incompressible infinite fluid.

The aim of the paper is to propose solutions to the problem of active control of the solid-fluid coupled system using linear optimal control theory,the objective being to minimize a mechanical energy in some domain of the structure to be protected.For the application,the minimization of a weighted vibration energy is achieved by means of two fixed point actuators.The cylin ders group is submitted to one fixed point perturbing excitation.

1. Introduction

Flow-induced vibrations is of particular interest in the design and analysis of nuclear reactor components.Interaction between fluid and solid has been recognized to be an important factor in the study of flow-induced vibrations and has studied by many authors [1], [2], [3] .Closed form solutions for simple geometries are available.In addition to nuclear fuel bundles,other structural components consisting of a group of circular cylinders ranging from heat exchanger tubes,piles,parallel pipelines and bundled transmission lines,frequently experience vortex-excited oscillations,fluidelastic instability,and other types of flow-induced vibrations.The equation of motion for a cylinders group include fluid coupling,damping,drag force,centrifugal force,Coriolis force,fluid pressure,gravity effect and axial tension.The group cylinder vibrations,is modelled from mathematical point of view as a parameters distributed system.The active control of these structure types constitutes an important field of interest.The aim of the paper is to propose solutions to the problem of active control of the solid-fluid coupled system using linear optimal control theory,the objective being to minimize a mechanical energy in some domain of the structure to be protected.Also, various protection criteria compatible with the state analysis are proposed and after presentation of the observer problem, the solution of the optimal regulator problem is developed.For the application,the minimization of a weighted vibration energy is achieved by means of two fixed point actuators.The cylinders group is submitted to one fixed point perturbing excitation with deterministic signal.The results allow one to proceed to a qualitative comparison of the reliability of chosen criteria.

2. Equations of motion of a group of circular cylinders in axial flow.

Consider a rod in a group of k circular cylindrical rods immersed in a fluid flowing at a velocity V parallel to the z axis(Fig.1).The rod has linear density(mass per unit length)m,flexural rigidity EI and total length l.

A small element dz of the rod is shown in fig.2,where the elastic forces,hydrodynamic forces,damping force and excitation forces associated with the motion in the x-z plane are given.The presure effect for nonuniform pressure on a cylindrical rod was determined in [2] ;the forces acting on a single rod set obliquely to a stream of fluid were discussed in [3] and the equations of motion for a group of k cylinders was given by S.S.Chen [1] :

$$\begin{aligned} & E_pI_p\frac{\partial^4 u_p}{\partial z^4}+\mu_pI_p\frac{\partial^5 u_p}{\partial t\partial z^4}+\sum_{q=1}^{2K}\delta_{pq}\left(\frac{\partial}{\partial t}+V\frac{\partial}{\partial z}\right)^2 u_q-\left[T_p(\ell)-(m_pg-\rho R_pV^2\bar{C}_p)(\ell-z)+\rho A_p\right]\frac{\partial^2 u_p}{\partial z^2} \\ & -\left(m_pg+\frac{\partial p}{\partial z}A_p\right)\frac{\partial u_p}{\partial z}+\rho R_pV\bar{C}_p\left(\frac{\partial u_p}{\partial t}+V\frac{\partial u_p}{\partial z}\right)+C_p\frac{\partial u_p}{\partial t}+m_p\frac{\partial^2 u_p}{\partial t^2}=f_p,\quad p,q=1,2,3,\dots,2K \end{aligned} \qquad (1)$$

where $u_p(z,t)$,E_pI_p,C_p and f_p are the displacement,flexural rigidity,damping coefficient,and excitation in the x direction,and the corresponding quantities in the y direction are u_{K+p} ,E_{K+p} I_{K+p} ,C_{K+p} and f_{K+p} ,T (l) is the axial tension at the downstream end,C_p is a drag coefficient associated with skin friction.For convenience,a subscript p is used for to denote the variables associated with cylinder p in the x-z plane,while p+k in the y-z plane.

