Feasibility Tests of Iterative Perturbation Method in Fluid-Structure Interaction Analysis

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Abstract

To deal with the flow-induced free vibration problems, many numerical methods have been proposed for efficient structural analysis. Assuming that fluid is linear and incompressible, an interaction between fluid and structure can be considered as an additive mass. However, since such additive mass is of full-matrix type, it is costly to make eigenvalue analysis. In the present method, the variations of eigenpairs due to coupling are calculated only by using eigendata in the uncoupled state. Hence we are free from the above eigenvalue problems with full matrices. The proposed approach is based on an iteration process, and its behaviors are dependent on the values of perturbations and parameters. In this paper, some feasibility tests are given to observe the influence of aspect ratio and the employed parameters on the natural frequncies in the coupled state.

1. Introduction

Coupling eigenvalue analysis is necessary to verify structural integrity against flow-induced vibrations. Since such an eigenvalue analysis is far more costly than the uncoupled one, several numerical approaches have been proposed to develop more efficient algorithms [1], [2], [3]. Most of them make less use of eigendata in the uncoupled state, and, furthermore, an alternative procedure is demanded to repeat these coupling eigenvalue analyses in the design phase without excessive increase of computational costs. The author [4], [5], [6] has proposed a numerical method for perturbation analysis of matrix eigenvalue problem arising in finite element calculations for free vibration of structures; applying this method to a flow-induced free vibration problem, eigenpairs in the updated or coupled state are obtained efficiently only by using eigendata of structures. Since the proposed method can still utilize the band property of mass and stiffness matrices, we can avoid an increase of computational costs due to consideration of an additive mass of full-matrix.

In the present paper, an outline of the present approach is shown with some comments on calculation techniques for iteration process. To see effectiveness of the proposed method, we take for a numerical example free vibration problem of a beam partially submerged in water chamber with trapezoidal cross-section. In the feasibility tests, we observe the sensitivity of shift parameter and dimension of subspace to iterative behaviors. In addition, eigenpairs are calculated for various values of aspect ratio to find out the change of natural frequencies against the values of perturbations.

2. Outline of fundamental concept

In the finite element analysis of flow-induced vibrations of structures, we often consider an algebraic eigenvalue problem,

$$[K]\{U\} = \lambda[M]\{U\} + [C]\{P\} \quad , \quad ([M] = \rho_S[M']) \quad , \quad (1) \qquad\qquad [H]\{P\} = \lambda\rho_F[C]^t\{U\} \quad , \quad (2)$$

where λ is the eigenvalue, and, $\{U\}$ and $\{P\}$ are the associated eigenvectors for nodal displacement of structure and pressure of fluid, respectively. $[K]$ and $[M]$ are the stiffness and mass matrices of order n for elastic structure, while $[H]$ and $[C]$ respectively denote the hydrodynamic mass matrix of fluid and the coupling coefficient matrix between structure and fluid. Both ρ_S and ρ_F are the density of mass for structure and fluid, respectively. No pressure terms appear in the right-hand side of eq. (2), since fluid is assumed to be linear; i.e. it is inviscous, incompressible and slow. Assuming that $[H]$ is invertible, then $\{P\}$ is expressed by

$$\{P\} = \lambda \rho_F [H]^{-1} [C]^t \{U\} \quad . \qquad (3)$$

Substituting eq. (3) into (1), we have

$$[K]\{U\} = \lambda\{ [M] + \rho_F[C][H]^{-1}[C]^t \}\{U\} \quad . \qquad (4)$$

The newly built matrix $[M_a] = \rho_F[C][H]^{-1}[C]^t$ is so-called an additive mass, which is symmetric and full, while $[K]$ and $[M]$ are symmetric band matrices. Hence, conventional numerical solvers are not so efficient for such eigenvalue problems with full matrices. Furthermore, it appears to be difficult to understand the influence of coupling interactions on the dynamic responses in coupled state, since the eigenpairs for coupled eigenvalue problems are calculated independently of those in the unperturbed or uncoupled state. Then, we need a certain method to make full use of eigendata in uncoupled state.

