Improved Strength Criteria for Anisotropic Brittle Graphites

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Abstract

The criterion of strength for anisotropic brittle media by Gol'denblat and Kopnov is adopted in this paper. The invariants of stress tensor for the strength function having the transversely isotropic symmetry are used to formulate a criterion of strength for the AGOT graphite materials. In the realization of the efficiency of the quadratic strength criteria proposed previously, a cubic strength criterion is proposed in this paper. The criterion contain more disposable coefficients which accommodate necessary physical properties. The strength for an AGOT graphite is tested by uniaxial and biaxial stress. The experimental data are used to validate the improved strength-criterion. This shows that the theoretical and experimental in results are close agreement.

1. Introduction

Fracture criteria and constitutive relations valid in the inelastic range bring a much greater degree of realism to the analysis and design of machines and structures than purely elastically responsive relations. Yet despite the many structural components that will fail by excessive plastic deformation in a moderate strain range, avoidance of failure by fracture is the greatest engineering challenge in particular for brittle like materials. Thus, for the purpose of material characterization and design, rational simple strength criteria for structural materials and composites are essential. As pointed out by Tsai and Wu [1], most of the criteria proposed do not include, or are limited in their ability to include, the anisotropic properties of materials, the difference of tensile and compressive strength, and the mutual effects of stress components. The actual strength criterion is some closed hypersurface in stress space, which signifies that the strength of materials is finite. An infinite failure stress is used as a convenient idealization. In order to remove such limitations, Gol'denblat and Kopnov [2] proposed a strength tensor criterion specially adapted for glass-reinforced plastics and verified its validity by experiment. Assuming the existence of a strength function which is expressible as a polynomial function of stress invariants, the first writer [3] proposed a quadratic strength tensor criterion for graphite (grade AGOT) and verified this experimentally. This graphite is a transversely isotropic brittle material which exhibits the Bauschinger effect and the property of compressibility.

The result of the proposed quadratic strength function shows that the predictions seem to match the test data approximately. In this paper, an improved strength criterion, which includes the cubic terms of stress invariants, is proposed.

2. Development of Strength Criteria

Consider a strength function

$$F(\sigma_{ij}) = 0 \tag{1}$$

which is required to be invariant under a group of transformations of coordinate $\{t_{ij}\}$, characterizing the material anisotropy, i.e.,

$$F(\bar{\sigma}_{ij}) \equiv F(\sigma_{ij}) \tag{2}$$

where $\bar{\sigma}_{ij} = t_{ir}\, t_{js}\, \sigma_{rs}$.

Therefore, any strength function is expressible as a polynomial function in the invariant quantities as

$$F(I_j^{(i)}) = 0 \tag{3}$$

where $I_j^{(i)}$ denotes the invariant quantity in the i-th degree and the j-th element. (All the practical yield criteria for metals and the failure criteria for brittle materials can easily be recognized as special cases of the general form of Eq. (3).) Invariant quantities for each system of anisotropic materials had been obtained by Huang [4].

The invariants for a transversely isotropic material, with the group of coordinate transformations $\{t_{ij}$: I, and $R_\alpha[\bar{x}_1 + i\bar{x}_2 = e^{-i\alpha}(x_1 + ix_2),\ \bar{x}_3 = x_3 \mid$ for all values of $\alpha]\}$, are

$$I_j^{(1)}:\quad \sigma_3,\ \sigma_1 + \sigma_2$$

$$I_j^{(2)}:\quad \sigma_4^2 + \sigma_5^2,\ \sigma_1\sigma_2 - \sigma_6^2$$

$$I_j^{(3)}:\quad \{\det|\sigma_i|\} \text{ or } 2\sigma_4\sigma_5\sigma_6 - \sigma_1\sigma_4^2 - \sigma_2\sigma_5^2 \tag{4}$$

where the contracted notation is used, i.e.,

$$(\sigma_{11}, \sigma_{22}, \sigma_{33}, \sigma_{23}, \sigma_{13}, \sigma_{12}) = (\sigma_1, \sigma_2, \sigma_3, \sigma_4, \sigma_5, \sigma_6).$$

For anisotropic brittle materials, the failure mechanisms or the failure modes may be classified as tensile (compressive) ruptures and shear yieldings, which can be identified by the observation of failure planes. It is of interest to note that the invariant groups given in Eq. (4) determine the failure modes of the transversely isotropic materials. The $I_j^{(1)}$ are the maximum or minimum axial and transverse normal stresses, which result the tensile (compressive) rupture. The $I_j^{(2)}$ are the square of the maximum axial and transverse shear stresses which determine the planes of yield failure. The cubic-term $I_j^{(3)}$ is the cubic power of stress under the combined stress state, which determines the orientation of failure planes. With Eq. (4) the cubic strength function F can be written as the following form.

