A Finite Element Method with Stress Transform in Viscoelastic Analysis of Graphite Components

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Abstract

This paper is concerned with numerical analysis of graphite components in nuclear reactor. Because creep response of irradiated graphite is related to operating temperature and accumulative dose of fast neutron, calculating procedure has been complicated. In order to solve the problem, a stress transformation is presented and finite element formulae is obtained. As an example, the problem of an irradiated, hollow cylinder which has heat source in it and convective heat transfer on its internal and external walls, was analysed. The results calculated by this method show that its solution agrees fairly well with analytic solution. It can simplify computing procedure and save calculating time.

For predication of behavior of irradiated graphite components using in HTGR, it is necessary to consider the effect of irradiation induced creep and dimensional changes. The material properties are considered to be temperature as well as fast neutron flux dependent. Irradiation induced dimensional changes are treated as equivalent thermal strain. Creep compliance of irradiated graphite is function of reactor operation temperature and fast neutron accumulative dose. That behavior can be regarded as viscoelastic problem, that is: for the linear viscoelastic solid, the constitutive relation of stress-strain in memory method can be written as:

\[\varepsilon(D) = \int_0^D J(D-D') \frac{\partial \sigma(D')}{\partial D'} dD'\]  

(1)

\[\sigma(D) = \int_0^D G(D-D') \frac{\partial \varepsilon(D')}{\partial D'} dD'\]  

(2)

where \(D\) - fast neutron accumulative dose, is fast neutron flux integration for time, \(\sigma\)-stress, \(\varepsilon\)-strain, \(J\)-creep compliance, \(G\)-relaxation modulus. \(J\) and \(G\) are given in following relationship,

\[\int_0^D J(D-D') \frac{\partial \sigma(D')}{\partial D'} dD' = \kappa(D)\]  

(3)

where \(\kappa\)-unit step function.

Now available viscoelastic finite element method is complicated. For
simplification of calculation a finite element method with stress transform in viscoelastic stress analysis of Graphite components is presented.

1. The principle of the stress transform.

We consider a linear viscoelastic solid, assuming small displacement deformation. The temperature distribution in it is time-independent.

In addition to consider irradiation strain $\varepsilon^\theta$ and the thermal strain $\varepsilon^0$, the equation (1), (2) are rewritten in matrix form, which describes three dimensional stress condition.

$$
(\varepsilon(D)) = [L] \int_0^D \int_D - D(D-D') G(D-D') \frac{\partial}{\partial (D')} \{ \varepsilon^0 (D') \} - \{ \varepsilon^0 (D') \} dD'
$$

$$
(\sigma (D)) = [H] \int_0^D \int_D - G(D-D') \frac{\partial}{\partial (D')} \{ \varepsilon(D') \} - \{ \varepsilon (D') \} \{ \varepsilon^0 (D') \} dD'
$$

where $[L]$, $[\varepsilon]$ are constant coefficient matrices, their detailed expression can be found in reference [1].

Applying Laplace transform to equations (4), (5) we obtain

$$
(\varepsilon (\xi)) = [L] \overline{J}(\xi) \varepsilon (\xi) + (\varepsilon^0 (\xi)) + (\varepsilon^0 (\xi))
$$

$$
(\sigma (\xi)) = [H] \overline{G}(\xi) \varepsilon (\xi) - (\varepsilon (\xi)) - (\varepsilon^0 (\xi))
$$

where $\xi$ -transform parameter.

The Laplace transform form of equation (3) is

$$
\overline{\sigma} (\xi) J(\xi) = 1/\xi
$$

Let

$$
\overline{\sigma} (\xi) = \overline{\varepsilon} J(\xi) \overline{\sigma} (\xi)
$$

$E_0$ is elastic modulus of unirradiated Graphite.

Substituting (9) into (6), (7), we have

$$
(\varepsilon (\xi)) = [C^\dagger] \overline{\varepsilon} J(\xi) + (\varepsilon^0 (\xi)) + (\varepsilon^0 (\xi))
$$

$$
(\overline{\sigma})^T (\xi) = [R^\dagger] J(\xi) (\varepsilon (\xi)) - (\varepsilon^0 (\xi)) - (\varepsilon^0 (\xi))
$$

where $[C^\dagger] = [L]/E_0$,  $[R^\dagger] = E_0 [H]$. 

