Seismic Design Response of Structure Subjected to Six Components of Earthquake

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Abstract
A complete description of the earthquake-induced ground motion requires definition of six components. These components, defined along a set of structural axes will be statistically correlated with each other. In this paper, the three rotational components are defined in terms of the translational components and the correlation between the six components is then analytically established. As the correlation varies with the orientation of the structure and affects the response, some orientations may induce larger response than others. A method is developed to obtain the largest response, irrespective of the structural orientation. The numerical results obtained for the maximum response are presented. It is shown that the effect of the rotational components can be quite large, especially for large structures.

1. Introduction
During an earthquake, a structure will, in general, experience all six components of the earthquake-induced ground motion: three translation and three rotational components. In practice, however, only translational components are considered. They are also assumed to be statistically independent. The components are, however, correlated and this correlation depends upon the orientation of the structure. In this paper, a response spectrum approach is developed for the calculation of the response of a structure subjected to six correlated components. Using this approach, the maximum response which can possibly occur, irrespective of the orientation of the structure, can be easily obtained. Numerical results showing the effect of the rotational components on a response, as well as the effect of the search for the maximum response are presented.

2. Correlation Between Earthquake Components
Assuming the existence of three uncorrelated translation components, called the principal components, the components in any other orientation can be defined as [1]:
\[ (E'_i) = [D]^{T} (E) \]  \hspace{1cm} (1)
where \((E)\) and \((E'_i)\) are the vectors of the uncorrelated and correlated translational components and \([D]\) is the transformation matrix the element of which are the direction cosines of the uncorrelated component axes with respect to the axes of the correlated components. To
include the rotational components, the approach initially proposed by Newmark [2] can be used. It can be shown [3] that the vector of six components, \( \{E'(t)\} \), can be written in terms of \( \{E\} \) as:

\[
\{E'(t)\} = \{G_1\}^T + \frac{1}{2c} \frac{d}{dt} \{G_2\}^T \{U\} \{E(t)\}
\]

where \( c \) is the shear wave velocity of propagation and

\[
\{E'(t)\}^T = \{x_1, x_2, x_3, \ddot{x}_4, \ddot{x}_5, \ddot{x}_6\}
\]

\[
\{G_1\} = \{[I], [0J], [G_2] \} = \{[U],[J]\}
\]

In Eq. 3, \( \ddot{x}_i \) and \( \ddot{\psi}_i \) respectively, are the translational and rotational components of accelerations; and \([0],[I]\) are 3×3 identity and null matrices, respectively, and

\[
[J] = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}
\]

Eq. 2 can be used to define the correlation matrix of excitation components as:

\[
\text{Ex}[\{E'(t_1)\} \{E'(t_2)\}^T] = \frac{1}{2} \int_\omega \Phi_k(\omega) e^{i\omega(t_1-t_2)}
\]

\[
\{[G_1]^T \{d_k\} \{U\} \{G_1\}^T \{G_2\}^T \{d_k\} \{d_k\}^T \{G_2\}^T \} + \frac{i\omega}{4c^2} \{G_1\}^T \{d_k\} \{U\} \{G_1\}^T \{G_2\}^T \{d_k\} \{d_k\}^T \{G_2\}^T \{G_1\}^T \{d_k\} \{d_k\}^T \{G_2\}^T \{G_1\}^T \{d_k\} \{d_k\}^T \{G_2\}^T \{G_1\}^T \{d_k\} \{d_k\}^T \{G_2\}^T \{G_1\}^T \{d_k\} \{d_k\}^T \{G_2\}^T \{G_1\}^T \{d_k\} \{d_k\}^T \{G_2\}^T \{G_1\}^T \{d_k\} \{d_k\}^T \{G_2\}^T \{G_1\}^T \{d_k\} \}
\]

where \( \text{Ex}(\cdot) \) = expected value of (\( \cdot \)); \( \Phi_k(\omega) \) = spectral density function for the \( k \)th principal components of excitation; \( \{d_k\} \) is the direction cosine vector for the \( k \)th component defined in the structural coordinate system; and \( \omega \) = frequency variable. In the development of Eq. 5, the earthquake components are assumed to be stationary random processes.

