Experimental and Analytical Instabilities of Cylinders Under Primary and Secondary Loads

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Abstract
The structure, which is a thin cylinder (e/r = 0.002, e thickness, R radius) can be subjected to three types of loadings: a tractive effort, an external pressure and a temperature gradient. During experiments program with thermal load only, the temperature gradient was increased up to the maximum and the cylinder did not buckle. Computations, with the bifurcation method using the tangent modulus formula, give a small critical gradient. On the other hand, incremental computations, with an initial modal imperfection give, in any case, a good representation of the structure behaviour.

I. INTRODUCTION
Experiments are realized by C.E.A./D.E.M.T. to study the instabilities of thin cylindrical shells (e/r = 0.002, e thickness, R radius) subjected to different combinations of mechanical and thermal loads. Until now, the studied instabilities are bifurcation instabilities under a critical load.
The objectives of these investigations are the following:
- determination of the effect of a secondary load on critical buckling loads,
- qualification or criticism of the design methodology based on the principle which consists to increase all the loads with the same coefficient,
- qualification of computer codes of the CASTEM System.
In this paper, we present the first experiments and comparisons with calculations.

II. EXPERIMENTAL DEVICE
The cylinder is built-in two stiff rings. It can be subjected to three types of loadings: a tractive effort, an external pressure and a temperature gradient (Figure 1). The tractive effort is applied by an hydraulic jack, which can pull just to 500 KN. The cylinder is surrounded by a plastic envelope, and a rubber joint warrants imperviousness between to the two shells so that a gas pressure can be applied. An inductance coil constituted by three spirals, supplied by an aperiodic generator and placed half-way up the cylinder can heat it on about 0.04 m high. The cylinder is full of water just to the first spiral in order to increase the axial gradient intensity. The temperature field shape along a meridian is measured with 20 thermocouples.
III. GEOMETRIC SHAPE MEASUREMENTS
Calculations and experiments of buckling on this type of cylinder loaded by an external pressure show up that for these structures, geometric imperfections can notably reduce the buckling load.
So, before any test, we make geometric shape measurements of the cylinder on several parallels using a rotating system. The defect amplitude is approximately equal to the thickness. The defect decomposition, obtained through a Fourier transformation, show up a great contribution of low order circumferential modes (Figure 2).

IV. TESTS WITH THERMAL LOAD ONLY
The results of the first tests realized with thermal load only are related in Table 1 [3]:

- 9 tests have been realized on cylinder number 0, increasing the thermal load from $T_{\text{max}} = 280^\circ \text{C}$ ($G = 150^\circ \text{C}/15 \text{ mm}$) just to $T_{\text{max}} = 1100^\circ \text{C}$ ($G = 880^\circ \text{C}/15\text{ mm}$). The heating period is from 2 to 45 s and the temperature gradient is applied on about 15 mm high, in any test. The temperature field shape is presented on Figure 3.

- 20 tests have been realized on cylinder number 1, with a constant thermal load of $T_{\text{max}} = 980^\circ \text{C}$ and $G = 760^\circ \text{C}/15 \text{ mm}$.

In spite of the very strong temperature gradients applied on the two cylinders, no buckling was observed. The geometric shape measurements, done after tests, show that there is no evolution of circumferential modes the order of which is greater than 4 (Figure 2). Nevertheless progressive deformation was observed, but the heating temperatures were very hard.

V. TESTS WITH THERMAL AND MECHANICAL LOADS
The results of the other tests realized with thermal and mechanical loads are related in Table 2:

- the objective of the first tests was to know if the cylinder can support the nominal loadings, corresponding to the operating conditions of the real structure, with a safety margin. Not any instability was observed.

- the objective of the following tests was to determine the critical pressure without and with a tractive effort.

VI. INSTABILITIES COMPUTATIONS
VI.1 - Calculations procedures
Calculations are made, using the Finite Element Method, with the CASTEM System Codes. Different numerical procedures are used [2]:

- Bifurcation computations on the perfect structure, taking into account geometrical and material non-linearities.

- Incremental computations on the imperfect structure, using the shell element with defect of INCA Code [1]. The shape of the imperfection is chosen parallel to the buckling mode of the perfect structure. The response is on the imperfection mode, and on its harmonics, orders 2 and 3.

Variations of the material properties with temperature are taken into account.

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VI.2 - Modellisation
For computation with thermal load, the cylinder is modelled by 135 thin shell elements, with a very fine modelisation of the gradient zone (Figure 4).
For computation with only mechanical loads, the cylinder is modelled by 20 thin shell elements regularly distributed on the cylinder height.

VI.3 - Results
The computations results are the following (Table 3)[4][5] :

- with only mechanical loads :
  On perfect structure, the critical load is rather higher than the experimental one.
  On structure with modal defect, the computation is conservative. Nevertheless, the incremental computations show up different structure behaviours against buckling in case of pressure load, and in case of pressure and traction load. In fact, with pressure only, the load displacement curve, resulting of the incremental computation decreases at Perit (Figure 5) ; therefore, in case of pressure and traction, the load displacement curve presents an inflexion at Perit, but increases again (Figure 6). While with pressure only, the structure becomes really unstable, in case of traction and pressure, buckling is not distinct. The same phenomenon is observed during tests : blisters appear but the pressure does not fail.

