Ground Acceleration Distribution of Montenegro Earthquake and Its Influence on Determination of Seismic Design Parameters

D. Petrovski

University of Skopje, Institute of Earthquake Engineering and Engineering Seismology, POB 101, 91000 Skopje, Yugoslavia

Abstract

Considering the recorded maximum accelerations of the Montenegro earthquake and the possition of the recording instruments around the earthquake origin, a relationship has been determined in this paper, which takes into account the different ground acceleration attenuation, both parallel and perpendicular to the main fault zone. Applying these attenuation relationships a detailed methodology for determination of the probability distribution function of the maximum ground acceleration, velocity or displacement for a considered site of a complex seismic activity model of the surrounding region has been developed. The design values can be determined based on the acceptable risk,that is regularly lower than the other risks which the considered site is exposed to.

1. Introduction

Records of the ground accelerations due to the catastrophic April 15, 1979 earthquake and the strongest May 24, 1979 aftershock were obtained at many sites both in the epicentral and the wider affected region. This represents a significant contribution to compiling instrumental data on the strong earthquake ground motion not only in Yugoslavia, but on a worldwide scale. The strong earthquake ground motion investigations carried out so far point to its complex character due to the complex earthquake source mechanism and generation of seismic energy in complex unisotropic media, the complex character of seismic wave propagation through the complex structure of the earth crust and the still insufficiently defined influence of local geology.

For the needs of investigations it the field of seismic risk, it is necessary to establich a functional relationship of the maximum acceleration at earh point on the ground surface and the characteristics of the earthquake with known position. However, considering the complex influence of the individual processes and the fact that they are not sufficiently known, the investigations in this field are brought to empirical studies which provide the empirical functional relationships of the maximum acceleration and only two parameters of essential influence, i.e., the magnitude and the epicentral, i.e., the hypocentral distance. In this study, considering the values of the recorded maximum accelerations of the Montenegro earthquake and their distribution around the earthquake source, a functional relationship has been established which also takes into account the uneven attenuation in two directions, parallel and normal to the main fault zone (Fig. 1). This functional relationship is:

$$\log a_{max} = c_1 + c_2 \log(R/\rho + c_3) + x \qquad (1)$$

$$\rho = \left(\frac{1 + tg^2\alpha}{a^{-2} + tg^2\alpha}\right)^{\frac{1}{2}} \qquad (2)$$

where a_{max} is the maximum acceleration, R is epicentral distance, x - random values with mean value zero and standard deviation σ, while c_1, c_2 and c_3 are regression coefficients. It is easy to prove that the isolines of the maximum acceleration a_{max} are similar to the ellipse with half-axis relationship a:1 in the direction and normally to the fault zone stretching direction. Applying the regression analysis, the coefficients c_1, c_2 and c_3; their standard deviations and confidential intervals as well as the conditional deviation σ can be obtained by which the maximum acceleration value a_{max} is evaluated.

2. Attenuation Relationships Applied in this Study

The elliptical shape of the maximum acceleration attenuation of the Montenegro earthquake points to the possibility for different shape of the attenuation functions for different seismic sources. Therefore, and for the convenience in the further analytical calculations, the following attenuation shape is suggested:

$$a = c_1 e^{c_2 m} \left(\frac{R}{\rho} + c_3\right)^{-c_4} \cdot \varepsilon \qquad (3)$$

where a is maximum value of acceleration, velocity or soil displacement at the site, m is a magnitude, while R represents the distance between the site and the epicentre. c_1, c_2 c_3 and c_4 are the empirical constants for the considered seismic source, while ρ is the function which determines the attenuation shape that can be of the general type given by the following expression:

$$\rho = \rho\ (\psi,\ b_1,\ b_2,\ b_3\ \ldots\ldots) \qquad (4)$$

It is obvious that the function ρ enables different attenuation levels for different angles ψ. ψ is the angle between the epicentre - location direction and the referent axis, in which respect the function ρ is given (Fig. 2). The number of the empirical constants of b_1, b_2, etc. type can be different for each seismic source depending on the complex shape of the attenuation. So for a circle $\rho = 1$, for an ellipse there is only b_1 (Fig. 1) and so on. In the studies so far, considerable scatter of data around the mean value has been observed. In the suggested attenuation expression it is provided by the parameter ε with a natural logarithm having a normal probability density function with mean value zero and standard deviation σ.

$$x = \ln \varepsilon \qquad (5)$$

$$f(x) = N(0,\sigma) = \frac{1}{\sqrt{2\pi}\ \sigma}\ \exp \left| -\frac{1}{2} \left(\frac{x}{\sigma}\right)^2 \right| \qquad (6)$$

Such attenuation relationship makes possible that by numerical procedure the attenuation of general shape with certain scatter be applied to the values around the mean value.