3. Structural Response

The equation of motion for a structure subjected to six earthquake components can be written as:

\[
[M]\dddot{u} + [C]\ddot{u} + [K]u = - [M] [r'] \{E'\}
\]

where \([M],[C],[K]\) are the mass, damping and stiffness matrices, respectively; \(u\) = relative displacement vector; \([r]\) = the influence coefficient matrix of size \( nxn \), the \( n \)th vector \([r_k]\) of which is the earthquake displacement influence vector for the \( k \)th component; and \( n \) = the degrees-of-freedom. The solution of this can be obtained both for classically as well as nonclassically damped systems. For classically damped systems, the damping matrix can be decoupled by the undamped modal shapes of systems, whereas for nonclassically damped the complex modes are required to be used. See Reference 3. Here, we will provide the formulation only for the classically damped systems.

A decoupled modal equation of Eq. 6 is as follows:

\[
\dddot{v}_j + 2\xi_j \omega_j \dot{v}_j + \omega_j^2 v_j = - (y_j)^T \{E'(t)\}
\]

\[
-488-
\]

\[
K 10/7
\]
where \( v_j \) = \( j \)th principal coordinate; \( \omega_j \) = \( j \)th natural frequency, \( \xi_j \) = \( j \)th modal damping ratio; \( \gamma_j \) = 6x1 vector of \( j \)th participation factors. A response quantity of interest which is linearly related to the displacement can be written in terms of the modal quantities as:

\[
S(t) = \frac{1}{2} \sum_{j=1}^{n} \xi_j \gamma_j(t) \int_0^t \gamma_j(t-\tau) d\tau
\]

(8)

where \( \xi_j \) = \( j \)th normal mode shape of the response quantity \( S(t) \), and \( \gamma_j(t) \) is the impulse response function of Eq. 7. For design purposes, we are interested in the maximum value of \( S(t) \). This value can be related to the mean square value which can be shown to be as follows [3]:

\[
\text{Ex}[S^2] = \sum_{k=1}^{3} \xi_k^2 \text{Tr} \left[ \gamma_k \right] \xi_k
\]

(9)

where \( \left[ \gamma_k \right] \) is a 3x3 matrix, called the response matrix, for the excitation component \( k \) which is defined as:

\[
\left[ \gamma_k \right] = \frac{1}{2} \frac{1}{N} \sum_{j=1}^{N} \xi_j^2 \left[ \gamma_{1jk}^T \right] \left[ \gamma_{2jk}^T \right] \left[ \gamma_{3jk}^T \right] \left[ \gamma_{4jk}^T \right] \frac{1}{N} \frac{1}{N} \sum_{j=1}^{N} \xi_j \xi_k
\]

(10)

where \( \gamma_{1jk}, \gamma_{2jk}, \gamma_{3jk}, \gamma_{4jk} \) are the coefficients of partial fractions, defined in Reference 3. The elements of matrices \( \left[ \gamma_{1jk} \right], \left[ \gamma_{2jk} \right] \) and \( \left[ \gamma_{3jk} \right] \) in their final simplified form are defined as follows:

\[
\gamma_{1jk} = \gamma_{nj} \gamma_{nk} + \gamma_{nj} \gamma_{mk}
\]

(11)

\[
\gamma_{2jk} = \frac{6}{n} \sum_{p=4}^{n} \gamma_{np} \gamma_{nq} (\gamma_{pq} \gamma_{pk} + \gamma_{qj} \gamma_{pq}) \frac{4c^2}{Ac^2}
\]

(12)

\[
\gamma_{3jk} = \frac{1}{2c^2} \left[ \sum_{q=4}^{n} \gamma_{np} \gamma_{nq} (\gamma_{nq} \gamma_{mk} - \gamma_{nj} \gamma_{nk}) \right] - \frac{6}{n} \sum_{p=4}^{n} \gamma_{np} (\gamma_{mj} \gamma_{mk} - \gamma_{uj} \gamma_{mk})
\]

(13)

in which \( m, n, 1, 2, 3 \) are the elements in the \( \mu \)th row and \( \nu \)th column of matrix \( \left[ \xi_j \right] \), \( \gamma_{nk} \) and \( \gamma_{nj} \) are the frequency integrals representing the mean square values of the relative displacement and relative velocity responses of an oscillator of frequency \( \omega_j \) and damping ratio \( \xi_j \) as follows:

\[
I_{nk}^\omega(\omega_j) = \int_{-\infty}^{\infty} \gamma_{nk}(w) \Phi_k(w) \gamma_{nk}(w) |H_j|^2 dw; \quad I_{nj}^\omega(\omega_j) = \int_{-\infty}^{\infty} \gamma_{nj}(w) \Phi_k(w) \gamma_{nj}(w) |H_j|^2 dw
\]