Let us solve the above eigenvalue problem (4), based on the eigenpairs $\{\lambda_j, \phi_j\}$ $(1 \le j \le r \le n)$ in the uncoupled state. Considering a specific eigenpair $\lambda_i, \{U_i\}$ $(1 \le i \le m \le r)$ in the coupling state, we can generally express them by

$$\lambda_i = \lambda^* + \mu_i, \qquad \{U_i\} = \sum_{j=1}^{r} \alpha_j\{\phi_j\} + \{\psi_i\}, \qquad (5)$$

where λ^* is a so-called shift parameter, which must be appropriately chosen, μ_i an unknown deviation of λ_i from λ^*, α_j an unknown scalar, and $\{\psi_i\}$ an unknown vector M-orthogonal to $\{\phi_j\}$'s. Substituting eq. (5) into (4), we find

$$([K]-\lambda^*[M])\{\psi_i\} = ([A]+\mu_i[B]) \sum_{j=1}^{r} \alpha_j\{\phi_j\} + (\lambda^*[M_a]+\mu_i[B])\{\psi_i\} \quad , \qquad (6)$$

where $[A] = \lambda^*([M]+[M_a])-[K]$ and $[B] = [M]+[M_a]$. Since λ^* is chosen not to coincide with any values of λ_j's, $[K]-\lambda^*[M]$ is invertible. Then $\{\psi_i\}$ is rewritten by

$$\{\psi_i\} = ([K]-\lambda^*[M])^{-1} \{([A]+\mu_i[B]) \sum_{j=1}^{r} \alpha_j\{\phi_j\} + (\lambda^*[M_a]+\mu_i[B])\{\psi_i\} \quad . \qquad (7)$$

Since $\{\phi_j\}^t[M]\{\psi_i\} = 0$ and $\{\phi_j\}^t[K]\{\psi_i\} = 0$, the right-hand side of eq. (6) is orthogonal to $\{\phi_j\}$'s. Hence the associated solvability condition of eq. (6) is given by

$$0 = [A^*]\{\alpha\} + \mu[B^*]\{\alpha\} + (\lambda^*+\mu)\{\beta\} \quad , \qquad (8)$$

where $[A^*]$ and $[B^*]$ are the r-th order matrices, whose (j,k)-th components are given by $\{\phi_j\}^t[A]\{\phi_k\}$, and $\{\phi_j\}^t[B]\{\phi_k\}$, respectively, $\{\beta\}$ the r-th order vector whose j-th component is $\{\phi_j\}^t[B]\{\psi_i\}$, and $\{\alpha\}$ the r-th order unknown vector defined by $\{\alpha\}^t = [\alpha_1\ \alpha_2 \ \ldots\ \alpha_r]$. It is to be noted that $\{\psi_i\}$ is small when the coupling is relatively weak, and then $\{\beta\}$ becomes small. Especially, in the uncoupled state, where $\{\beta\} = 0$, and eq. (8) coincides with the eigenvalue problem of the r-th order:

$$[A^*]\{\alpha\} = \mu^*[B^*]\{\alpha\} \quad (\mu^* = -\mu) \ . \tag{9}$$

Here, let us use a subspace spanned by the eigenpairs $\mu_j, \{\chi_j\}$ $(1 \leq j \leq r)$ of problem (9). Imposing a normalization condition on α(or U), we can explicitly express the unknown parameters μ and α:

$$\mu_i = -(\mu_i^* + \lambda^*\beta_i^*) \ / \ (1+\beta_i^*) \ , \quad (10) \qquad \alpha_{ii}^* = 1 \ ; \quad \alpha_{ij}^* = -(\lambda^*+\mu_i) \ / \ (\mu_j^* + \mu\beta_j^*) \ , \quad (11)$$

where a scalar α_{ij}^* $(= \{\chi_j\}^t[B^*]\{\alpha_i\})$ coincides with a coeffcient of formal series expanded by $\{\chi_j\}$'s for $\{\alpha\}$, and β_j^* is given by $\{\chi_j\}\{\beta\}$. Here, $\alpha_{ii}^* = 1$ corresponds to the normalization condition to eliminate the indefiniteness of obtained eigenvectors in the coupled state. Since μ_i and α_{ij}^* are functions in unknown vector $\{\psi_i\}$, $\{\psi_i\}$ is still undeterminable directly from eq. (6). Then we consider an iteration process to solve the implicit relation (6) in $\{\psi_i\}$.