Taking Laplace transform of equilibrium equations, we obtain following result that is expressed in tensor form and convention of summation:

$$
\overline{\sigma}_{ij} f_j (\xi) + \overline{F}_j (\xi) = 0
$$

where $i, j = 1, 2, 3$—three coordinate axis respectively, $'i$ -differential to $i$ coordinate. According to equation (9), we have

$$
\overline{\sigma}_{ij} = E_0 J(\xi) \overline{\sigma}_{ij} (\xi) + E_0 J, j (\xi) \overline{\sigma}_{ij} (\xi)
$$

Then following equation can be obtained by substituting equation (13) into equation (12):
\[ \bar{c}_{ij,j}(\xi) + \bar{g}_j(\xi) \bar{f}_{ij}(\xi) - \bar{g}_i \bar{J}_{ij}(\xi) = 0 \]  \hspace{1cm} (14)

or:

\[ \bar{c}_{ij,j}(\xi) + \bar{p}^{+}_{ij}(\xi) = 0 \]  \hspace{1cm} (15)

where

\[ \bar{p}^{+}_{ij}(\xi) = \bar{g}_i \bar{J}_{ij}(\xi) - \bar{g}_j \bar{J}_{ij}(\xi) \]  \hspace{1cm} (16)

Applying Laplace transform to strain-displacement relation, then we have:

\[ \bar{\varepsilon}_{ij}(\xi) = \frac{1}{2} \{ \bar{U}_{ij,j}(\xi) + \bar{U}_{ij,j}(\xi) \} \]  \hspace{1cm} (17)

Similarly for boundary condition, following can be given:

\[ \bar{c}_{ij,j}(\xi) + \bar{g}_j(\xi) \bar{f}_{ij}(\xi) = \bar{p}^{+}_{ij}(\xi) \]  \hspace{1cm} (18)

\[ \bar{U}_{ij}(\xi) = \bar{U}_{ij}(\xi) \]  \hspace{1cm} (19)

In equation (17), (18), (19), \( U \) is displacement vector, \( p_j \) is components of the reaction vector applied on the boundary, \( I_d \) is unit outward normal.

It is seen in (10), (11), (15), (18) and (19) that the above established equations for this problem is similar mathematically to the equations for the elastic problem. This direct analogy leads to an associated potential energy that exist in the transformed space. It can be written as follows:

\[ \bar{w} = \int_V \frac{1}{2} \left[ \{ \bar{R} \} (\bar{\varepsilon}(\xi)) \right]^T \{ \bar{\varepsilon}(\xi) \} dV - \int_V \left[ \left\{ \{ \bar{R} \} (\bar{\varepsilon}_0(\xi)) + \{ \bar{R} \} (\bar{\varepsilon}(\xi)) \right\} \right]^T \{ \bar{\varepsilon}(\xi) \} dV - \int_V \{ \bar{R} \} (\bar{\varepsilon}(\xi))^T \bar{\varepsilon}(\xi) dV - \int_S \{ \bar{R} \} (\bar{\varepsilon}(\xi))^T \bar{\varepsilon}(\xi) ds \]  \hspace{1cm} (20)

In equation (15), \( V \) is volume of solid, in which the body forces \( F \) apply, \( S \) is boundary on which surface tractions \( P \) apply, \( (\cdot)^T \) denotes the transpose of the vector.

It can be seen from above equations that when we solve this problem by displacement method we'll obtain true displacements though the stresses are transformed.

12. Finite Element Formulation

Let the solid in transformed space be divided into a number of small closed element \( \Delta V \). In each element the displacement function is chosen. The displacement \( U \) and strain \( \varepsilon \) in the element is expressed by node displacement \( \bar{U}_k \). After Applying Laplace Transform, they can be written

\[ \{ \bar{U}(\xi) \} = [N] \{ \bar{U}_k(\xi) \} \]  \hspace{1cm} (21)

\[ \{ \bar{\varepsilon}(\xi) \} = [\bar{B}] \{ \bar{U}_k(\xi) \} \]  \hspace{1cm} (22)

In above equations \([N],[\bar{B}]\) are constant matrices.
Substituting (21), (22) into (20), taking the variation of \( \overline{\pi} \), that is \( \delta \pi = 0 \), it leads to \( \partial \pi / \partial w = 0 \). After putting in order and taking applying inverse Laplace transform into the equations, which are given after variation, finally we obtain the finite element formulation as follows

\[
[K](U_k(D)) = f(D)
\]  
(23)

where \([K]\) is total stiffness matrix after transformation. \(U_k\) is node displacement vector. For an element its stiffness matrix is

\[
[K]_e = \int_\Omega [B]^T [R^T] [B] \, dv
\]  
(24)

\(f\) is equivalent force vector. An element equivalent force vector can be written as the sum of following four terms:

\[
(f(D))_e = (\Theta(D))_e + (Q(D))_e + (F(D))_e + (P(D))_e
\]  
(25)