3. Mathematical Model of Seismic Sources

In the scope of this study seismic sources of random shape are considered. The depth of the earthquake source is taken indirectly by determinating the attenuation relationships for each seismic source, separately. By small interventions in the analytical derivation of the equations other geometrical shapes of the seismic source can be also introduced.

To determine the probability for occurrence of an earthquake with certain magnitude it is necessary to apply the well known magnitude-to-frequency diagrams for each seismic source. The typical diagram of the magnitude and frequency for earthquake occurrence is presented as a streignt line, while the functional relationship is of the shape:

$$\log n(m) = a - vm \tag{7}$$

where n(m) is the cumulative number of earthquakes with magnitudes higher than m, while a and b are empirical constants. Such a simple functional relationship is quite sufficient for determination of the design parameters with relatively high probability for exceeding (low protection level). In the determination of the design parameters for nuclear power plants, whose values should be determined based on a very low acceptable probability for exceeding (high protection level) the shape of the magnitude-to-frequency diagram in the range of maximum probable matnitudes is very important. In this case, more reliable results can be obtained using the functional relationship with the shape of higher order polynomial or by using a series of several linear functions. For this study a magnitude-to-frequency diagram is used which is represented by a series of linear functions (Fig. 3).

In this case the relationship is determined by n linear functions of the shape:

$$\log n(m) = a_i - b_i m \ ; \ m_{i-1} < m < m_i \tag{8}$$

for i = 1, 2, 3 ... n

By replacing

$$\alpha_i = a_i l_n 10 \qquad \beta_i = b_i l_n 10 \tag{9}$$

the frequency of the earthquakes n(m) with a magnitudes larger than m is:

$$n(m) = \exp (\alpha_i - \beta_i m) \ ; \ m_{i-1} < m < m_i \tag{10}$$

for i = 1, 2, 3 ... n

It should be considered that m_o is the magnitude of the earthquake which doės not rise any engineering interest, while m_n is the maximum probable magnitude of the considered seismic source. The mean value of the number of occurred earthquakes in a time unit is obtained as

$$\nu = T^{-1} \left| \exp (\alpha_1 - \beta_1 m_o) - \exp (\alpha_n - \beta_n m_n) \right| \tag{11}$$

where, T is the time period for which the magnitude-frequency diagram is drawn. In the case of a single earthquake originating from this seismic source it is, also, possible to determine the possibility that the earthquake magnitude is larger than m, when the following n+2 relationships are obtained:

$$\hat{p} (M > m) = 1 \ ; \ -\infty < m < m_o \tag{12}$$

$$\hat{p} (M > m) = 1 - h \ \left| 1 - \exp (\alpha_i - \beta_i m - \alpha_1 + \beta_1 m_o \right| \tag{13}$$

for $m_{i-1} < m < m_i$; where $h = 1-\exp(\alpha_n - \beta_n m_n - \alpha_1 + \beta_1 m_o)^{-1}$; where, i = 1, 2, 3 ... n

$$\hat{p} (M > m) = 0 \ ; \ m_n < m < \infty \tag{14}$$

It should be mentioned here that the same relationship apply to the case when it is required to determine the probability for exceeding the magnitude m for an earthquake originated from the elementary part of the seismic source.

