Numerical Analysis of Hydride Blisters in Pressure Tubes

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Abstract

A stress analysis of pressure tubes was performed to study stresses produced by volume expansion associated with the precipitation of zirconium-hydrides in a zirconium matrix in the form of 'blisters'. Parametric elastic analyses were performed to identify the relative importance of geometrical and material parameters on stresses associated with the blisters. Results show a pronounced effect of the postulated hydride concentration distribution and the ratio of blister diameter to depth.

1. Introduction

Fast fracture of primary system piping is a failure mode that is rigorously analysed in all water-cooled reactor systems because of the potential for fuel damage through inadequate cooling. Such a fracture occurred in a Zircaloy-2 pressure tube of Pickering-2 CANDU (Canada Deuterium Uranium) reactor in August 1983, yet no damage to the fuel sheaths occurred during the fracture, and consequently there was no radiation release. The reactor was shut down with normal operating systems and procedures, without invoking any of the safety systems [1].

Figure 1 shows a simplified description of a CANDU fuel channel. The 1983 accident was associated with the growth of semi-ellipsoidal solid hydride zones at points of contact between the pressure tube and the calandria tube owing to the shifting of a close-coiled helical spring spacer from its design location. These zones, as shown in Figure 2, are termed 'blisters' because they are raised above the surface and the normal blue-grey colour of the surface oxide is thicker and whiter. These hydride blisters grow as a result of the thermal diffusion of hydrogen in the zirconium alloy: a mechanism in which hydrogen diffuses down a temperature gradient. If the dissolved hydrogen concentration exceeds the terminal solid solubility at the colder region, zirconium-hydride will be precipitated. Subsequent investigations on Zr-Nb pressure tubes, however, have shown that zirconium-hydride blistering is not expected to be a problem in the CANDU power reactors apart from those having Zircaloy-2 pressure tubes [2].
The effect of blisters on pressure tube stresses was unknown. This prompted a project to determine stresses produced by volume expansion of hydride blisters. The objective of this paper is to identify the relative importance of geometrical and material parameters on stresses associated with the blisters.

2. Method Of Analysis

2.1 Geometry

Typical pressure tube dimensions are as follows: 104 mm ID, 5.0 mm wall thickness and 6.3 m tube length. Blisters detected in Pickering-2 were typically 2.1 mm in radius and 1.3 mm in depth. Since the blisters are relatively small in comparison to tube dimensions, an axisymmetric model was used in most cases to study the local stresses. Notionally, a small disk of materials is cut from the tube as shown in Figure 3 (a). It was assumed in the analysis that the blister is circular (plan view). Tube curvature and material anisotropy were neglected in the analysis. Figure 3 (b) shows the directional conventions used in this paper. The 'Z' (vertical axis) corresponds to the axis of symmetry which is in the through-thickness direction.

2.2 Modelling of Expansion

As hydrogen comes out of solution, zirconium-hydrides are formed. The new material swells but because it is surrounded by a matrix which otherwise would not expand, the swelling is suppressed to some extent, producing compressive stresses within the blister and placing the surrounding material in tension. The concentration of precipitated hydrides apparently varies axially. It was assumed that the average expansion at a point within the blister is proportional to the local hydride concentration and that the expansion is isotropic.

The analysis was based on the 'initial strain' method which is more typically used for thermal stress analysis in which the (isotropic) expansion strains are:

\[
e_{ij} = \begin{cases} 
\alpha \Delta T & \text{(normal components)} \\
0 & \text{(shear components)} 
\end{cases}
\]  

eq. (1)

Expansion due to hydride precipitation was treated by taking as \(\alpha = 1\) and \(\Delta T = \text{maximum linear expansion times the ratio of local to maximum hydride concentration.}\)

3. Exact Solutions

3.1 Effect of Expansion Distribution

Micrographic sections of actual blisters \([1]\) suggest that there is not a well-delineated interface between the blister and tube material, but rather a solid zirconium-hydride core surrounded by a two-phase region in which the proportion of
The analytical model is of an entire sphere (radius b) surrounding a spherical core (radius a) in which the assumed initial strain distribution is one of the following: i) constant, ii) linear and iii) cosine. Timoshenko [3] gives for a solid sphere:

\[ \sigma_r = \frac{2E \varepsilon}{1-\nu} \left\{ \frac{1}{b^3} \int_0^b T(r) r^2 dr - \frac{1}{b^3} \int_0^b T(r) r^2 dr \right\} \]

\[ \sigma_\theta = \frac{E \varepsilon}{1-\nu} \left\{ \frac{2}{b^3} \int_0^b T(r) r^2 dr + \frac{1}{b^3} \int_0^b T(r) r^2 dr - T(r) \right\} \]
eq (2)

in which

\[ T = T_0 \quad \text{for constant distribution} \]

\[ T = T_0 (1-r/a) \quad \text{for linear distribution} \quad a < r < a \]

\[ T = T_0 (1+\cos\pi r/a)/2 \quad \text{for cosine distribution} \]

\[ T = 0 \quad \text{for all distributions} \quad a < r < b \]