At the first stage of this iteration process, we must assume the starting function $\{\psi_i^{(0)}\}$ of $\{\psi\}$. Once $\{\psi_i^{(0)}\}$ is provided, μ_i and $\{\alpha_i\}$ can be determined from eqs. (10) and (11). For example, $\{\psi_i^{(0)}\}$ is employed like $\{\psi_i^{(0)}\} = 0$, and then the first approximation $\mu^{(1)}$ and $\{\alpha_i^{(1)}\}$ of μ_i and $\{\alpha\}$ are respectively described by

$$\mu_i^{(1)} = -\mu_i^* \ , \quad \alpha_{ii}^{*(1)} = 1 \ ; \quad \alpha_{ij}^{*(1)} = -(\lambda^*+\mu_i^*) \ / \ \mu_j^* \quad (i \neq j) \ . \tag{12}$$

Using the obtained parameters $\mu_i^{(1)}$ and $\{\alpha_i^{(1)}\}$, we can determine $\{\psi_i^{(1)}\}$ from eq. (6):

$$\{\psi_i^{(1)}\} = ([K]-\lambda^*[M])^{-1} \{ \sum_{j=1}^{r} \alpha_j^{(1)} ([A]+\mu^{(1)}[B])\{\phi_j\} + (\lambda^*[M_a]+\mu^{(1)}[B])\{\psi^{(0)}\} \}. \tag{13}$$

At the successive stages, we calculate the k-th approximation of μ , $\{\alpha\}$ and $\{\psi\}$ by using Eqs. (10), (11) and (6) in order. The iteration process is stopped for a certain stage by some termination conditions, and the obtained approximations are employed as convergent solutions. As may be proved easily, the k-th approximations $\{\psi_i^{(k)}\}$ of $\{\psi_i\}$ $(k \geq 2)$ are M-orthogonal to $\{\phi_j\}$'s if $\{\psi_i^{(0)}\}$ is so, and the approximate eigenpair converges to an exact finite element solution if additive mass is sufficiently small, that is, if we consider sufficiently weak coupling interactions between structure and fluid.

3. Calculation techniques

The proposed method is compared with some conventional method, where coupled eigenvalue problem is solved independently of the uncoupled one. The usual methods must directly consider an algebraic eigenvalue problem with full coefficient matrix, while an additive mass of full-matrix appears only in the multiplications between matrix and vector for the present method. Moreover, we can still utilize the band property of stiffness and mass matrices for necessary inverse operation of $([K]-\lambda^*[M])$ in eq. (7). To deal with such inverse operations, we can use the conventional solution of linear simultaneous equations; Gauss elimination technique. In order to repeat coupled eigenvalue analyses for various values of geometric parameters, we may solve the uncoupled problem only once, and hence the common inverse matrix for $([K]-\lambda^*[M])$ is available in all the cases.

To show the effectiveness of the present method, let us evaluate the dominant numbers of multiplications and divisions by comparing with a standard method. Here, we employ as such a method the subspace iteration method, which utilizes a subspace technique like the present method. Based on the rough estimates of the number of main operations [4], [5], [6], the proposed method becomes more efficient than the subspace iteration method when $n > 4\ell$. Since n is always much larger than ℓ , and, one and only one triangulation process is required for this approach, the present method turns out superior to such a conventional

method. In addition, efficiency of the present algorithm may be improved by an appropriate choice of λ^* and dimension r of subspace. For practical computations, some feasibility tests are necessary to find the effects of λ^* and r on the numerical results. In the above discussion, we assume that vectors $[M]\{\phi_j\}$ and $[M_a]\{\phi_j\}(1\leq j\leq r)$ are calculated beforehand for each perturbation, and they are stored as $\{X_j\} = [M]\{\phi_j\}$ and $\{Y_j\} = [M_a]\{\phi_j\}$. Then, the unknowns $\{\beta\}$ and $\{\psi\}$ in the k-th iteration step are rewritten by

$$\{\beta\} = [\ Y_1\ Y_2\ \ldots\ Y_r\ ]^t\ \{\psi_i\}\ , \tag{14}$$

$$\psi = ([K]-\lambda^*[M])^{-1}\ (\ \sum_{j=1}^{r} \alpha_j\{(\lambda^*-\lambda_j+\mu)\{X_j\}+(\lambda^*+\mu)\{Y_j\}\ \} + (\lambda^*+\mu)[M_a]\{\psi\} + \mu[M]\{\psi\}). \tag{15}$$

Since most of calculations are inner-products and multiplications between vectors and matrices, computational cost is further reduced.