(14)

These integrals can be defined in terms of the pseudo acceleration and relative velocity response spectra and their respective peak factors. Thus, Eq. 10 and the response defined by Eq. 9 can also be obtained in terms of ground response spectra directly.
4. Worst-Case Response

Eq. 9 can be used to obtain the mean square value if the orientation of the principal components is known. This is, however, very unlikely for a design. In absence of this information, we can still obtain the maximum possible value of the mean square response and the corresponding direction of the impinging seismic waves. This can be done by employing the Lagrange multiplier approach, as described in References [3,4]. This approach requires the solution of the following eigenvalue problem

$$[K_x]d_x - \lambda d_x = 0, \quad x = 1,2,3$$ (15)

with the constraint $d_x d_x^T = 1$. This problem is solved for each principal component separately. Each solution gives a set of three eigenvalues and corresponding eigenvectors. It can be shown that each eigenvalue is precisely the mean square response when the component is applied along the direction defined by the corresponding eigenvector. The complete solution defines nine eigenvalues and a set of eigenvectors for the three excitation components. These values can then be used to identify the globally maximum response out of several (in fact 18) locally maximum values by using the systematic search outlined in Ref. 3. One of such possible 18 values, for example, is as follows:

$$E[S^2] = \lambda_{11} + [d_1^{(2)}]^T [K_x] [d_1^{(2)}] + [d_1^{(3)}]^T [K_x] [d_1^{(3)}]$$ (16)

This is the mean square response due to excitations 1, 2 and 3 respectively, applied along the directions of eigenvectors $[d_1^{(1)}]$, $[d_1^{(2)}]$ and $[d_1^{(3)}]$ obtained for the eigenvalue problem of excitation [1].

Herein, this globally maximum mean square value is called as the worst-case response. This when multiplied by the peak factor will give the maximum design response which can possibly occur irrespective of the orientation of the structure. This value can be obtained for any given intensities and frequency characteristics of the principal components. These characteristics can be defined in terms of ground response spectra, and the element of response matrices $[K_x]$ can be obtained by Eq. 10, which is in essence a response spectrum type of approach.

5. Numerical Results

To show the effects of the rotational components and making a search for the maximum response, the numerical results obtained for an arbitrary space frame structure subjected to six components are presented. The principal components are defined in terms of three spectral density functions of Kanai-Tajimi form as are given in the report by Ghafary-Ashtiani and Singh [3]. The space frame structure is shown in Fig. 1. The masses are assumed to be lumped at the nodes, and each mass has six degrees-of-freedom.

Table 1 shows the results obtained for the member bending moments about one of the member axes. The shear wave velocity was taken to be $c = 2000$ ft. per second. The Table shows the results obtained with six components, with three components and with and without making a search for the worst-case response. The results reported in Col. (2) and (4), respectively, are the worst-case response values obtained with six and three components, respectively. Their ratio is shown in Col. (6). This ratio shows the effect of omitting the
rotational components. It is seen that this effect can be quite large. The results obtained for other structures [see Ref. 3] also show similar trend. These effects are more drastic for large structures.

Columns (3) and (5) show the results obtained when the principal components are applied along the structural axes and the responses due to various components are combined by the square-root-of-the-sum-of-squares method; thus no search for the worst-case response is made. For this structure, the results show that response is likely to be underestimated by about 10% if no search for the maximum is made. Depending on the type of structure, especially the symmetry, this effect could be small or large.

6. References


TABLE 1: BENDING MOMENTS ABOUT X3-AXIS IN SPACE FRAME OF FIG. 1 FOR C = 2000 FPS.

<table>
<thead>
<tr>
<th>MEMBER NO.</th>
<th>SIX COMPONENTS EXCITATION</th>
<th>THREE TRANSLATIONAL EXCITATION</th>
</tr>
</thead>
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<tr>
<td></td>
<td>MAX. RESP.</td>
<td>SKSS MAX. RESP.</td>
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<tr>
<td>(1)</td>
<td>130.4</td>
<td>0.846</td>
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<tr>
<td>2</td>
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<td>0.839</td>
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<tr>
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<tr>
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</tr>
<tr>
<td>8</td>
<td>58.8</td>
<td>0.909</td>
</tr>
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**Figure 1:** A SPACE FRAME STRUCTURE