A Finite Element Analysis of R.C. Water Pressure Tube

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Abstract

This paper describes the application of finite element technique to the solution of a practical 2-D nonlinear problem. An existing high pressure hydraulic reinforced concrete pipe is analysed to ascertain the reasons for the observed cracking and rupture of the structural concrete.

1. Introduction

A large number of numerical models and techniques for the analysis and design of reinforced concrete structures has been developed in the last two decades or so, and the literature abounds with examples of their implementation and application (See for example the "state-of-the-art" Proceedings of ICC-Split'88/1). Numerous models with considerable simplifying assumptions are proposed as well as a number of very sophisticated models. In many cases the accuracy of these elaborated models far outweighs the accuracy of available material properties, and they often require extensive computations. In addition, the complexity of practical engineering problems makes numerical analysis more expensive.

In this paper, the applicability of a computationally simple finite element model but capable to simulate the dominant nonlinear behaviour of reinforced concrete is illustrated by the results from a practical analysis (Damjaníć/2,3/). An existing buried high pressure hydraulic pipe is analysed to ascertain the reasons for the observed cracking and rupture of the structural concrete. The results obtained, especially the cracking patterns, are discussed and compared with real observations.

2. Outlines of Material Modelling and Solution Procedure

The finite element model, employed in the present study, was earlier developed (Damjaníć/4/) and the detailed formulations have been provided elsewhere (Damjaníć/5/, Owen et al./6/). Therefore, only essential assumptions are outlined here.

Both a perfect and strain-hardening viscoplasticity approach are employed to model the multiaxial compressive behaviour of concrete. A dual criterion for yield and crushing in terms of stresses and strains is employed, which is complemented with a maximum tensile stress criterion. A smeared representation for cracked concrete, more convenient for finite element formulation, is assumed, which implies that cracks are not discrete but distributed within a sampling volume. The post-cracking tensile behaviour is modelled by a special adopted proce-
dure. In order to improve the solution accuracy some practical suggestions for the post-cracking model are proposed by Damjančić & Owen [7].

The same numerical model, updated with the corresponding soil properties is employed to simulate the soil behaviour, while the steel behaviour is idealised by an uniaxial elastoviscoplastic model (Owen & Hinton [8]) resisting only the axial force in the reinforcing bars (Phillips & Zienkiewicz [9]).

The 8-node serendipity isoparametric elements and reduced integration rule are exclusively employed in the present analysis. The two-dimensional elements are used to idealize concrete structure and soil, while special one-dimensional elements are employed to simulate reinforcing bars (Phillips & Zienkiewicz [9]).

The above outlined model is incorporated in the general nonlinear finite element solution technique (Owen & Hinton [8]). An incremental and iterative "initial stiffness" procedure is used to the solution of the problem described in the following section.

3. Problem Description

A 5.0 km long high pressure hydraulic reinforced concrete pipe as a part of the "N.Tesla" hydro-power plant system (built in Yugoslavia 35 years ago) is considered. The cross-sectional topology and details of the pipe together with the location of the hoop reinforcement are given in Fig. 1. Internal water pressure of the existing pipe was varied according to the operational cycle of the hydro-system which finally resulted in the radial cracks and rupture of the structural concrete. Maximum design pressure was $p_{\text{max}} = 0.19$ MPa. It is necessary to determine an internal pressure when initial cracking occurs and the critical internal pressure which causes the structure failure. The pipe and the surrounding soil are considered as two dimensional plane strain problem and Fig. 1 shows the finite element idealization. For the analysis only one half of the cross-section is considered due to symmetry. The concrete, reinforcement and soil properties are listed in Table I.

4. Results of Analysis

The radial cracks and ruptures are located where zones of defective concrete were observed as well as along parts of the pipe of sound structural concrete. Therefore, two selections of numerical examples are presented: Example I considers the structural response of the sound concrete profiles, while results of Example II illustrate the structural behaviour of pipe profiles with zones of defective concrete.

4.1 Example I:

The model is subjected to both the gravity load and an increasing internal pressure. After an initial elastic response, the first cracked concrete zone is predicted in the region confined to the bottom of the pipe at the pressure $p = 0.348$ MPa (Fig. 2(a)). The development of tensile cracking zones with increasing internal pressure is illustrated in Fig. 2(a)-(d). The structural failure (Fig. 2(d)) is predicted at the pressure $p = 0.511$ MPa. At this stage the cracking zones (only radial cracks) are spreaded out in the bottom and on the side of the pipe and particularly at the top where the rupture is expected. This description compares favourably with real cracking pattern. The max. stress in reinforcing bars at the rupture is $q_e = 113.3$ MPa which is far less then the yield stress. This numerical result is in good agreement with observations in situ. Namely, experimentally tested bars from the damaged pipe still behaves elastically. Fig. 3 shows the distribution of principal stress ($q_1$) across re-
presentative sections with increasing internal pressure, and the stress variation in the re-
inforcing bar is shown in Fig. 4. Significantly higher stress in reinforcement through the 
cracked concrete zones is evident.