To consider total efficiency of the present approach, we must make some comments on the deduction process from problem (1) and (2) to (4), where the inverse operation of hydrodynamic mass matrix [H] is necessary to obtain the additive mass. In the present scheme, since [H] is of a band type and [C] is a full matrix with r x n components, usual band-solvers of linear simultaneous equations can be also utilized with the i-th column vector of the order n ($1\leq i\leq n$) of [C] regarded as each right-hand side vector. Though we must solve such simultaneous equations r-times, one triangulation is commonly used to reduce excessive computational costs. Besides the above procedure, we can make use of some lumping technique for hydrodynamic matrix in order to reduce more costs. To find out the appropriate lumping technique, we also need feasibility tests in practice.

4. Numerical examples for feasibility tests

To verify the effects of λ^* and r on the convergence behavior and to see the variation of eigenpairs for various perturbations, some numerical results are shown in what follows. The necessary eigendata in uncoupled state must be obtained with sufficient accuracy by the subspace iteration method. For the iteration, we employ the following starting and stopping conditions:

$$\mu_i^{(0)} = 0\ ,\ \{\psi_i^{(0)}\} = 0\ , \tag{16}$$

$$|\ \mu_i^{(k)} - \mu_i^{(k-1)}\ |\ /\ |\lambda^* + \mu_i^{(k)}\ | \leq \varepsilon\ (=10^{-8}), \tag{17}$$

The iteration process is stopped at a certain step when eq. (17) is first satisfied, or, if eq. (17) is not satisfied within 100 iterations. Beam elements are used for lateral displacement, and 3-node triangular elements for pressure. The employed subdivisions are nearly uniform, and matching of nodal values is considered at the fluid-structure interface.

A numerical example is on the free vibration of a uniform beam structure partially submerged in a fluid chamber with trapezoidal cross-section, as shown in Fig. 1. The beam is assumed to be simply supported at the top of chamber and rotationally restrained at the bottom. The fluid is surrounded by a chamber except the free surface, where the pressure is assumed to be zero. This assumption assures that the hydrodynamic mass matrix is invertible, and that the additive mass of fluid can be determined. The chamber depth or beam span is denoted by b, compartment width at the top and at the bottom by a_2 and a_1, respectively, cross-sectional area by A, the bending rigidity by EI, and the head of fluid by h. When the beam is completely submerged in fluid (h=b) and the cross-section is rectangular ($a_1 = a_2 = a$), the exact solutions can be obtained; the n-th eigenvalue λ_n (n=1, 2, ...) is given by $\lambda_n = \beta_n^4 EI/(\rho_S A+\rho_F \tilde{A}^*)$ where $\beta_n = (2n-1)\pi/2b$ and $\tilde{A}^* = 2\beta_n^{-1}\coth(\beta_n a)$, but the associated eigenfunction is still unchanged for lateral displacement. For simplicity,

λ^* is chosen $\lambda^* = 0$. In this case, the present solution is compared with the exact and the finite element solutions. Table 1 shows that the present solution gives the finite element solution when it converges, and further that the second approximation in iteration process is nearly equal to the exact solution, since $\{\psi_i^{(1)}\}=0$ and $\{\alpha_i^{(1)}\}$ is unchanged. Next, we observe the variation of non-dimensional natural frequencies $\bar{\alpha}$ of the first three modes with a ratio (b-h)/b. The beam in vacuo is employed as an uncoupled state, and the value of perturbation is controlled by the number of subdivisions (see Fig. 2). As depicted in Fig. 3, $\bar{\alpha}$ decreases monotonically with (b-h)/b for all modes. Tables 2 and 3 show that the convergency of the present method is improved by the subspace technique and it can calculate the solutions which have failed in convergence. Also, it is found from Table 4 that the convergence is improved with r. The variation of $\bar{\alpha}_i$ against a_2/a_1 is calculated for i = 1, 2, 3 when (b-h)/b=0.5 and r=3. As shown in Fig. 4 and Table 5, this method can easily trace the change of $\bar{\alpha}$ with a_2/a_1.

5. Conclusion

It is found that coupled eigenvalue analysis with additive mass can be efficiently solved by the present method. Further feasibility tests are taking place, and their results will be reported in near future.

