Elastic Follow Up in Piping Systems How to Specify the Creep Use-Fraction Factor

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Abstract

A shortcut is proposed to take into account the elastic follow up effect due to thermal expansion of piping systems (expansion stress $P_e$). To the use fraction sum associated with the primary stress, is added an use fraction sum associated to the expansion stress. An expression of this one is given. In addition a simple way to use conventional analysis of piping system is included.

1. Introduction - What is elastic follow up - Aim of the paper

1.1 To the best of author's knowledge, the first study about elastic follow up was made by Robinson [1]. Then construction codes like NA14 [2][17] asked designers to give a great attention to this phenomenon. Nevertheless definition of elastic follow up is a subject of discussion. As a general rule, it is said that elastic follow up occurs when slightly loaded parts of a piping system act as a spring on the heavily stressed parts, where the bulk of the deformation is concentrated. Practically these effects are of concern for designers. The first one is related to low cycle fatigue analysis: strain ranges (or rather curvature range) under cyclic thermal expansion are higher than computed on a linear elastic base. This effect will not be considered here because it is a conventional part of fatigue analysis.

1.2 The second effect appears in piping systems operating at elevated temperature where creep damage is significant. It is an undesirable accumulation of creep occurring in the weak zones which could result in fracture. In elevated temperature piping systems, expansion stresses $P_e$ (deformation controlled load) acting with primary stress $P_m$ (due to dead weight for instance) may cause such an accumulation of creep and reduce the creep life to lower values that estimated for primary stress $P_m$ acting alone. The scope of this paper is limited to this effect.

1.3 Below the creep range it is possible to limit the expansion stress range $P_e$ in order to obtain safe down, therefore the concentration of plastic deformation is small and expansion stresses cannot exhaust material ductility. This is not true any longer: in the creep region and expansion stresses can contribute to exhaust material ductility in the weak parts of piping systems. Practical methods to appraise elastic follow up have already by given [3], [4], [5], [6], but it seems that a shortcut would be useful. The aim of this paper is to propose one.

1.4 To return to the meaning of elastic follow up - it is the author's opinion - it is mainly a drawback coming from elastic computations. A partition of computed stress between primary and secondary parts is needed for practical application. Unfortunately this process is not obvious and attention must be paid to real stress and strain fields. A suggested in [3] this can be done with the help of KATCHANOV's approximation and HORTON'S law. A creep strain concentration factor $K_e$ can be appraised from elastic computation results. This creep SCF express the ratio of the "local" stress relaxation rate to the global one (of the piping line). It must be pointed out that this hypothesis assumes that there is no stress...
redistribution within the piping line and that stresses relax as a whole. Hence the piping line is considered as a statically determinate structure, which is conservative for strain appraisal.

2. Modifications of the use fraction sum to take elastic follow up into account

2.1 The current practice is to limit the use fraction sum associated with the general primary membrane stress $P_m$ for all increments during levels A, B and C loading [2][7]

$$\frac{W_p}{1} < 1$$  \hspace{1cm} (1)

where $W_p = E \cdot t_i / t_m$ ($t_i$ total duration of $P_m$, $t_m$ maximum allowed time under $P_m$). Obviously, such a requirement would not take elastic follow up into account. In order to correct this drawback, expansion stress $P_e$ due to thermal expansion of piping are often considered as general primary stress. This practice is safe, but is overconservative and very costly.

In the method presented here, a second use fraction sum $W_e$ is associated with expansion stress $P_e$ (due to thermal expansion of the piping system) and equation (1) becomes

$$\frac{W_p + W_e}{1} < 1$$  \hspace{1cm} (2)

$W_p$ use fraction sum associated with general primary stress,
$W_e$ use fraction sum associated with general expansion stress.

3. Basic assumptions of the proposed method

3.1 Obviously, some hypotheses are needed to write such a rule. It will be assumed that the creep use factor is proportional to equivalent creep strain and will reach one when this strain reaches a fixed value $e_c$ (about 1%). This hypothesis is conservative for material exhibiting a ductile creep rupture (strain to rupture being larger than the product of secondary strain rate by time to rupture).

3.2 Another assumption is that full relaxation will occur (this is conservative, for creep straining will be over estimated).

3.3 With the help of the two preceding assumptions, it is possible to compute the use fraction sum due to the expansion stress acting alone. Unfortunately, as creep straining is not a linear function of applied load, the use fraction sum due to $P_m + P_e$ is not merely the sum of the use fractions associated with $P_m$ and $P_e$. Nevertheless it is possible to show that an upper bound of the total use fraction sum is the sum of the use fraction associated with $P_m$ and $S$ time the use fraction associated with $P_e$.

A conservative value of $S$ can be derived from the two bars model (where $S$ value is between 1 and 2 depending of the expansion stress due to the temperature difference of the two bars which are also loaded by a constant primary load).

3.4 Finally, to perform the estimation, computation of strain concentration factor in the studied piping system must be made (more exactly an estimation of $K_c = e_c / e_o$ where $e_o$ is the strain (or curvature) due to thermal expansion computed or a linear elastic basis and $e_c$ the same strain (or curvature) computed on the assumption that $e_c = B \cdot \theta$). Such a creep concentration factor can be extracted from the results of the conventional elastic beam type analysis of the piping line (see [3]) and the annex.