The appropriate boundary condition associated with the equations of motion are:

for z = 0

$$k_pu_p+E_pI_p(\partial^3u_p/\partial z^3)=0;\quad c_p(\partial^2u_p/\partial z^2)=0 \qquad (2)$$

for z = 1

$$k'_pu_p-E_pI_p(\partial^3u_p/\partial z^3)=0;\quad c'_p(\partial u_p/\partial z)+E_pI_p(\partial^2u_p/\partial z^2)=0$$

where c_p and c'_p are torsional spring constants and k_p and k'_p are displacement spring constants.Eq.(1) include the fluid coupling;hence,it can be used for studies of both stability and response problems of fuel bundles.

In this analysis,it is assumed that all cylinders are of the same length and have the same type of boundary conditions.Let

$$u_p(z,t)=\sum_{n=1}^{\infty}h_{pn}(t)\,\psi_n(z) \qquad (3)$$

where $\psi_n(z)$ is the nth orthonormal function of the cylinders in vacuo,i.e.

$$\frac{1}{\ell}\int_0^{\ell}\psi_m(z)\,\psi_n(z)\,dz=\delta_{mn} \qquad (4)$$

where l is the length of the cylinders.Using eqs.(1),(3) and (4) gives

$$a_{pnqm}\ddot{h}_{qm}+b_{pnqm}\dot{h}_{qm}+c_{pnqm}h_{qm}=f_{pn}\,,\quad p,q=1,2,3,\dots,2K;\; mn=1,2,\dots,\infty \qquad (5)$$

where

$$a_{pnqm} = (m_p \delta_{pq} + \gamma_{pq})\delta_{mn} \; ; \quad b_{pnqm} = [(\mu_p/E_p) m_p \omega_{pn}^2 + \rho R_p V \bar{C}_p + C_p]\delta_{pq}\delta_{mn} + 2V\gamma_{pq} c_{mn} \; ;$$

$$c_{pnqm} = m_p \omega_{pn}^2 \delta_{pq}\delta_{mn} - \{[T_p(\ell) + p_0 A_p - (m_p g - \rho R_p V^2 \bar{C}_p)\ell] d_{mn} + [m_p g - \rho R_p V^2 \bar{C}_p - p_1 A_p] e_{mn}\} \cdot \delta_{pq} + V^2 \gamma_{pq} d_{mn} + (\rho R_p V^2 \bar{C}_p - m_p g + p_1 A_p)\delta_{pq} c_{mn} \; ; \tag{6}$$

$$c_{mn} = \frac{1}{\ell}\int_0^\ell \frac{\partial \psi_m}{\partial z}\psi_n dz \, , \quad d_{mn} = \frac{1}{\ell}\int_0^\ell \frac{\partial^2 \psi_m}{\partial z^2}\psi_n dz \, , \quad e_{mn} = \frac{1}{\ell}\int_0^\ell \frac{\partial^2 \psi_m}{\partial z^2}\psi_n z dz, \quad f_{pn} = \frac{1}{\ell}\int_0^\ell f_p \psi_n dz$$

and ω_{pn} is the nth natural frequency of cylinder p in vacuo.The fluid pressure has been assumed to have the form $p = p_0 - p_1 z$,where p_0 corresponds to the fluid pressure at the upstream end and p_1 is the pressure gradient.Eq.(5)consists of an infinite number of differential equations.However,typically only a finite number of equations are selected from case to case according to the desired accuracy.It is assumed that the maximum value for m and n in eqs.(5) is taken to be N.Eq.(5) may be written in matrix form:

$$[M]\{\ddot{U}(t)\} + [D]\{\dot{U}(t)\} + [K]\{U\} = [F]\{f(t)\} \tag{7}$$

where $[M]$, $[D]$ and $[K]$ are square matrices and $\{U\}$, $\{f\}$ are vectors.The dimension is (2k×N).When the fluid is flowing,$[K]$ is not necessarily symmetric and positive definite and $[D]$ is not necessarily proportional to $[M]$ or $[K]$.