$$\left\{F_1(\sigma_1 + \sigma_2) + F_3\sigma_3\right\}^\alpha + \left\{F_{11}(\sigma_1^2 + \sigma_2^2) + F_{33}\sigma_3^2 + 2F_{13}[\sigma_3(\sigma_1 + \sigma_2)] + 2F_{12}\sigma_1\sigma_2 + F_{55}(\sigma_4^2 + \sigma_5^2) + 2(F_{11} - F_{12})\,\sigma_6^2\right\}^\beta + \left\{F_{333}\sigma_3^3 + F_{111}(\sigma_1 + \sigma_2)^3 + F_{113}(\sigma_1 + \sigma_2)^2\,\sigma_3 + F_{133}(\sigma_1 + \sigma_2)\sigma_3^2 + F_{316}[(\sigma_1\sigma_2 - \sigma_6^2)\sigma_3] + F_{355}[(\sigma_4^2 + \sigma_5^2)\sigma_3] + F_{116}[(\sigma_1 + \sigma_2)(\sigma_1\sigma_2 - \sigma_6^2)] + F_{155}[(\sigma_1 + \sigma_2)(\sigma_4^2 + \sigma_5^2)] + F_{666}[\det(\sigma_i)]\right\}^\gamma = 1 \tag{5}$$

where F_i, F_{ij}, and F_{ijk} are the components of strength tensors, σ_i ($i=1, 2, \ldots, 6$) is the matrix for of the stress components, and α, β, and γ are material parameters.

For the plane stress problems in (x_1, x_3), Eq. (5) yields:

$$(F_1\sigma_1 + F_3\sigma_3)^\alpha + (F_{11}\sigma_1^2 + F_{33}\sigma_3^2 + 2F_{13}\sigma_1\sigma_3 + F_{55}\sigma_5^2)^\beta + (F_{111}\sigma_1^3 + F_{333}\sigma_3^3 + F_{113}\sigma_1^2\sigma_3 + F_{133}\sigma_1\sigma_3^2 + F_{355}\sigma_3\sigma_5^2 + F_{155}\sigma_1\sigma_5^2)^\beta = 1 \tag{6}$$

The function given in Eq. (5) or Eq. (6) is invariant under transformation of coordinates. With $\alpha = \beta = 1$, the cubic terms in Eq. (6) are deleted, and the quadratic strength criterion given by Tsai and Wu [1] is obtained. Also, by imposing conditions of incompressibility, Eq. (6) can be reduced to the form of the yield criterion proposed by Hill [5] for transversely isotropic material, such as sheet metals.

The strength criteria, proposed in Ref [3], are quadratic in stress. It should be emphasized that the choice of quadratic forms is based on curve fitting considerations and not on physical reasoning, for convenience and simplicity. It is unfortunate that the yielding plane occurring at a point in a continuous medium is not a priori identified, and will occur on the plane on which the maximum shearing stress results. The quadratic approximation of strength tensor theory does not permit to determination of the orientations of the failure planes. Third-order approximations contain more disposable coefficients, are therefore more flexible and improve on the curve fitting. Since the strength function F of Eq. (1) must be a convex function [6], the components of strength tensors F_i, F_{ij} and F_{ijk} cannot be arbitrary. They are subjected to a set of inequality constraints. Let σ_{ij} and σ_{ij}' be two distance stress states in a stress space. Then, their combination

$$\bar{\sigma}_{ij} = \varepsilon\,\sigma_{ij} + (1-\varepsilon)\,\sigma_{ij}' \quad , \quad 0 \leq \varepsilon \leq 1$$

must remain in the space and satisfy the inequality

$$F(\bar{\sigma}_{ij}) \leq \varepsilon\,F(\sigma_{ij}) + (1-\varepsilon)\,F(\sigma_{ij}')$$

The convexity condition for the third-order approximation is very complicated and is not considered in the paper. In this paper the cubic terms σ_1^3 and σ_3^3 are judiciously discarded, and the proposed cubic form of the strength function in the principal stress space is

$$F_1\sigma_1 + F_3\sigma_3 + F_{11}\sigma_1^2 + F_{33}\sigma_3^2 + 2F_{13}\sigma_1\sigma_3$$

$$+ F_{133}\sigma_1\sigma_3^2 + F_{113}\sigma_1^2\sigma_3 - 1 = 0 \qquad (7)$$

where the strength coefficients F_{13}, F_{133} and F_{113} characterize the interactions of the principal normal stresses σ_1 and σ_3. Certainly these coefficients can be determined only by experimental multiaxial stress tests.

