An Application of the Concept of the Increasing Sequence of Macromodels (ISM) to the Optimization of the Large Discrete Non-Linear Systems

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Abstract

A set of new algorithms for the effective calculations of an optimum large non-linear mechanical system, i.e. the system giving the extremum of the quality functional on the properties measure density space, using the combined incremental and Newton-Raphson methods as well as including the author's own idea of the increasing sequence of macromodels /ISM/, has been proposed. In these algorithms the author's own algorithm of the current modification of the eigen vector basis of macromodels has been included. The algebraic operations to two level hypermatrices using the author's own block-frontal approach and package of subroutines have been applied. These ideas seem to be very efficient to compute the optimum very large non-linear mechanical systems on relatively small computers /even on personal computers/.

1. Introduction

The analysis of the physically and/or geometrically non-linear mechanical systems, using the finite element technique, has recently become the subject of a large number of papers /e.g. [1 ÷ 9]/. The most important methods of the calculations of such systems are: the combined incremental and /modified/ Newton-Raphson methods /I and N-R/, /I and mN-R/, the Aitken acceleration method /A/, the BFGS method /BFGS/, the Newton-Lanczos method /N-L/.

In all the algorithms, mentioned above, there are two important difficulties which arise with the increase of the number of degree of freedom /DOF/: the considerable increase of the central processor unit time and the large amount of data.

The above mentioned computing difficulties lead to the application of the concept of the increasing sequence of macromodels /ISM/ to the mechanics of very large discrete systems. The idea of the increasing sequence of macromodels in the increasing sequence of the eigen subspaces of the Z displacement space, being the Hilbert separable space, has been proposed by PRZYBYŁO [10]. This idea consists of the generalization of the modal approach proposed by B.O.ALMROTH et al. in [11] to the solution of the large linear and non-linear problems. The idea, shown in paper [11] , has been applied to various computations of very large, linear and non-linear, statical and dynamical systems /e.g. [2],[3],[6],[7]/. In the paper /W.PRZYBYŁO [12]/ the concept of the increasing sequence of macromodels /ISM/ as well as the current modification of the finite Schauder basis, have been included in the N-R, mN-R, I and mN-R methods. In this paper we apply these modified methods to form a set of new algorithms for the effective calculations of optimum large non-linear mechanical systems. In these algorithms we shall apply the algebraic operations to two level hypermatrices using the author's own block-frontal approach and the author's own package of subroutines /[10],[13]/.

The author's own software, forming the system generator for computer aided research and engineering, including the relational database management system [13] , makes it possible to generate the structural mechanics software systems having nearly all properties described by G.A.STROHKORB and A.K.NOOR [14].

2. The definition of the large mechanical system

The formal definition of a large mechanical system has been given in the papers [10] and [12]. Here we notice that the $Z(V)$ displacement space, defined on the V reference configuration, has been the Hilbert separable space having the $B(V) \subset Z(V)$ Schauder basis. The energy family has been derived from the common relation /Lebesgue integral/:

$$E_V = \frac{1}{2} \cdot \int_V (D^s z)^T \cdot M_V \cdot (D^s z)\, dV = \frac{1}{2} \cdot \langle z^s, z^s \rangle_V , \qquad (1)$$

where $z(r;t) \in Z$ denotes the displacement field, D^s denotes the proper matrix differential operator, M_V denotes the proper measure density matrix, being the element of the properties measure density space - $\mathcal{M}(V)$.

3. The mechanical systems class

We define the mechanical systems class as being a set of mechanical systems, defined in Section 2. The two systems are defined on the same displacement space and differ from each other only by their mechanical properties measure densities.

4. The quality functional of the mechanical systems class

The quality functional is assumed to be the arbitrary functional $J : \mathcal{M} \dashrightarrow R$ defined on the $\mathcal{M}$ mechanical properties measure density space of the mechanical systems class.

5. The optimum mechanical system

The optimum mechanical system is assumed to be the admissible system, belonging to the mechanical systems class, having the $M \in \mathcal{M}_a \subset \mathcal{M}$ properties measure density and giving the extremum of the J functional: $U^{opt}(M)$:
$M^{opt} \in \mathcal{M}_a \subset \mathcal{M}$, $J(M^{opt}) = \mathrm{extr}(J(M))$, $M \in \mathcal{M}_a \subset \mathcal{M}$.