Numerical results are plotted in Figure 4. Dimensions were arbitrarily selected taking an outer sphere of 5 mm radius and an expansion region of 1 mm radius. Figure 4 shows that the radial stress is compressive decaying with radius and the circumferential stress is compressive throughout most of the expanding (blister) region; tensile in the surrounding region. Stresses arising from the linear and cosine expansion distributions differ only slightly, but the constant expansion case differs markedly.

The significance of this analysis is that it demonstrates the pronounced effect of the assumed expansion distribution. Of the three assumed distributions, the maximum \( \sigma_\theta \) occurs at the blister/matrix interface boundary for the constant expansion case (eg in which as much of the region is expanding as much as possible).

3.2 Decay Distance of Expansion Stresses

The distance over which the blister expansion influences stresses in the surrounding region was estimated by comparing stresses for the two cases: (i) the ratio \( b/a \) of the radius of the outer sphere to expanding core is finite, and (ii) the ratio is infinite (eg the core is embedded in an unbounded region).

For any expansion distribution within the core, the ratio of the radial stress of the finite to infinite region is:

\[ -473 - \]
\[
\sigma_r / \sigma_r^o = 1 - (a/b)^3 \tag{3}
\]

at the blister/matrix interface, and similarly, for the circumferential component:

\[
\sigma_\theta / \sigma_\theta^o = 1 + 2(a/b)^3 \tag{4}
\]

Values are tabulated below

<table>
<thead>
<tr>
<th>Effect of Blister Expansion on Stress</th>
<th>b/a</th>
<th>(\sigma_r / \sigma_r^o)</th>
<th>(\sigma_\theta / \sigma_\theta^o)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2.0</td>
<td>0.88</td>
<td>1.25</td>
</tr>
<tr>
<td></td>
<td>2.5</td>
<td>0.94</td>
<td>1.13</td>
</tr>
<tr>
<td></td>
<td>3.0</td>
<td>0.96</td>
<td>1.07</td>
</tr>
<tr>
<td></td>
<td>4.0</td>
<td>0.98</td>
<td>1.03</td>
</tr>
<tr>
<td></td>
<td>5.0</td>
<td>0.99</td>
<td>1.02</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The largest blisters observed have dimensions (comparable to "a" above) in the order of 1.3 mm depth and 2.1 mm radius and the tube wall thickness (comparable to "b") are 5.0 mm. Consequently, it is estimated that the effect of blister expansion on stresses will be insignificant at distances of about one wall thickness from the blister center.

3.3 Effect of Free-Surface

An exact elastic solution for the case when the blister is not completely surrounded by tube material, but is exposed to the surface was derived to provide an additional means to verify the finite element model. The main geometrical differences between the previous model and actual blisters are: (i) blisters have a free surface, (ii) the blister region is not necessarily spherical or hemispherical, and (iii) blisters are embedded in the tube wall rather than being surrounded by a spherical region.

Because stresses due to expansion decay quickly, (iii) above probably has only a secondary effect. Assuming this is so, an exact elastic solution for a hemispherical blister in a semi-infinite medium can be constructed by superimposing the known solutions for the infinite sphere (eq'n 252 of [3]) and an elastic "half space" (eq'n 92.7 of [4]) where surface is acted upon by forces equal to the reversed circumferential stresses of the sphere, as illustrated in Figure 5. Unfortunately, the exact solution contains a series of extremely complicated definite integrals which can only be evaluated numerically in the general case. However, an exact evaluation giving the vertical deflections along the axis of symmetry (Z-axis in Figure 3) was obtained as:
\[ \omega = \frac{2}{3} \left( \frac{1+v}{1-v} \right) \alpha T \left\{ \frac{Z^2}{a^2 + z^2} - \frac{a^3}{2z} \left( 2 - \frac{\sqrt{a^2 + z^2}}{a} - \frac{a}{\sqrt{a^2 + z^2}} \right) \right\} \\
+ \frac{4}{3} (1+v) \alpha T \left\{ \frac{Z - \sqrt{a^2 + z^2}}{a} - \frac{a^3}{2z} \left( 1 - \frac{\sqrt{a^2 + z^2}}{a} \right) \right\} + \omega_0 \]

where

\[ \omega_0 = \begin{cases} 
\frac{1}{3} \left( \frac{1+v}{1-v} \right) \alpha r \text{Tr} & \text{for } 0 < r < a \\
\frac{1}{3} \left( \frac{1+v}{1-v} \right) \alpha r \frac{a^3}{z^2} & \text{for } r > a
\end{cases} \]

As shown in Figure 6, this solution is in good agreement with finite element results.