- with thermal load :
The bifurcation computation give a small critical load $G_{crit} = 168^\circ/15$ mm (Figure 8). Therefore, the incremental calculation with modal defect computed just to $G = 280^\circ/15$ mm shows up that the structure presents a stable behaviour (Figure 7).
Notice that the bifurcation computation are made with the tangent modulus formula, for taking into account the plasticity.

VII. CONCLUSIONS
In spite of the very strong temperature gradients applied on the structure, no buckling was observed in tests with thermal load.
The bifurcation computation, using the tangent modulus formula, is too conservative, when there is a thermal load. A more realistic critical load could perhaps be estimated using an other method for taking into account the plasticity.
The incremental calculation, with an initial modal imperfection gives, in all cases, a good representation of the structure behaviour.

REFERENCES

[1] O. GARUTI, A. COMBESCREU
"A New Periodic Imperfect Quasi Axisymmetric Shell Element"
Paper L1/3/5 - SMIRT 7th - CHICAGO (1983)

"Calculations Methodology of the Instabilities For Thin Shell Structures"
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"Calculs d'interprétation des essais de flambage thermique - 1ère phase"
Rapport à paraître

FIGURE 1 : Experimental set-up

FIGURE 2 : Amplitude and modal contribution of the deflections before and after test
### Table 1

**Tests with Thermal Load Only**

<table>
<thead>
<tr>
<th>Cylinder Number</th>
<th>Test Number</th>
<th>Thermal Load</th>
<th>Heating Period</th>
<th>Buckling</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td>200°C</td>
<td>20°C/15 mm</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>370°C</td>
<td>260°C/15 mm</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>450°C</td>
<td>350°C/15 mm</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>580°C</td>
<td>350°C/15 mm</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>630°C</td>
<td>450°C/15 mm</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>700°C</td>
<td>500°C/15 mm</td>
<td>no</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>880°C</td>
<td>600°C/15 mm</td>
<td>yes</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>980°C</td>
<td>700°C/15 mm</td>
<td>yes</td>
</tr>
<tr>
<td>1</td>
<td>1 to 20</td>
<td>760°C</td>
<td>760°C/15 mm</td>
<td>yes</td>
</tr>
</tbody>
</table>

### Table 2

**Tests with Thermal and Mechanical Loads**

<table>
<thead>
<tr>
<th>Cylinder Number</th>
<th>Test Number</th>
<th>Loading</th>
<th>Traction</th>
<th>Thermal Load</th>
<th>Buckling</th>
<th>Eventually Critical Pressure</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>1</td>
<td>0.0032 MPa</td>
<td>3.88 kN</td>
<td>415°C/15 mm</td>
<td>yes</td>
<td>yes* 0.021 MPa</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0.0060 MPa</td>
<td>3.88 kN</td>
<td>415°C/15 mm</td>
<td>no</td>
<td>yes* 0.021 MPa</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0.0160 MPa</td>
<td>11.4 kN</td>
<td>415°C/15 mm</td>
<td>no</td>
<td>yes* 0.07 MPa</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>8.58 kN</td>
<td>-</td>
<td>-</td>
<td>yes*</td>
<td>0.028 MPa</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>8.58 kN</td>
<td>-</td>
<td>-</td>
<td>yes*</td>
<td>0.032 MPa</td>
</tr>
</tbody>
</table>

*Smears formation but no pressure fall.

### Table 3

**Computations Results**

<table>
<thead>
<tr>
<th>Loading Type</th>
<th>Computation Type</th>
<th>Method</th>
<th>Buckling</th>
<th>Eventually Critical Load and Elastofracture Node</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pressure</td>
<td>Perfect</td>
<td>Bifurcation</td>
<td>Yes</td>
<td>( P_{crit} = 0.016 \text{ MPa} ) ( (\text{node 13}) )</td>
</tr>
<tr>
<td>Pressure + Traction ( T = 180 \text{ kN} )</td>
<td>Perfect</td>
<td>Bifurcation</td>
<td>Yes*</td>
<td>( P_{crit} = 0.030 \text{ MPa} ) ( (\text{node 12}) )</td>
</tr>
<tr>
<td>Thermal Load</td>
<td>Perfect</td>
<td>Bifurcation</td>
<td>Yes</td>
<td>( T_{crit} = 185°C ) ( (\text{node 15}) )</td>
</tr>
<tr>
<td>Pressure + Traction ( T = 260 \text{ kN} )</td>
<td>Perfect</td>
<td>Bifurcation</td>
<td>Yes*</td>
<td>( T_{crit} = 260°C ) ( (\text{node 22}) )</td>
</tr>
<tr>
<td>Thermal Load</td>
<td>( \delta_0 = 0.1 )</td>
<td>Incremental</td>
<td>No</td>
<td></td>
</tr>
</tbody>
</table>

\( \delta_0 / \text{e} \) : Initial imperfection amplitude related to the thickness.

\( m \) : no distinct buckling.