6. The notions of a discrete non-linear mechanical system

Let us formulate the main notions of the physically or/and geometrically non-linear discrete mechanical system - the U_d system [12]. We assume that the discretization of the continuous non-linear system - the U system [10] - has been made using the finite element technique [1].

Let us introduce the following notions and symbols: Q - the displacement space /$q \in Q$/, P - the force space /$p \in P$/, $\dim(q) = \dim(p) = N$. Q and P are assumed to be the Banach vector spaces. Let us introduce the non-linear continuous differentiable algebraic operator $S : Q \longrightarrow P$ and its Frechet differential $K(q) = dS/dq$. The symbol $S(q)$ denotes the vector of the internal forces /$\dim(S) = N$/ and the symbol $K(q)$ is interpreted as the tangential stiffness matrix. Let as also introduce the vector of the residual forces in the nodes: $\psi(q) = S(q) - p = S(q) - \lambda_m \cdot p_o$ /$\dim(\psi) = N$/.

Having introduced the above notions in the following Sections we shall formulate the equilibrium equations system of the U_d system.

7. The initial and current algebraic eigen problems

Let us formulate the current algebraic eigen problem /see [12]/

$$\left({}^N\!\left(B_m^n\right)_k^l\right)^T \cdot \left(K_m^n\right)_k^l \cdot \left({}^N\!\left(B_m^n\right)_k^l\right) = {}^N\!\left(\Lambda_m^n\right)_k^l = {}^N\!\left(K_m^{*n}\right)_k^l \quad , \qquad (2)$$

in which $\left(K_m^n\right)_k^l$ is the current tangential stiffness matrix, ${}^N\!\left(\Lambda_m^n\right)_k^l$ - the current diagonal eigen value matrix and ${}^N\!\left(B_m^n\right)_k^l$ - the current finite Schauder basis in the Q displacement space. The ${}^N\!\left(B_1^o\right)_1^o$ is assumed to be the initial Schauder basis of the whole computational process and ${}^N\!\left(B_1^o\right)_k^o$ - of the k direction process.

Let us denote by ${}^i\!\left(B_m^n\right)_k^l$ the subset of the ${}^N\!\left(B_m^n\right)_k^l$ finite Schauder basis and by ${}^iQ$ the "i" subspace of the Q space, defined by the relations

$${}^i\!\left(B_m^n\right)_k^l = \left[{}^j\!\left(B_m^n\right)_k^l\right] , \quad j \in [1,N] , \quad {}^iQ = \mathrm{lin}\left({}^i\!\left(B_m^n\right)_k^l\right) \subset Q \; . \qquad (3)$$

Using the formulae (3) we can construct the increasing sequence of subspaces of the Q space: ${}^1Q \subset {}^2Q \subset \ldots \subset {}^iQ \subset {}^{(i+1)}Q \subset \ldots \subset {}^NQ = Q$.

8. The equations of the increasing sequence of the discrete macromodels

Let us introduce the vector ${}^i\!\left(r_m^n\right)_k^l$ of the displacement of the ${}^jD_d$-macromodel by the formula

$${}^j\!\left(q_m^n\right)_k^l = {}^j\!\left(B_m^n\right)_k^l \cdot {}^j\!\left(r_m^n\right)_k^l \qquad (4)$$

and then let us formulate the Lagrange equations system of the ${}^jD_d$-macromodel in the incremental form /see [1] and [12]/:

$$\left({}^j\!\left(B_m^n\right)_k^l\right)^T \cdot \left(K_m^n\right)_k^l \cdot \left({}^j\!\left(B_m^n\right)_k^l\right) = {}^j\!\left(K_m^{*n}\right)_k^l , \quad \left({}^j\!\left(B_m^n\right)_k^l\right)^T \cdot \left(\psi_m^n\right)_k^l = {}^j\!\left(\psi_m^{*n}\right)_k^l ,$$
$${}^j\!\left(K_m^{*n}\right)_k^l \cdot {}^j\!\left(\Delta r_m^n\right)_k^l + {}^j\!\left(\psi_m^{*n}\right)_k^l = 0 , \quad {}^j\!\left(\Delta q_m^n\right)_k^l = {}^j\!\left(B_m^n\right)_k^l \cdot {}^j\!\left(\Delta r_m^n\right)_k^l \; . \qquad (5)$$

