New Method of Dynamic Analysis of Piping Systems

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ABSTRACT

Increasing attention is being paid to the vibrational and seismic problems of piping systems in nuclear power plants. In order to minimize the consequences of seismic and other dynamic loadings, the piping design decisively depends on the quality of available analytical procedures.

This paper presents the concept of a new dynamic analysis that produces more complete and integrated results. It is based on the transfer function mathematical procedure combined with mechanical impedance and control engineering techniques. The transfer function approach and application on a heat removal loop of a nuclear reactor are described.

The main advantages foreseen are higher reliability and safety of the design, lower construction cost and better plant availability.

1. INTRODUCTION

In recent years, considerable attention has been given to the vibration problems of piping in nuclear power plants. This focus exists not only as a precaution against the always-present threat of earthquakes but also because oscillations have been observed during operation in some piping systems. Especially in the vicinity of dynamically active components, such as pumps, fans, heat exchangers or valves, the vibration has to be controlled.

To ensure high reliability of piping performance during all identified events, even those designated as extremely unlikely to occur, complicated analytical procedures have been developed. They evaluate the structural behavior of piping systems and adjacent mechanical equipment in order to secure efficient layout and optimum support arrangement. Computer-aided analyses are considering not only general technical aspects of stress and strength evaluation but, additionally, many requirements of regulating codes as well. The reliability of the piping operation depends decisively on the quality of the design and capability of available analytical tools.

The purpose of this study is to introduce a new method of dynamic analysis of piping systems which produces more complete and integrated results than the current methods. The method is based on the transfer function mathematical approach combined with mechanical impedance and control engineering techniques. The transfer function concept allows for the solution of the main vibrational equation with the damping member included. At the same time, the phase angle (lag) during the transfer of vibrational energy through the structures is established and used advantageously for modal summation. Experimental data from testing real structures can be combined with the analytical procedure.

For the first-time presentation, only the basic, typical characteristics are described. This paper will try to narrow the gap between the discussions in the area of structural dynamic theory and the practical application on the piping production analysis.

2. TRANSFER FUNCTION CONCEPT

The transfer function is defined as the ratio of the Laplace-transformed output to the Laplace-transformed input. It enables the function of a real variable to be transformed into a function of a complex variable and an original set of differential equations to be transformed into a set of algebraic equations.

For the case of damped vibration, the basic differential equation

$$m\ddot{Y}(t) + C\dot{Y}(t) + KY(t) = F(t), \qquad \text{(Eq. 1)}$$

will be transformed and solved for the response Y(s) with the result:

$$Y(s) = F(s)/(K(s^2/\omega + 2\zeta s/\omega + 1)), \qquad \text{(Eq. 2)}$$

where $\underline{m}$ is the mass, $\underline{K}$ stiffness, $\underline{C}$ damping, $\underline{F(t)}$ and $\underline{F(s)}$ force vectors, $\underline{Y(t)}$ and $\underline{Y(s)}$ response vectors, ω circular frequency of excitation by the force (or by seismic input); symbol ζ is the coefficient of critical damping. The Laplace operator is denoted by $\underline{s}$. The damping as a basic property of each component is considered in the basic equation.

For multi-element piping system, the transfer function between the response at one point ($\underline{Y}_1$) resulting from an applied loading at a second point ($\underline{F}_2$) will have the following form:

$$G_{1,2} \equiv Y_1/F_2 = \sum_{r=1}^{R} (C^r_{1,2}/(K^r(1-\omega/\omega^r)^2 + j(2\zeta^r\omega/\omega^r))). \qquad \text{(Eq. 3)}$$

The symbols $\underline{K}^r$, $\underline{\omega}^r$, and ζ^r denote the stiffness, natural frequency and damping coefficient for the r-th mode, respectively. Coefficient $\underline{C}^r$ expresses the contribution of mode shapes. For each piping transfer function, there will be as many $\underline{C}^r$ coefficients as the number of natural frequencies $\underline{R}$ found in a preselected dynamic range.

The equation is developed by substituting the Laplace operator $\underline{s}$ back into the transformed solution for $\underline{Y}(s)$ and by breaking the denominator down into its real and imaginary parts. Two terms corresponding to the first two rigid body translational and rotational modes are omitted here. Physically, the equation for $\underline{G}_{1,2}$ establishes how the vibration or seismic oscillation is amplified (or attenuated) during the transfer through the structure between the points $\underline{1}$ and $\underline{2}$.