4. Probability for Occurrence of Maximum Ground Motion Value from the Elementary Part of the Seismic Source

As mentioned before, for the elementary part of the seismic source, eq. 12 to eq. 14 apply and they determine the probability that the magnitude is larger than m in case of an earthquake originating from the same elementary part. To determine the probability for occurrence of the characteristic maximum ground motion at a site in case of an earthquake originating from single elementary part, it is necessary to apply besides the known equations the attenuation equations for the seismic source to which the considered elementary part belongs.

$$\hat{p}\,(M > m) = \hat{p}\,(A > a/x) \tag{15}$$

i.e., the probability for exceeding the magnitude m is equal to the conditional probability for exceeding the characteristic maximum value of a in respect to x. Replacing the applied values, the following conditional probability equations can be obtained analogue to eq. 12 to eq. 14:

$$\hat{p}(A > a/x) = 1 \;;\; x_o < x < \infty \tag{16}$$

$$\hat{p}(A > a/x) = 1 - h\,\{[1 - \exp((\alpha_i - \frac{\beta_i}{c_2}\{l_n \frac{a}{c_1} (\frac{R}{\rho} + x_3)^{c_4} - x\} - \alpha_1 + \beta_1 m_o))]\} \tag{17}$$

where, $h = |1 - \exp(\alpha_n - \beta_n m_n - \alpha_1 + \beta_1 m_o)|^{-1}$; for $x_i < x < x_{i-1}$

where, $i = 1, 2, 3 \,\ldots\ldots\, n$

$$\hat{p}(A > a/x) = 0 \;;\; -\infty < x < x_n \tag{18}$$

All these $n + 2$ equations are valid in certain range along x :

$$x_i = l_n a - l_n \left| c_1 e^{c_2 m_i} (\frac{R}{\rho} + s_3)^{-c_4} \right| \;; \tag{19}$$

where, $i = 1, 2, 3 \ldots n$

The probability that $A > a$ is determined by integration of the conditional probability by the whole range of possible values of x.

$$\hat{p}(A > a) = \int_{-\infty}^{\infty} \hat{p}(a > a/x)\, f(x)\,dx \tag{20}$$

Replacing (n + 2) eq. 16 to eq. 18 for $P(A > a/x)$ and eq. 6 for $f(x)$, the following final final integration result is obtained:

$$\hat{p}(A > a) = (1-h)\Phi^*(\frac{x_n}{\sigma}) + h\,\Phi^*(\frac{x_o}{\sigma}) + h\exp(-\alpha_1 + \beta_1 m_o) \sum_{i=1}^{n} \exp(\sigma_i + \frac{\beta_i^2 \sigma^2}{2c_2^2}) \cdot (\frac{a}{c_1})^{-\frac{\beta_i}{c_2}} \cdot (\frac{R}{\rho} + c_3)^{-\frac{\beta_i c_4}{c_2}} \cdot \left| \Phi^*(\frac{x_i}{\sigma} - \frac{\beta_i \sigma}{c_2}) - \Phi^*(\frac{x_{i-1}}{\sigma} - \frac{\beta_i \sigma}{c_2}) \right| \tag{21}$$

Φ^* denotes complementary cumulative distribution of the Gaus random variable. This equation determines the probability for occurrence of the characteristic maximum ground motion at the site in case of an earthquake originating from the elementary part of the seismic source.

To determine the probability for occurrence of the same maximum ground motion with certain time duration of exposition t of the site to the effect of the earthquake originating from the elementary source, it is necessary to apply the probability model of earthquake generation by time. Applying the Poison's model for earthquake generation and eq. 21, it is obtained:

$$\hat{p}(A < a,\ t) = \exp\left|-\ \hat{\nu}t\ \hat{p}(A > a)\right| \tag{22}$$

This equation is the probability distribution function for occurrence of parameter a at the site exposed to the effect of the earthquake from the elementary seismic source for time t. In this equation, ν is the mean value of the number of earthquakes, for a unit of time, from the elementary seismic source.