Equations (4) and (5) are assumed to be the ${}^{j}D_d$-macromodel in the ${}^{j}Q$ subspace of the Q space. By analogy to the notions described in W.PRZYBYŁO [12] we can formulate the increasing sequence of the discrete macromodels:

$$ {}^{1}D_d \prec {}^{2}D_d \prec \dots \prec {}^{j}D_d \prec \dots \prec {}^{i}D_d \prec \dots \prec {}^{N}D_d \quad . \tag{6} $$

In the equations (2) ÷ (5) m is the subscript enumerating the increments of the λ_m load vector parameter, n - the superscript enumerating the iterations within the m load increment step, k - the subscript enumerating the directions of the search of the extremum of the J functional in the $\mathcal{M}$ space, l - the number of a step on the m direction, $(q_m^n)_k^l$ - the full node displacement vector at the (m,n) step of the (k,l) searching state, $(\Delta q_m^n)_k^l$ - the increment of the node displacement vector at the (m,n) step of the (k,l) searching state, $(\psi_m^n)_k^l = \psi\left((q_m^n)_k^l\right)$ - the residual force vector at the (m,n) step of the (k,l) searching state, $(S_m^n)_k^l = S\left((q_m^n)_k^l\right)$ - the vector of the internal forces at the (m,n) step of the (k,l) searching state, $(K_m^n)_k^l = K\left((q_m^n)_k^l\right)$ - the tangential stiffness matrix at the (m,n) step of the (k,l) searching state /$(K_m^n)_k^l = K_m^n(M_k^l)$, $M_k^l \in \mathcal{M}$ /, p_o - the reference load vector.

9. The modification of the ${}^{i}(B_m^n)_k^l$ finite Schauder basis

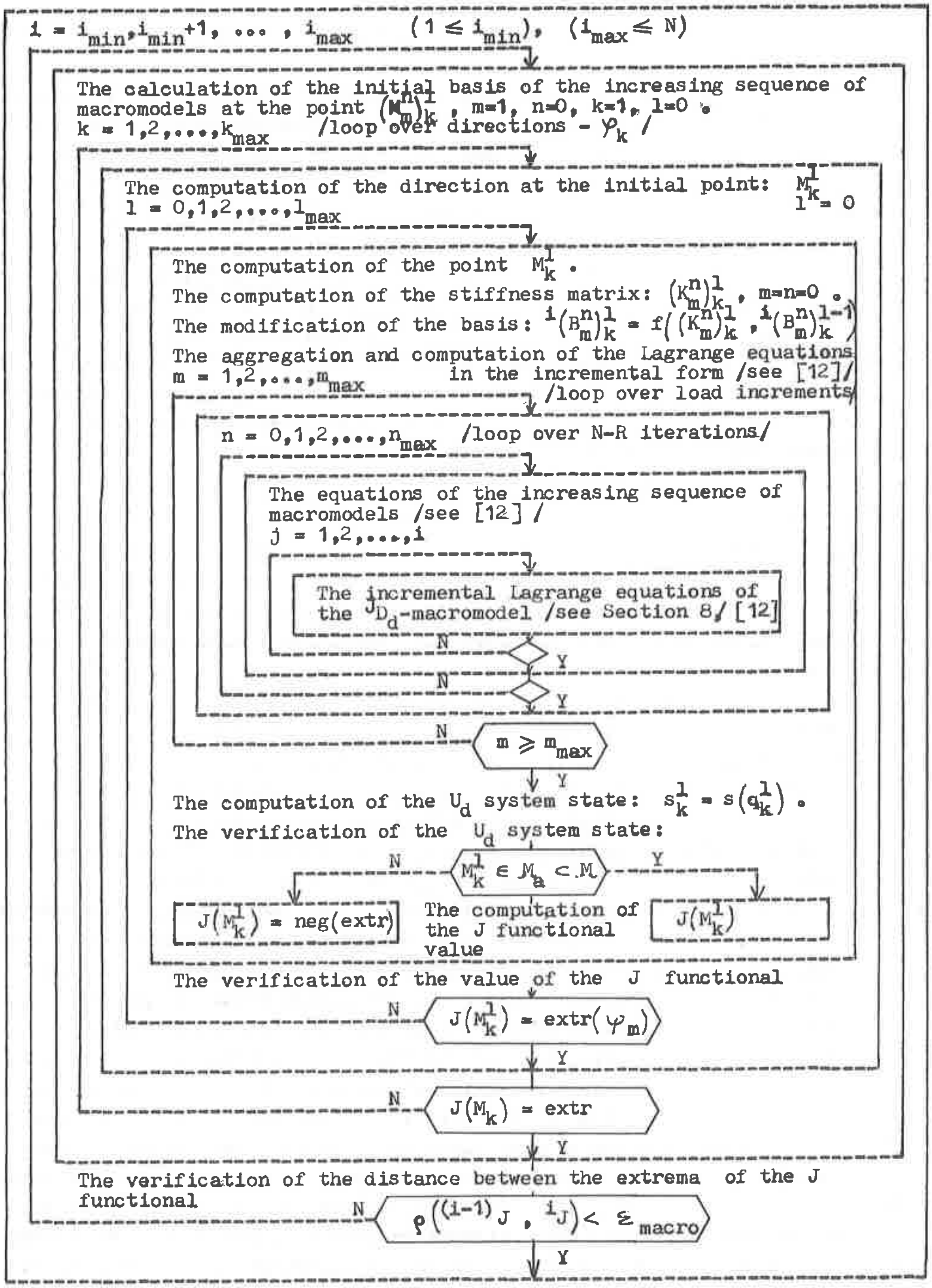
Let us consider the step $\binom{n\ l}{m\ k}$ of the computational process in which we know the ${}^{i}(B_m^n)_k^l$ Schauder basis, calculated from the relation (2). Using this basis we compute the new point $M_k^{l+1} \in \mathcal{M}$ or the new stiffness matrix $(K_m^{n+1})_k^l$. The algorithm of the computation of the new ${}^{i}(B_m^n)_k^{l+1}$ /${}^{i}(B_m^{n+1})_k^l$/ Schauder basis being the function of the ${}^{i}(B_m^n)_k^l$ and $(K_m^n)_k^{l+1}$ /$(K_m^{n+1})_k^l$/ matrices, according to the equation (2) , has been given by W.PRZYBYŁO [12].