The phase angle between the input at $\underline{1}$ and output at $\underline{2}$, established according to Equation 3, represents new information which is important for the modal summation. Each mode will have its own phase angle, which is similar to having an algebraic sign. By considering the relative summation sign, the resulting loading becomes significantly smaller than loading from an analysis which can calculate the magnitudes only. The position of the ASME B&PV Code in this matter is stated in Paragraph NB 3653.1 of Section III: "If the method of earthquake analysis is such that only magnitudes (without relative algebraic signs) are obtained, the most disadvantageous combination must be used."

The feature of phase angle will ultimately prevent oversupporting of piping systems with resulting lower construction cost. Equation 3 is already of practical importance for computer analytical procedures [Ref. 1,2,3].

3. PIPING SYSTEM ANALYSIS

To illustrate the application of the transfer function method on piping system analysis, a loop of the heat removal system of a nuclear reactor is chosen. Figure 1 shows, in a simplified way, typical components such as the vessel ($\underline{1}$), valve ($\underline{2}$), flued head ($\underline{3}$), support ($\underline{4}$) and the coolant pump ($\underline{5}$).

The system is composed from four structural groups, namely

a) piping runs,

b) components having a mass that is significant compared to the mass of the piping; e.g., vessels, tanks, pumps, heat exchangers, fans; these components display their own dynamic properties (natural frequencies, mode shapes, loads) which are transferred into the piping through the anchor points,

c) components with relatively small mass; e.g., supports, flued heads, bellows,

d) components in which the effect of flow-induced vibration is located; e.g., valves.

The transformation into a dynamic model is shown in Figure 2. For each component, the stiffness (translational and rotational) and damping properties are denoted by $\underline{K}_x$, $\underline{K}_o$, $\underline{C}$, respectively. The symbols $\underline{F}^E$ represent earthquake and external loadings. The effect of flow-induced transients localized at the valve (or at the elbows or strainers) is denoted by $\underline{F}^F$.

Each component can be analyzed separately by choosing the most convenient analytical or experimental (testing) procedure. The main objective is to obtain the transfer function characteristics needed for incorporation into the total assembly equations. This approach parallels the design process in which major structural components are designed or tested by different engineering groups.

The testing methods are new for piping systems, although they were sufficiently developed for some other mechanical systems; e.g., for a rotating equipment system. The necessary devices such as electro-hydraulic exciter or hand-held electro-inductive exciter are available. For lightly damped structures, like piping, special shock hammers equipped with force transducers can be used. The Transfer Function Analyzers or combination dual-channel analyzers along with measuring instrumentation (accelerometers, non-contacting probes) are commonly marketed. The author of this study performed extensive transfer function evaluation for rotor-bearing-housing systems [Ref. 4, 5].

The measured data are read from printouts and diagrams or can be synthesized by computer procedures. Especially useful are the Bode diagrams which show two graphs in the frequency domain: the transfer function curve and the corresponding phase angle values. From the shape of resonant peaks, the damping coefficient can be derived separately for each mode [Ref. 6,7].

The testing data either complement the mathematical analysis or verify some results obtained purely analytically. In any case, the inclusion of transfer functions from testing the real structures improves the reliability of the overall design.

Transfer function data at the attachment points on the supporting structure yield the seismic anchor movements which are required to be evaluated by regulating codes. However, this time, the data have dynamic character. Methods that analyze the movements by static procedure, similar to thermal analysis, have to consider the resulting stress as secondary stress only. The influence of the anchors on the connected piping is reduced there to a static deflection.

For developing the total system assembly matrix, the free body diagram is constructed in Figure 3. The dynamic responses at the two main components (reactor vessel and pump) are denoted by R_1^C and R_2^C respectively. The symbols R_1^S and R_2^S stand for the responses at the supports, both forces and moments. The vector $\underline{Y}$ indicates here the translations and rotations.

Finally, the assembly matrix for the chosen reactor loop will interconnect the individual components of the whole system. In Equation 4, the symbols $\underline{G}^C$ are transfer functions between the piping and the reactor or the pump; $\underline{H}^S$ are similar functions for the supporting devices; $\underline{K}$ are translational and rotational stiffnesses of components and supports; $\underline{G}^E$ are the transfer functions for piping at the locations of earthquake input or external loading; $\underline{G}^F$ expresses the relationship between the piping and flow-induced transients.

$$\begin{Bmatrix} Y_T \\ R^C \\ R^S \\ F^E \\ F^F \end{Bmatrix} \times \begin{bmatrix} 1 & G^C & H^S & G^E & G^F \\ K^C & 1 & 0 & 0 & 0 \\ K^S & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} = \begin{Bmatrix} 0 \\ 0 \\ 0 \\ F^E \\ F^F \end{Bmatrix} \qquad \text{(Eq. 4)}$$

$$G^C = \begin{bmatrix} G^C_{XF} & G^C_{XM} \\ G^C_{\Theta F} & G^C_{\Theta M} \end{bmatrix} \qquad \text{(Eq. 5)}$$

$$K^C = \begin{bmatrix} K^C_{FX} & K^C_{F\Theta} \\ K^C_{MX} & K^C_{M\Theta} \end{bmatrix} \qquad \text{(Eq. 6)}$$

The submatrices should include all important locations, such as connecting points and points of forcing disturbances. The mutual interaction indicates how the dynamic properties of individual members will be modified as they couple with the overall arrangement.