Considerable simplification can be made when fluid is stationary and internal damping,gravity effect and fluid pressure are neglected.In this case,eq.(1)become

$$E_p I_p \frac{\partial^4 u_p}{\partial z^4} + C_p \frac{\partial u_p}{\partial t} + m_p \frac{\partial^2 u_p}{\partial t^2} + \sum_{q=1}^{2k} \gamma_{pq} \frac{\partial^2 u_q}{\partial t^2} = f_p \qquad p,q = 1,2,3,\ldots,2k \tag{8}$$

Following the procedure as in case of flowing fluid,using eqs.(1),(2) and (8) yields

$$m_p \ddot{h}_{pn} + \sum_{q=1}^{2k} \gamma_{pq} \ddot{h}_{qn} + 2 m_p \xi_{pn} \omega_{pn} \dot{h}_{pn} + m_p \omega_{pn}^2 h_{pn} = f_{pn} \; , \quad p,q = 1,2,3,\ldots,2k \quad n = 1,2,3,\ldots,\infty \tag{9}$$

where ω_{pn} ,as defined previously,is the nth frequency of cylinder p in vacuo and

$$\xi_{pn} = C_p / 2 m_p \omega_{pn} \quad , \quad f_{pn} = \frac{1}{\ell}\int_0^\ell f_p \psi_n dz \tag{10}$$

Note that eqs.(9) can be applied to all values of n.For each n,there are 2k equations which are coupled.However,there is no coupling among the equations for different n.This is true for a group of cylinders with the same type of boundary conditions and of the same length.

3. Active protection problem.

From eq.(7) one can write the state equations of the structure where the structural state vector is constituted by U(t) and U(t):

$$\begin{bmatrix} \ddot{U}(t) \\ \dot{U}(t) \end{bmatrix} = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}D \end{bmatrix} \begin{bmatrix} U(t) \\ \dot{U}(t) \end{bmatrix} + \begin{bmatrix} 0 \\ F \end{bmatrix} [f(t)] \tag{11}$$

The mechanical system is assumed to be subjected to y distinct perturbation sources.The state equation of its ith component can be written as

$$\begin{bmatrix}\dot{\bar{f}}^i(t)\\ \dot{\bar{f}}_1^i(t)\\ \dot{f}^i(t)\end{bmatrix}=\begin{bmatrix}0 & I & 0\\ -F^i & -E^i & V^i\\ 0 & 0 & A^i\end{bmatrix}\begin{bmatrix}\bar{f}^i(t)\\ \bar{f}_1^i(t)\\ f^i(t)\end{bmatrix}+\begin{bmatrix}C^i\\ B^i\\ 0\end{bmatrix}w_1^i(t),\ i=1,y \quad (12)$$

where $f^i(t)$ denotes the ith deterministic perturbation signal with d_i dimension, $\bar{f}_1^i(t)$ denotes a first order Markov process output vector, $\bar{f}^i(t)$ denotes the output of the Markov process; it represents the ith perturbation signal on the structure. A^i is a square matrix defining the shape of the ith deterministic signal $f^i(t)$ according to some given initial conditions, d_i is related to the desired approximation quality of the identification process, E^i and F^i are square matrices determined by the configuration of the perturbation device or by the identification process, V^i is a rectangular matrix weighting the contribution of the deterministic signal onto the output vector of the Markov process, B^i and C^i are square matrices determined by the autocorrelation matrix of the actual stochastic perturbation signal $\bar{f}^i(t)$, w_1^i is a vector whose components are uncorrelated white Gaussian noise with intensity $\mathcal{D}_1^i$. For the ith perturbation device, the equivalent simulated system representation of Fig.3 is proposed. In order to define the contribution of each of the $f^i(t)$ vector components on the structure to be protected, one must determine a load matrix F^i according to the nature and distribution of the load device on the structure; F^i is a rectangular matrix.