4. Expression of the use fraction associated to thermal expansion stresses

On the basis of the preceding assumptions it is easy to compute the use fraction sum $W_e$ associated with $P_e$.

Due to cold springing, part of the expansion stress is elastic, therefore only $P_e = \alpha \cdot g_r$ can lead to creep straining. If no creep strain concentration occurs, the complete relaxation causes a strain equal to $K_p (P_e - \alpha \cdot g_r) / E$, corresponding to a real use fraction $K_p (P_e - \alpha \cdot g_r) / E$. In order to take care of the non linear combination with the use fraction associated with $P_m$, one must write:

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\[ W_p = \frac{\beta K_p (P_p - a \sigma_y)}{E \epsilon_p} \]  

\( P_p \) general expansion stress  
\( \sigma_y \) yield strength  
\( E \) Young's Modulus  
\( \epsilon_p \) allowable creep strain  
\( a \) cold springing efficiency  
\( \beta \) non-linearity factor (depending of \( P_p/\sigma_y \))  
\( K_p \) creep strain concentration factor (see annex)  
(good values are \( \beta = 2 \) and \( \epsilon_c \), between 1 and 2 \%).

Example with \( \sigma_y = 200 \text{ MPa} \), \( P_p = 400 \text{ MPa} \), \( E = 200 \text{ 000 MPa} \), \( \epsilon = 1 \text{ %} \), \( a = 1 \), \( \beta = 2 \) and \( K_p = 1 \), equation (3) gives \( W_p = 0.8 \) and only 0.2 can be left to \( W_p \) associated with the primary stress.

5. Conclusions

With simple assumptions it is possible to write a practical rule to consider elastic follow up. An use fraction part \( W_p \) is associated with the expansion stress \( P_p \) caused by thermal expansion of the piping system. This use fraction part must be added to the one associated with the general primary stress, and the sum must not exceed the admissible use fraction factor. (This rule is condenced in equations 2 and 3). An upper bound of the creep SCP can be extracted of piping conventional analysis.

References
2 ASME Code Case N47.  
7 A.F.C.E.N. "Règles de construction et de conception des matériaux mécaniques RNB", provisional edition 1983 (it is planned that the next edition will be available in 1985 from the French Standardization Society AFNOR).

ANNEX

HOW TO OBTAIN \( K_p \) (CREEP SCP) FROM A CONVENTIONAL ELASTIC COMPUTATION

A.1 Introduction

\( K_p \) can be obtained by creep-plastic computation, but it is easier to extract an upper bound from the results of the conventional elastic computation of the piping system. For doing that KATCHANOV's approximation is very useful. Most of this annex was included in [3] or is an easy development of [3].

A.2 Stress relaxation

KATCHANOV made the assumption that the stress field is decreasing on the same way in every point of the structure (piping system)
\[ \sigma(t) = \sigma_0 \phi(t) \]

where \( \phi = 1 \) for \( t = 0 \) and decreases toward zero when time \( t \) is increasing toward inhomogeneity values.

With the NORTON's law, the strain rate is given by
\[ \dot{\varepsilon} = \frac{\dot{\sigma}}{E + B \sigma^m} \]
where \( m = 2 \) for the global relaxation time equal to the initial value of the ratio of elastic energy by dissipation power.

**A.3 Expression of \( K_p \)**

As \( \phi = -2 \theta \varepsilon \), the strain rate expression can be written \( \dot{\varepsilon} = \dot{\varepsilon}(\varepsilon_0 - 2 \theta \varepsilon) \), the integration of it is obvious

\[ \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} = \dot{\theta}(1 - \frac{\varepsilon}{\varepsilon_0}) + \frac{\theta}{\varepsilon_0} \]

where \( \theta = \varepsilon_0/2 \dot{\varepsilon}_0 = \frac{1}{E} \) is the local relaxation time which can be defined like the global one \( \dot{\theta} \).

As \( \varepsilon/\varepsilon_0 \) is the creep SGF (initial strain \( \varepsilon_0 \) is the elastic one and \( \varepsilon \) is the real one), when full relaxation is assumed (\( \phi = 0 \)) the following value is obtained:

\[ K_p = \frac{\theta}{\varepsilon_0} \]  

**A.4 Practical expression in piping system**

In a cross section of a piping line, the local relaxation time can be computed on the basis of the reference stress \( \sigma_R \) of the section

\[ \frac{\theta}{\varepsilon_0} = \frac{\sigma_R/E}{2 B \sigma_R^m} \]

and it is obvious that the global time \( \dot{\theta} \) can be obtained from the local values by integration along the piping line

\[ \frac{1}{\dot{\theta}} = \int \frac{\sigma_R^m}{\sigma_R^m + \sigma_R^m} \, d \varepsilon \]

This reference stress can be deduced from the expressions of limit bending (or twist) moment. For instance, in plane bending \( \sigma_\text{R} = H \frac{R^2}{R^2} \), where \( H = 1 \) for straight parts and \( (t^2/r)^3 \) for elbows (but \( r \approx 1 \)).

*This expression can be corrected if partial relaxation is taken into account (in using the value of \( \phi \)) (see [3]).

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