4. Elastic Finite Element Analysis

4.1 Effect of Size and Shape

A parametric elastic analysis was performed to study the effect of blister size and shape. All models were axisymmetric with elliptical blister cross sections. Combinations of the following parameters were considered: (i) maximum blister depth of 0.325, 0.650, 0.95 and 1.3 mm, (ii) aspect ratio (ratio of blister radius to maximum depth) of 1.0, 1.5, 2.0 and 2.5, and (iii) initial strain gradient of constant and cosine distribution.

The resulting stresses were distributed in a similar manner to exact solutions for spherical blisters. Figure 7 shows the peak tensile stress (under the blister at the deepest point) versus maximum blister depth for the various aspect ratios considered. As before, the assumption of constant blister expansion results in stresses much larger than those based on a cosine distribution.

For both initial strain distributions, the maximum stress increased with aspect ratio. The effect is more pronounced in the cosine than constant hydride concentration case. For a given aspect ratio, the maximum stress is almost independent of maximum blister depth for the cosine distribution case while it increases slightly with maximum blister depth for the constant expansion case. For given blister radius, the maximum stress decreases with increasing blister depth for the cosine distribution case while it is unchanged for the constant expansion case.

In the cosine distribution case, for a given blister radius, a shallow blister (i.e., higher aspect ratio) produces the higher expansion stress and somewhat surprisingly, for
a given aspect ratio, the blister size has little impact on expansion stresses. In explanation, it is suggested that the distance between location of the point of maximum stress (the deepest point under the blister) and the point of maximum expansion influences the maximum stress. In other words, in the constant expansion case, the distance between the two points is zero, and the maximum stress is reasonably constant regardless of blister depth (for a given radius). For the cosine distribution case, however, as the blister becomes shallower, the distance between the two points decreases and consequently, stresses for a given diameter increases.

4.2 Effect of Blister Spacing

The effect of blister spacing was studied using plane strain elastic models in which blister centers were separated by 6.3, 8.4, 12.6, 17.85, 14.0 mm and infinite distances. Constant initial strains were postulated in the blister region. The most significant tensile stresses occurred under the blister, as shown in Figure 8. The magnitude of the tensile peak decreased with decreasing spacing. This implies that blister spacing is not an important factor in increasing stresses.

5. Conclusion

The possibility that stresses might be produced as a consequence of expansion resulting from the transformation of zirconium to zirconium-hydride in the form of blisters was investigated. Exact solutions for a simplified blister geometry were examined and elastic finite element analysis of semi-ellipsoidal blister was performed.

Results indicated that expansion stresses decay to near zero at about one wall thickness away from a blister and highest stresses are developed under the blister, at the deepest point. Having assumed that the expansion at a point is proportional to the hydride concentration, results were strongly influenced by the postulated concentration distribution (which is presently unknown). The largest stresses (under the blister) increase with the ratio of blister diameter to depth, but for a given ratio are relatively insensitive to depth.

References


FIG. 1: SIMPLIFIED DESCRIPTION OF A CANDU FUEL CHANNEL

FIG. 2: CROSS SECTION THROUGH ZIRCONIUM HYDRIIDE BLISTER

FIG. 3A: AXI SYMMETRIC MODEL: FLATTENED DISK "CUT" FROM PRESSURE TUBE

FIG. 3B: DIRECTION CONVENTION FOR AXI SYMMETRIC MODEL

FIG. 4A: EXACT ELASTIC SOLUTIONS: SOLID SPHERE (IS INITIAL STRAIN)

NOTE: STRESSES ARE PRODUCED BY BLISTER EXPANSION IN CASES 1) AND 2) AND BY SURFACE FORCES ONLY IN 3)

FIG. 5: APPROXIMATE ELASTIC SUPERPOSITION TO DETERMINE EFFECT OF FREE SURFACE
FIG. 6: COMPARISON OF EXACT AND FINITE ELEMENT RESULTS - ELASTIC VERTICAL DEFLECTIONS ALONG Z-AXIS (15% INITIAL STRAIN, CONSTANT WITHIN BLISTER)

FIG. 7: EFFECT OF BLISTER SIZE AND SHAPE ON MAXIMUM ELASTIC STRESS

FIG. 8: EFFECT OF SPACING ON ELASTIC (PLANE STRAIN) STRESSES