3. Experiments

The multi-axial testing of the brittle AGOT Graphite is made, and has been conducted by the writer at KSU for studying on the material properties of the AGOT Graphite. The Uni-axial Compression stress-strain curves for the parallel and transverse directions have been tested and the partial results are shown in Ref. [7]. The experimental work is directed toward, but not limited to, the biaxial stress states [8]. The biaxial stress state tests have been carried out by varying different radial loading paths using the large size, hollow cylinders subject to an axial load and either an external or internal hydrostatic pressure.

In specimen fabrication, the surface finishing of specimen and the contact pressure between the grinding tool and the workpiece must be properly controlled and selected for avoiding the machining damage on the surface of specimen. According to the Industrial Graphite Handbook [9], surface finishing of 63 and 250 micro inches can be achieved at cutting depths of 0.0005" to 0.005" with cutting speeds of 1,000 ft/min.

The strength test results are summarized in the Fig. 1 with the data points.

4. Numerical Example

In order to verify the proposed improved strength criterion, the experimental results obtained for the grade AGOT graphite are used. Unlike the quadratic strength criteria, the explicit analytical solutions of strength coefficients cannot be obtained for the cubic strength criterion Eq. (7). The least square curve fitting method is used to obtain the optimum values of F_i, F_{ij} and F_{ijk}. The results are summarized in the Table 1.

5. Concluding Remarks

From the invariants of the stress tensor, a strength function $F(\sigma_{ij})$ in the form of a polynomial function for anisotropic brittle materials can be easily established. The components of strength tensors are determined experimentally from basic strength data. The proposed cubic strength criterion and the test data are plotted in Fig. 1, For comparison, two quadratic strength criteria [3] are also included in the figure. It can been seen that results using the proposed cubic strength criterion fits the data points more closely in all states of stress than the results of the quadratic criteria. Such a result is expected, because the higher-order approximations contain more disposable parameters.

6. References

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Table 1. Strength Coefficients for

PSI	Modified Quadratic Curve (A)	Quadratic Curve (B)	Cubic Curve (C)
F_1	5.433×10^{-4}	5.433×10^{-4}	4.464×10^{-4}
F_3	3.509×10^{-4}	3.509×10^{-4}	2.5×10^{-4}
F_{11}	1.221×10^{-7}	1.221×10^{-7}	1.116×10^{-7}
F_{33}	9.234×10^{-8}	9.234×10^{-8}	6.944×10^{-8}
F_{13}	-1.534×10^{-8}	-1.006×10^{-8}	-8.59×10^{-9}
F_{113}	------	------	1.728×10^{-12}
F_{133}	------	------	-1.722×10^{-12}

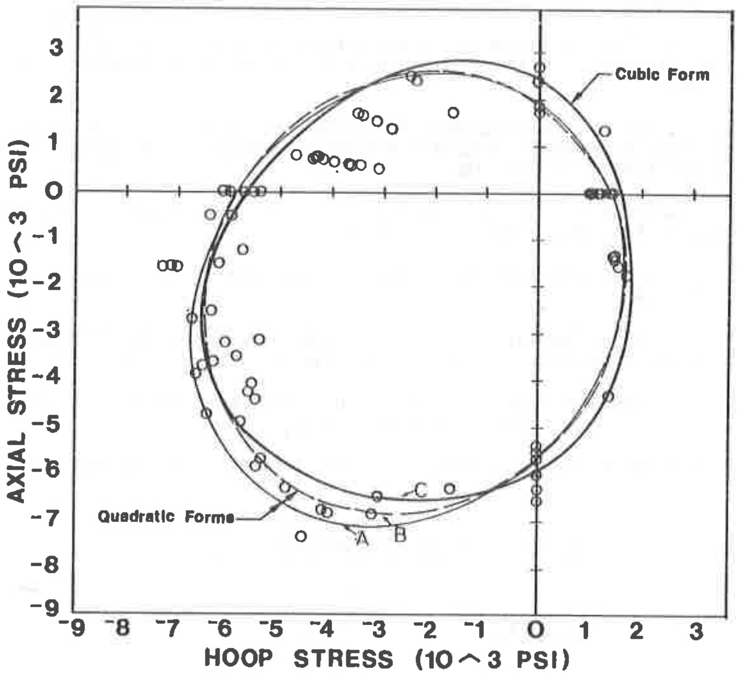

Figure 1. Improved Strength Function (Cubic Form).