Mixed Mode Blunt Crack Instability Using a Path Independent Contour Integral

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Abstract

The reported implementation of the smeared crack model, in conjunction with its fracture mechanics characterization, are first reviewed. Then the various techniques which could be used to extract the stress intensity factors from a finite element mesh with no stress singularity are presented. And finally, one such attempt to extract stress intensity factors from a "smeared" crack using a contour integral method is presented.

1. Introduction

It is well known that crack instability and propagation play important roles in the nonlinear structural behavior of concrete structures, specially for unreinforced or lightly reinforced concrete. In the finite element analysis of cracked concrete structures, fracture can be modeled as: a) Discrete, interelement discontinuity; or as b) "smeared" over entire elements. While the former was the first model used by the pioneering work of Ngo and Scordelis, it was quickly dropped in favor of the smeared crack model, proposed by Rashid, when crack propagation studies were first performed. Despite some recent work by Saouma and Ingraffea [1], and Ingraffea and Saouma [2] which proved the viability of discrete cracking with modern computing capabilities, it should be recognized that this approach may not be appropriate for all cases. Furthermore one should "capitalize" on the extensive smeared crack based software available, and develop techniques which would enable smeared crack propagation to be entirely governed by fracture mechanics concepts.

Bazant and Cedolin [3] were the first to recognize that need, and proposed an approach which had the crack growth governed by a "modified" tensile strength based on the critical energy release rate (which in turn is related to the fracture toughness), and the direction of crack growth remained governed by the direction of principal stresses.

To the best of the authors knowledge, most of the published applications of the blunt crack model were based on implementation and enhancements performed at Argonne. When implementing the previously described blunt crack model, Marchetas, et al. [4] found that: 1) Cracks tend to propagate into elements which possessed the largest principal stress; and 2) Crack path was mesh dependent (cracks tended to propagate along the inclined plane of the discretization). As such they concluded that the direction of the principal stresses can not be used to guide the direction of crack propagation, and used an error accumulation technique. While this technique was found to be an improvement over the previous one, effect of discretization were not entirely eliminated. It should be noted that all three reported examples were cracks under tension (unidirectional crack extension) but with skewed meshes.
Pan and Kennedy [5] compared the values of Rice's J integral for cracks under mode I loading for blunt and discrete cracks. It was found that the J* values were very close. However, they encountered difficulties in implementing the approach into a general purpose finite element code due to the absence of a general procedure to accurately predict the direction of crack growth. As such they investigated the direction of crack propagation using a modified J integral as defined by Hellen and Blackburn [7]. Their examples was a crack subjected to mixed mode loading. It was found that to predict the angle of crack extension with an accuracy of ten percent, the ratio of the cracked element to the regular element has to be in the order of 10**-3, which is very unpractical. Pfeiffer et al. [8] used the equivalent tensile strength, along with principal stress direction to compute the stress intensity factors, and angle of crack extension of 1) center-cracked panel (with skew mesh), and 2) curved cracks in a three-point beam. It was found that the SIF of the first example compared favourably with known analytical solution. However, the authors noted the absence of 'exact' solution to the second problem. Thus the accuracy of the method could not be evaluated for this general case. Finally, Pan et al. [9] have used the blunt crack and the discrete crack to compute the inelastic J* integral as defined by Blackburn [10], for a three point bend test specimen. It was found that the J* values for the two crack models compare favourably.

In light of those reported studies and implementation, one can conclude that while a technique for the determination of SIF of blunt cracks is highly desirable, it is yet unavailable. Such techniques could be directly used for "large" structures where LEFM is most likely applicable. As to bodies with "short" cracks where the size of the process zone is not negligible, than SIF ratios or "pseudo SIF" ratios could be used to assess the direction of crack growth [11].

2. Alternative Solutions

The fracture mechanics literature includes numerous methods for determining SIF of cracks which singularity is not necessarily modelled, and thus potentially applicable to smeared cracks. Hughes and Blackburn [7] have developed a procedure to determine the two stress intensity factors from two virtual crack extensions. Ishikawa [12] determines G1 and G2 from one single analysis, and then determine K1 and KII. Saouma et al. [13] developed a technique which combines a global smeared crack analysis with a local discrete crack one. Babuska & Miller [14] have presented a technique which uses a surface integral for stress intensity evaluation (preliminary results based on this method were very encouraging). This study is based on the work of Stern, Becker, and Dunham [15].

Stern et al. have presented a path independent contour integral formula for the distinct calculation of combined mixed mode SIF calculation. This method is based on the concept of Betti's law, and has the advantage of directly determining the two SIF from a single analysis.


Once the stress intensity factors have been computed and the material fracture toughness known, a fracture initiation criterion encompassing these variables is sought. This criterion will: 1) Determine the angle of incipient propagation with respect to the crack axis. 2) Determine if the stress intensity factors are in such a critical combination as to render the crack locally unstable and force it to propagate. The local stability of a crack loaded in mixed mode is approached through the substitution of the computed K1 and KII values into a theoretical interaction formula.

4. Analysis.

While the good accuracy reported by Stern [12] was easily duplicated for slant discrete cracks, and mode I smeared cracks, the method failed for mixed mode cracks (slant crack in a plate under
uniform tension. Fig. 1 shows the FE mesh along with the selected contour path and its normals at the various gauge points. Further insight into the problem revealed that the error could be attributed to the [D] matrix of the cracked elements. As such it was speculated that the standard procedure of setting E1 (in the direction normal to the crack axis) equal to zero, E2 equal to E, and G as a certain percentage of the initial shear modulus is not satisfactory for SIF extraction under mixed-mode loading.

An alternative model to the cracked element stiffness matrix was suggested by Rebora [16]. It consists in “breaking” the cracked element into two separate sub-elements, the stiffness of which are separately evaluated, and subsequently entirely lumped to the nodes adjacent to the crack axis (thus two mid-side nodes will have zero stiffnesses, and would have to be constrained). Preliminary studies indicated an improvement over the initial ones, however more analyses should be performed before the accuracy of this method is fully ascertained. Comprehensive results will be presented during the conference.

5. Acknowledgments

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6. References


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