4.2 Example II:

Certain parts of the pipe have zones of defective concrete due to casting conditions. 
Series of models with different positions and sizes of such the zones were examined, and here, 
the results of three examples (IIa, IIb, IIc) are only presented. Tensile strength of defective 
concrete is assumed as 50% of the sound concrete strength. Development of cracking zones with 
increasing internal pressure is shown in Figs. 5-7. There are also indicated the positions of 
defective concrete zones. Naturally, cracks and rupture are mainly concentrated within these 
zones.

5. Discussion of Results and Conclusions

The presented numerical results illustrate the applicability of the technique for the 
solution of practical engineering problems. The results are summarized in Table II showing 
the initial crack and rupture safety coefficients related to the max. design pressure. It is 
obvious that the lower coefficients are related to parts of the pipe with zones of defective 
concrete. However, neither the pipe made by a fully sound concrete could resist an occasional 
pressure (up to 1.0 MPa) due to water hammer effects established by Jović /10/. The cracking 
zones favourably compares with observed cracking patterns.

6. References

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/ 6/ OWEN, D.R.J., FIGUEIRAS, J.A., DAMJANIĆ, F., "Finite Element Analysis of Reinforced 

/ 7/ DAMJANIĆ, F., OWEN, D.R.J., "Practical Considerations for Modelling of Post-Cracking 
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/ 8/ OWEN, D.R.J., HINTON, E., Finite Element in Plasticity: Theory and Practice, 

/ 9/ PHILLIPS, D.V., ZIENKIEMICZ, O.C., "Finite Element Non-linear Analysis of Concrete 

/10/ JOVIĆ, V., "Hydrodynamic analysis of the "M.Tesla" hydro-power plant" (In Croatian), 
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### Table I: Material properties

<table>
<thead>
<tr>
<th>Concrete</th>
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<tbody>
<tr>
<td>Young's modulus, $E_c$</td>
<td>22,000.00 MPa</td>
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<tr>
<td>Poisson's ratio, $v_c$</td>
<td>0.18</td>
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<tr>
<td>Compressive strength, $f_{c'}$</td>
<td>29.60 MPa</td>
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<td>Tensile strength, $f_{t'}$</td>
<td>2.96 MPa</td>
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<tr>
<td>Mass density, $\rho$</td>
<td>2,500.00 kg/m³</td>
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<table>
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<tr>
<td>Young's modulus, $E_s$</td>
<td>210,000.00 MPa</td>
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<tr>
<td>Yield stress, $\sigma_y$</td>
<td>240.00 MPa</td>
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<tr>
<td>Ultimate stress, $\sigma_u$</td>
<td>360.00 MPa</td>
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<tr>
<td>Area of reinforcement, $A_s$</td>
<td>2.54 cm²/10m²</td>
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<td>Young's modulus, $E_s$</td>
<td>25.00 MPa</td>
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<td>Poisson's ratio, $v_s$</td>
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<td>Compressive strength, $f_{c'}$</td>
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<td>Tensile strength, $f_{t'}$</td>
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<td>Mass density, $\rho$</td>
<td>1,800.00 kg/m³</td>
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### Table II: Initial crack and rupture safety coeff.

<table>
<thead>
<tr>
<th>Case</th>
<th>Initial crack pressure, $p_1$ (MPa)</th>
<th>Rupture pressure, $p_2$ (MPa)</th>
<th>Safety coeff. $V_1$</th>
<th>Safety coeff. $V_2$</th>
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<tbody>
<tr>
<td>A0</td>
<td>0.348</td>
<td>0.511</td>
<td>1.83</td>
<td>2.69</td>
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<tr>
<td>A1</td>
<td>0.280</td>
<td>0.460</td>
<td>1.47</td>
<td>2.42</td>
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<tr>
<td>A2</td>
<td>0.230</td>
<td>0.310</td>
<td>1.47</td>
<td>1.63</td>
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<tr>
<td>B3</td>
<td>0.165</td>
<td>0.357</td>
<td>0.87</td>
<td>1.88</td>
</tr>
</tbody>
</table>

![Buried R.C. pipe showing problem details and finite element idealization.](image)

Fig. 1

Fig. 2(a)-(d) Development of cracking zones with increasing internal pressure (Example II)

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Fig. 3 Principal stress ($\sigma_1$) distribution across representative sections with increasing internal pressure.

Fig. 4 Stress variation in the reinforcing bar with increasing internal pressure.

Fig. 5(a)-(d) Development of cracking zones with increasing internal pressure (Example IIa)

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Fig. 6(a)-(b) Development of cracking zones with increasing internal pressure (Example III)

Fig. 7(a)-(d) Development of cracking zones with increasing internal pressure (Example IIc)