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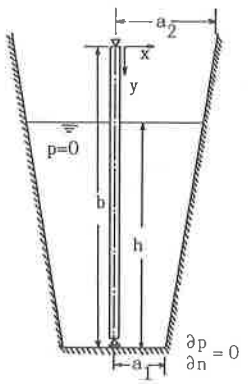

Fig. 1 A beam partially submerged in fluid chamber with trapezoidal cross-section

Table 1 Comparison of the present solutions with the exact and the finite element solutions
(i: modal number; ℓ: number of iterations)

		i=1	i=2	i=3	ℓ
Present solution	3 x 6	0.76604	3.48756	6.59305	2
	3 x 12	0.76523	3.45739	6.46010	2
Finite Element solution	3 x 6	0.76604	3.48756	6.59305	
	3 x 12	0.76523	3.45739	6.46010	
Exact solution		0.76469	3.43756	6.37317	

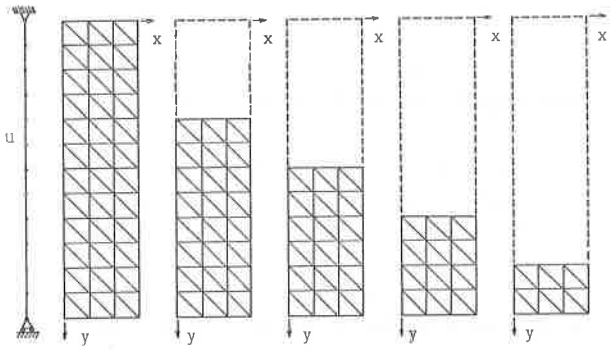

Fig. 2 Various values of perturbations corresponding to the change of depth (h/b)

Table 2 Number of iterations required for convergence ($\lambda^* = 0$, $r = 3$, $N = 12$)

(b-h)/b	i=1	i=2	i=3
0.0	2	2	2
1/12	3	5	10
2/12	3	5	11
4/12	3	5	12
6/12	3	5	12
8/12	3	6	12
10/12	3	6	12
11/12	3	5	10
1.0			

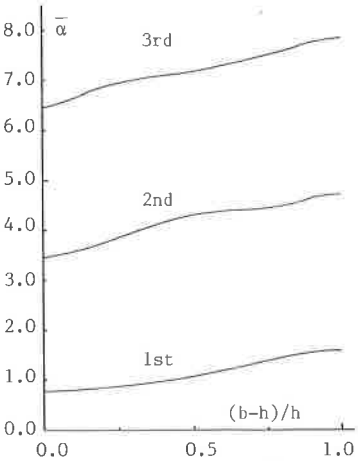

Fig. 3 Variation of $\bar{\alpha}$ against (b-h)/b (p=3, λ^*=0, N=12)

Table 3 Number of iterations required for convergence (r = m = 1)

(b-h)/b	i=1	i=2	i=3
0.0	2	2	2
1/12	4	—	—
2/12	4	—	—
4/12	5	—	—
6/12	5	—	—
8/12	5	47	—
10/12	4	9	17
11/12	4	6	7
1.0			

Table 4 Sensitivity of dimension r of subspace to the convergence

p	i=1	i=2	i=3
3	3	5	12
4	3	4	6
5	3	3	5
6	3	3	4

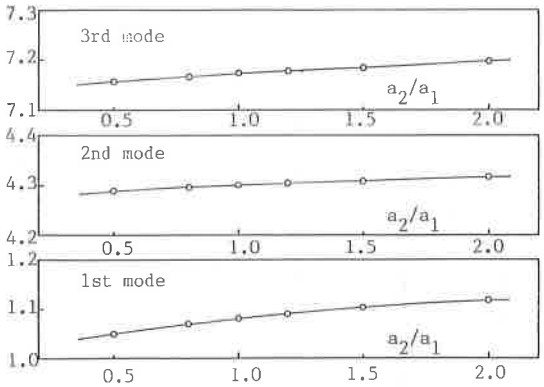

Fig. 4 Variation of non-dimensional natural frequency $\bar{\alpha}$ against a_2/a_1 (h/b=0.5,r=3,N=12)

Table 5 Number of iterations required for convergence (λ^*=0, r=3, N=12, h/b=0.5)

a_2/a_1	i=1	i=2	i=3
2.0	3	5	12
1.5	3	5	12
1.2	3	4	12
1.0	3	5	12
0.8	3	5	12
0.5	3	5	12