Each $\underline{G}$ function actually contains four members as is shown in Equation 5. The composition of $\underline{K}$ matrices is shown in Equation 6.

The solution of the assembly matrix equation (Equation 4) is obtained by the usual methods of matrix algebra. The total system response vector $\{\underline{Y}_T\}$ is computed by inverting the square matrix $[\underline{G}]$ as follows:

$$\{\underline{Y}_T\} = [\underline{G}]^{-1} \times \{\underline{F}\}. \qquad \text{(Eq. 7)}$$

The vector $\underline{Y}_T$ contains the information necessary to predict the frequency response behavior of the piping under the presumed operational conditions. It can be called the "state vector" because it generally consists of four state variables: displacement $\underline{X}$ in inches (mm), rotation or slope $\underline{\theta}$ in in/rad (mm/rad), force $\underline{F}$ in lb (daN) and moment $\underline{M}$ in in-lb (daN-mm). The variables are used to compute the required types of stresses according to the piping geometry, the material properties, and the applicable code regulations.

4. CONCLUSIONS

This paper has tried to demonstrate the main characteristics of applying the transfer function method on the analysis of piping systems. The advantages of the proposed new method can be summarized as follows:

a) the inclusion of damping into the main equation controls the transfer of vibrational energy by the dissipation effect, which results in a more complete evaluation of the dynamic structural behavior;

b) the establishment of phase angle allows to attribute each mode an algebraic sign and perform the modal summation in a less conservative (though in complete agreement with the ASME B&PV Code intentions) way;

c) flow-induced vibration is included in the solution of the total system where it benefits from the presence of damping;

d) the influence of the adjacent anchoring equipment on the piping is treated dynamically; the members of the system mutually affect not only the mode shapes and loading but the development of resonances as well;

e) data from testing of actual structures can be included into the model, thus improving the reliability of the design.

The first three items help to prevent unnecessary oversupporting of the piping systems and may lower the construction cost. High numbers of supports and heavy steel supporting structures are not a guarantee for better design especially under critical dynamic (seismic) conditions. Items d) and e) contribute to more reliable and safer design and may ultimately improve the plant availability.

5. REFERENCES

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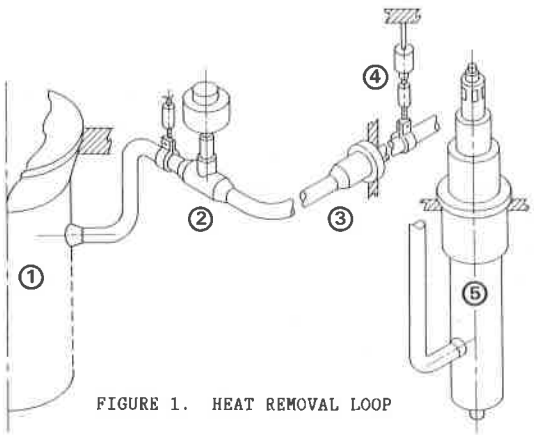

FIGURE 1. HEAT REMOVAL LOOP

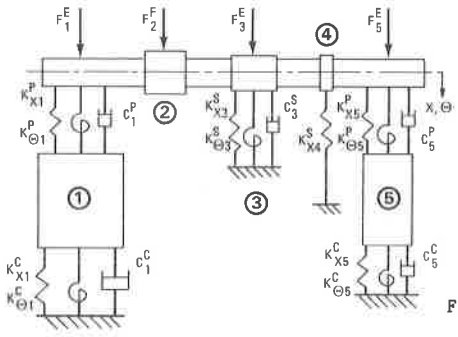

FIGURE 2. DYNAMIC MODEL

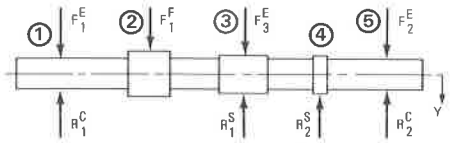

FIGURE 3. FREE BODY SKETCH