3.1 Control system.

One can define the augmented state equation from eqs.(11) and (12)

$$\dot{X}(t) = A\,X(t) + B\,u(t) + N\,w_1(t) \quad (13)$$

where u(t) is the vector of the actuator force intensities, or control vector S is a **rectangular** submatrix of the matrix B determined by the nature and the distribution of the actuator loads on the structure, it being assumed that there are a number e of actuator loads. The subsystem representation corresponding to the substate $[U\ \dot{U}]^T$ given by eq.(11) is completely controllable for the control vector u(t) given by eq.(13) if and only if the rank of the controllability matrix is 2n. Here the controllability matrix can be written

$$\left[\begin{bmatrix}0\\S\end{bmatrix}\ \vdots\ (A_1)\begin{bmatrix}0\\S\end{bmatrix}\cdots(A_1)^{2n-1}\begin{bmatrix}0\\S\end{bmatrix}\right], \qquad A_1=\left[\begin{array}{c|c}0 & I\\ \hline -M^{-1}K & -M^{-1}V\end{array}\right]$$

3.2 Protection criteria

The controlled vector is defined by $Z(t)=H_1U(t) + H_2\dot{U}(t)$. The matrices H_1 and H_2 are determined by the protection criteria defined on the structure [4] . Therefore, one can define the performance index to be minimized by using the Pontryagin maximum principle:

$$\mathcal{E}(t_2) = E\left[\int_{t_1}^{t_2}(\mathcal{L}(Z)+u^TRu)dt + X^T(t_2)P_0X(t_2)\right] \quad (14)$$

where R is a positive definite symmetric matrix of control regulation and P_0 is a non-negative definite matrix which allows one to give minimization conditions on the final substate $[U(t_2),\dot{U}(t_2)]$ [5], E[..] denotes the mathematical expectation, $\mathcal{L}(Z)$ is a quadratic form of Z(t):

$$\mathcal{L}(Z) = [U^T; \dot{U}^T] \left[\begin{array}{c|c} L_{11} & L_{12} \\ \hline L_{21} & L_{22} \end{array}\right] \left[\begin{array}{c} U \\ \hline \dot{U} \end{array}\right] \qquad L = \left[\begin{array}{c|c} L_{11} & L_{12} \\ \hline L_{21} & L_{22} \end{array}\right]$$

L being the non-negative definite protection matrix.The weighted energy to be minimized on the structure is defined by the elements of this matrix.If one intends to protect some domain of a two-dimensional structure against its vibrations,one can define the time dependent quadratic form to be minimized as

$$\mathcal{L}(Z(t)) = \iint_{\Omega} \left(\rho_1 u_n^2 + \rho_2 \left(\frac{\partial u}{\partial \eta}\right)^2 + \rho_3 \left(\frac{\partial u}{\partial \mu}\right)^2 + \phi_1 \dot{u}_n^2 + \phi_2 \left(\frac{\partial \dot{u}_n}{\partial \eta}\right)^2 + \phi_3 \left(\frac{\partial \dot{u}_n}{\partial \mu}\right)^2 \right) d\eta\, d\mu$$

Therefore the protection matrix can be written as

$$[L_{11}]_j^i = \iint_{\Omega} \left[\rho_1 e_i e_j + \rho_2 \frac{\partial e_i}{\partial \eta} \frac{\partial e_j}{\partial \eta} + \rho_3 \frac{\partial e_i}{\partial \mu} \frac{\partial e_j}{\partial \mu} \right] d\eta\, d\mu = [H_1^T H_1]_j^i$$

$$[L_{22}]_j^i = \iint_{\Omega} \left[\phi_1 e_i e_j + \phi_2 \frac{\partial e_i}{\partial \eta} \frac{\partial e_j}{\partial \eta} + \phi_3 \frac{\partial e_i}{\partial \mu} \frac{\partial e_j}{\partial \mu} \right] d\eta\, d\mu = [H_2^T H_2]_j^i$$

$$[L_{12}]_j^i = [L_{21}]_j^i = 0 \quad \text{for} \quad i = 1,n \quad \text{and} \quad j = 1,n$$

Here η and μ denote the co-ordinates defined for the structure configuration, ρ_i and ϕ_i are non-negative definite weighting functions.The functions ρ_i are relative to the displacements and the functions ϕ_i are relative to the velocities.e_i are the eigenfunctions of the structure,linear independently values. The optimal solution of the stochastic linear optimal regulator problem is to choose the input according to the linear control law

$$u(t) = - F^{o}(t)\hat{X}(t)$$

where the optimal control matrix is given by

$$F^{o}(t) = (R)^{-1} \mathcal{B}\, P(t)$$

Here P(t) is the solution of the differential Riccati equation

$$-\dot{P}(t) = D - P(t)\mathcal{B}(R)^{-1}\mathcal{B}^T(t) + A^T P(t) + P(t)A$$

and $P(t_2) = P_0$ is given.