10. The formulation of the new algorithms family for the calculations of the optimum large discrete non-linear mechanical systems

The various algorithms for optimization have been formulated /e.g. [15]/. The common property of these algorithms consists of the distinction of two kinds of operations: /a/ computation of the φ_m direction of the search for the M^{extr} point in the given $M_m \in \mathcal{M}$ point, /b/ the search, on the previously computed direction, for the point M^{extr} in which the J functional has become the extremum.

In the table 1 the new algorithm family for the calculation of the optimum U_d system, using the concept of the increasing sequence of the macromodels in the ${}^{j}Q$ /j=1,2,...,i/ eigen subspaces of the ${}^{N}Q = Q$ displacement space has been given. The subscript k denotes the k-th /φ_k/ direction of the search at the given point $M_k^o \in \mathcal{M}_a \subset \mathcal{M}$, and the superscript l - the l point on the φ_k direction. The superscript i enumerates the trajectories which start from the common point $M_1^o \in \mathcal{M}$ and lead to the point M^{extr} giving the extremum of the J functional. In the first algorithm the change of the ${}^{i}(B_m^n)_k^l$ basis is executed after each change of the M_k^l element. In the second algorithm the change of the ${}^{i}(B_m^n)_k^l$ basis is executed after each change of

Table I. The algorithm family for the calculation of the optimum U_d system



the direction of the search, starting from the extremum M_k^1 element on the previous direction. In the third algorithm there is no change of the initial ${}^1B_1^o$ basis. This algorithm seems to be the cheapest one, but the convergence of the computational process would be worse, causing more iteration steps in comparison with the previous algorithms. It is obvious that the local changes of the ${}^i(B_m^n)_k^1$ Schauder basis within the analysis computational processes, can be made according to the algorithms, described by W.PRZYBYŁO in [12] .

In all the algorithms, mentioned above, one should solve the full initial algebraic eigen problem at the initial point $M_1^o \in \mathcal{M}$. By analogy to other computational processes of the structural mechanics, in which the methods of the modal analysis have been applied /e.g. [2],[3],[6],[7]/, we assume the hypothesis that the maximum number "i" of the ${}^iB$ Schauder basis of the increasing sequence of the macromodels $[{}^jD_d]_1^i$ ($i \leq N$, giving the technically useful approximation of the value ${}^iM^{extr} \approx M^{extr}$, is much smaller than "N" - the number of degrees of freedom of the U_d discrete mechanical system. This assumption makes the proposed algorithms very attractive from the computational point of view.

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