5. Probability for Occurrence of the Maximum Ground Motion at the Site from the Complex Seismic Activity Model of the Region

The complex seismic activity model of the region around the site is represented by a large number of seismic sources of different geometrical shapes, different seismic activity and different attenuation characteristics. For numerical determination of the probability for occurrence of the characteristic ground motion parameter at the site, it is necessary to divide these sources into elementary parts. The activity of each elementary part can be determined based on cetrain probability distribution of earthquake occurrence by each seismic source. Assuming the independent character of earthquake occurrence by seismic sources and their parts, the distribution function of the characteristic parameter $P(A < a,t)$ is determined as a product of the probabilities of all the elementary parts $P(A < a,t)$

$$p(A < a,t) = \Pi\ \hat{p}(A < a,t) \tag{23}$$

The product in eq. 23 is determined applying eq. 21 and eq. 22 introducing two more indeces j and k. Index k denotes the seismic source and its maximum value can be N, which is the total number of seismic sources. Index j denotes the elementary part of k-th seismic source and its maximum value can be N_k, which is the total number of elementary parts of k-th seismic source. Index i remains the same as in eq. 21, i.e., denotes the linear portion of the magnitude-to-frequancy diagram of k-th seismic source. The maximum value of index i for k-th seismic source can be M_k, which is the total number of the linear portions of the diagram for the same seismic source. Finally, it is determined the probability distribution function for occurrence of the characteristic ground motion parameter at the site exposed to the seismic effect of all the seismic sources in the region for t year period of time.

$$P(A < a,t) = \exp(-t \sum_{k=1}^{N} \sum_{j=1}^{N_k} \nu_{j,k} \cdot P_{j,k}) \tag{24}$$

In this equation, the characteristics of the seismic sources, which determine the seismic activity ($m_{i,k}$, $s_{2,k}$, $c_{3,k}$, $c_{4,k}$, $\alpha_{i,k}$, $\beta_{i,k}$ N_k, h_k, ν_k, T_k), attenuation relationship ($c_{1,k}$, $c_{2,k}$, $c_{3,k}$, $c_{4,k}$, σ_k, $b_{1,k}$, $b_{2,k}$, $b_{3,k}$...) as well as the geometrical position (γ_k) are denoted by the symbols as in the equations for the elementary part of a seismic source. The difference is in the added indeces j and k which determine the exact position of each elementary part within the integral seismicity model of the region. Therefore, here it is not necessary to repeat the meaning of the individual constants applied in eq. 24.

6. Conclusions

Applying the approach of item 5 it is possible to determine the probability distribution function for occurrence of the characteristic ground motion parameter at the site. The value of these design parameters should be defined based on the acceptable seismic risk, which is regularly lower than the other risks at which the structure is exposed. In this way, it is possible to determine the required design parameters depending on available data for the considered region. The design response spectrum can be obtained based on the maximum acceleration, velocity and displacement values, determined by the suggested probability method.

7. References

/1/ CORNELL, H.A., "Engineering Seismic Risk Analysis", Bulletin of the Sėismological Society of America, Vol. 5, 1968.

/2/ ALGERMISSEN, S.T., "Seismic Risk Studies in the United States", Proceedings of the 4th World Conference on Earthquake Engineering, Santiago, Chile, 1969.

/3/ CORNELL, S.A., "Probabilistic Analysis of Damage to Structures under Seismic Loads", Dynamic Waves in Civil Engineering, D. A., Howels, I. P. Haigh and C. Taylor, Editors, John Willey, London 1971.

/4/ PETROVSKI, D,S., NAUMOSKI, N., "Probabilistic Evaluation of Earthquake Design Parameters of Nuclear Power Plants", 6th International Conference on Structural Mechanics in Reactor Technology, Paris, France, 1981.

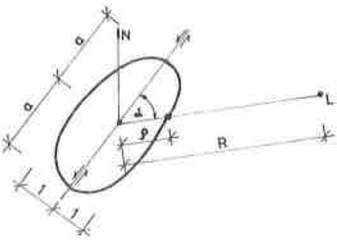

Fig. 1

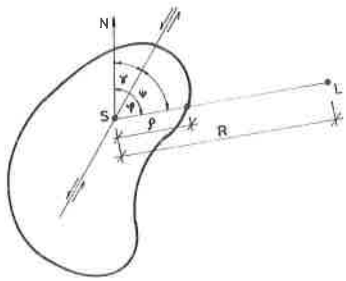

Fig. 2

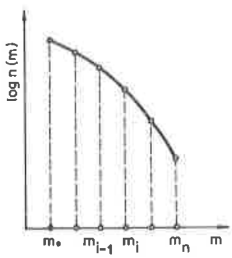

Fig. 3