\(\Theta(D)\)_e, \(Q(D)\)_e are equivalent force vectors, which are induced from thermal deformation and irradiation deformations, and are given by

\[
(\Theta(D))_e = \int_\Delta \nu [B]^T [R^T] [\epsilon \Theta(D)] \, dv
\]  
(26)

\[
(Q(D))_e = \int_\Delta \nu [B]^T [R^T] [\epsilon Q(D)] \, dv
\]  
(27)

\((F(D))_e\) is also equivalent force vector, which is induced from transformed body force and is written as follows

\[
(F(D))_e = \int_\Delta [N]^T (F^T(D)) \, dv
\]  
(28)

where three components of \( F^T \) are

\[
F^T(D) = \int_\Delta \nu J(D-D') F_{j, D'}(D') \, dd'
\]  
(29)

\((P(D))_e\) is given by

\[
(P(D))_e = \int_\Delta [N]^T (p^T(D)) \, ds
\]  
(30)

where three components of \( p^T \) can be written:

\[
p^T(D) = \int_\Delta [N]^T (p(D)) \, ds
\]  
(31)

The equation (23), as we know, represents a set of linear integral differential equation, which can be solved by numerical procedure.

3. Numerical example and its result.

As an example, the problem of an irradiated, thick and hollow cylinder with time-independent temperature distribution in its wall was analysed, as place strain deformation. The temperature field and fast neutron flux are time-independent throughout irradiation history.

A simple viscoelastic model is used in the calculation. Its mathematical express is

\[
J(T, D) - 1 / F(T) + K(T) D
\]  
(32)

where \( T \)-temperature, \( K \)-creep coefficient, and
\[ K(T) = (5.3 - 1.45 \times 10^{-7} T + 1.4 \times 10^{-5} T^2) \times 10^{-2.7} \]  

(33)

In the calculating \( E \) is taken as constant, that is \( E = E_0 = 1.2 \times 10^3 \text{kg/m}^2 \).

Irradiation strain can be described as:

\[ \varepsilon^0 = A(T)D \times 10^{-22} + B(T)D^2 \times 10^{-46} \]  

(34)

where

\[ A(T) = -2/3(0.11 - 7.0 \times 10^{-5} T)/(5.7 - 6.0 \times 10^{-3} T) \]
\[ B(T) = 1/3(0.11 - 7.0 \times 10^{-5} T)/(5.7 - 6.0 \times 10^{-3} T)^2 \]

Cylinder internal diameter \( d_i = 7.5 \text{mm} \)
Cylinder external diameter \( d_o = 50 \text{mm} \)
Temperature coefficient \( \alpha = 6 \times 10^{-6} \degree \text{C}^{-1} \)
Poson coefficient \( \mu = 0.27 \)
Thermal conductivity \( \lambda = 1.4654 \text{cal/(cm} \cdot \text{s} \cdot \degree \text{C)} \)
Transfer heat coefficient \( H = 0.7327 \text{cal/(cm}^2 \cdot \text{s} \cdot \degree \text{C)} \)
heat source \( Q = 37.6812 \text{cal/(cm}^3 \cdot \text{s)} \)
Cooler temperature \( T_0 = 600 \degree \text{C} \)

According to the finite element method that is derived above, triangle element is applied. The calculation finished on computer UNIVAC-1100. The results calculated describe in figure 1 and 2.

The results calculated by the finite element are compared with that by analytical method. The stress calculated by this method agrees fairly well with that by analytical solution throughout the irradiation history. This computing also shows that choose of calculating step is dependent on change of irradiation deformation the equation (34) describes.

It is not necessary to establish stiffness matrix again in each computing step except the first step because the stiffness matrix is constant throughout the computing process. It can be seen again that the creep compliance \( J \) that is experimentally obtained is just used in calculation. Therefore there is no need to solve relaxation modulus \( G \) numerically. And the latter calculation is time-consuming because the viscoelastic mode of irradiated graphite has generally more complicated form. To integrate experiment with calculation is an important advantage of this method. Thus it is possible to simplify computing procedure and save calculating time.

The above analysis shows that the finite element method with stress transform is most effective to be used in the stress analysis of reactor graphite components under irradiation condition. In addition this method can also be used to other materials exhibiting similar behavior.
4. References


2/ Novy,D, Berechnung der Spannungsfelder in Seitenreflektor der Kugelhaufen-reactors, Jül-1224

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![Graph 1](image1.png)

**Figure 1.** The change of circumferential stress in external wall with accumulative dose increase.

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![Graph 2](image2.png)

**Figure 2.** Radial temperature distribution. Circumferential stress distribution along radial just where accumulative dose equals $3.1 \times 10^{22} \text{/cm}^2$. 