4. Numerical simulation.

If the matricial equation for stationary fluid,similarly to (7) after the substitution $\{h\} = [P]\{W\}$ is premultiplying by the transpose P^T it is reduced to that of a single oscillator.It is assumed that the group of cylinders are assimilated with a beam simply supported at its boundary points.One point perturbing force f_1 is applied on the beam at the abscissa $z_1 = 0.30$.The corresponding excitation signal is defined as $f_1(t) = e^{-t}\sin 20t$,the corres-period is 0.1π s. and it is located between the two first eigenperiods of the beam.The actuator forces are realized by two point forces f_2 and f_3 located respectively at abscissa z_2=0.43 and z_3=0.57(Fig.4): $u(t) = [f_2(t) \quad f_3(t)]^T$

It is intended to protect domain D_p limited by the abscissae z_3 and 1.The controlled wector is defined as $Z(t)=H_1 U(t)$ where $[H_1^T H_1]_j^i = L_{11}$ [4],[5] ;it is intended to minimize the perturbation energy relative to the average deflection magnitude in domain D_p .(Global protection in D_p).The results are presented in Fig.5.

5. Conclusions.

The object of this study is to determine a general model to calcula·

te an optimal control system for a coupled solid-fluid model consists of a group of circular parallel cylinders in order to satisfy a given protection criteria.The energy which is required to regulate the vibration over the domain of protection D_p may be minimized.It is necessary to make a studies for a more realistic models.

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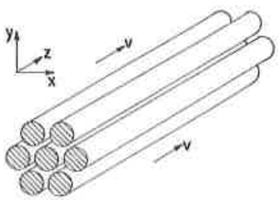


Fig. 1

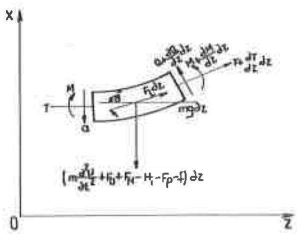


Fig. 2

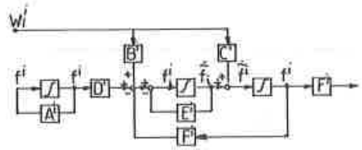
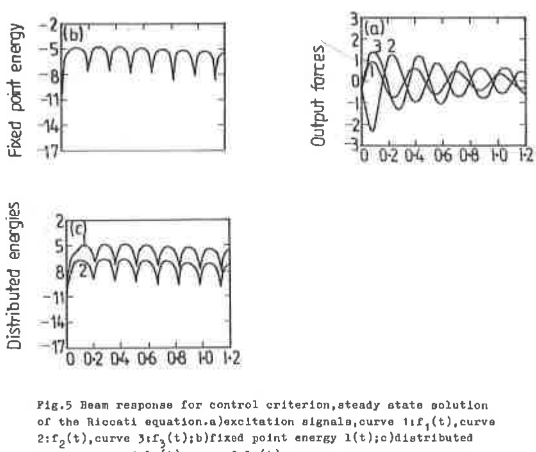


Fig. 3



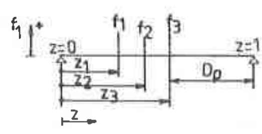


Fig. 4

Fig.5 Beam response for control criterion,steady state solution of the Riccati equation.a)excitation signals,curve 1:$f_1(t)$,curve 2:$f_2(t)$,curve 3:$f_3(t)$;b)fixed point energy l(t);c)distributed energy,curve 1:$l_1(t)$,curve 2:$l_2(t)$.