The Evaluation System of the Stiffness of Nuclear Power Plant

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ABSTRACTS

This report describes the outline of the evaluation system of the stiffness of large scale structures and shows the example of this procedure.

In the dynamic analysis of large scale structures, Matrix Condensation Method which is adapted in the evaluation of the normal mode of them is effective to replace the three dimensional finite element model with the dynamically equivalent lumped mass model.

The flexural stiffness and the shear stiffness of them is given by Strain Energy Method, according to the first mode obtained in Matrix Condensation Method.

1. INTRODUCTION

Recently, in the large scale and complicate structures such as a nuclear power plant, non-linear dynamic analysis is required in respect of the security with the progress of larger and high speed computers. The eigenvalue solution using the three dimensional finite element model has a problem of multi-degrees of freedom. Therefore, in the dynamic analysis of them, usually lumped mass model which is correctly evaluated from the three dimensional finite element model of structures is used.

This procedure has two peculiarities, one is that, in the eigenvalue solution of the three dimensional finite element model of them, local unwanted effects are added to the normal mode of the main structure. It is difficult to separate the additional unwanted mode from the normal mode when replacing the structure with the equivalent lumped mass model. Therefore, using Matrix Condensation Method to the eigenvalue equation on the master degree of freedom related to the normal mode of the structure, we are possible to separate the unwanted slave mode from the normal structure mode. The other is that the evaluation of the flexural rigidity or shear rigidity is conducted using Strain Energy Method according to the first mode obtained in abovementioned Matrix Condensation Method, and the lumped mass model to the structure for the dynamic analysis is made using those stiffnesses. The flow chart of this procedure is shown in Fig. 1.

2. MATRIX CONDENSATION METHOD

KM Matrix Condensation Method proposed by J. Guyan and M. Irons is used to the reduction of the unwanted slave degrees of freedom from the full-matrix of the structures.

Stiffness matrix, mass matrix, displacement vector and external force vector are respectively in the next
forms;

\[ [K] = \begin{bmatrix} K_{aa} & K_{ac} \\ K_{ca} & K_{cc} \end{bmatrix}, \quad \{d\} = \begin{bmatrix} \frac{da}{dc} \\ \frac{dc}{dc} \end{bmatrix}, \quad \{\dot{d}\} = \begin{bmatrix} \frac{\ddot{da}}{dc} \\ \frac{\ddot{dc}}{dc} \end{bmatrix} \]

\[ \{f\} = \begin{bmatrix} f_a \\ f_c \end{bmatrix} \]

\[ \ldots \ldots (1) \]

\[ [\bar{M}] = \begin{bmatrix} M_{aa} & M_{ac} \\ M_{ca} & M_{cc} \end{bmatrix}, \quad \{\dot{f}\} = \begin{bmatrix} \frac{\dot{f}_a}{dc} \\ \frac{\dot{f}_c}{dc} \end{bmatrix} \]

\[ \ldots \ldots (2) \]

where, under-subscript \(a\) and \(c\) mean the master and the slave degree of freedom respectively.

Thus, stiffness condensation matrix \([\bar{K}]\) and mass condensation matrix \([\bar{M}]\) are written as follows;

\[ [\bar{K}] = [K_{aa}] - [K_{ac}] [K_{cc}]^{-1} [K_{ca}] \]

\[ [\bar{M}] = [M_{aa}] - [K_{ca}]^T [K_{cc}]^{-1} [M_{ca}] - [M_{ac}] [K_{cc}]^{-1} [K_{ca}] 
+ [K_{ca}]^T [K_{cc}]^{-1} [M_{cc}] [K_{cc}]^{-1} [K_{ca}] \]

\[ \ldots \ldots (2) \]

3. STRAIN ENERGY METHOD

The stiffness evaluation procedure which we present is as follows; Firstly the eigenvalue solution is performed to get the normal mode of the structure using Matrix Condensation Method. Secondly the static analysis is performed using the three dimensional finite element model, where the static force to each layer is applied so that the distribution is similar to the first mode obtained by Matrix Condensation Method.

Applied external force is given by the formula as follows;

\[ P_i = \beta_i U_i W_i \]

\[ \ldots \ldots (3) \]

where \(P_i\): external force in the i-th layer

\(\beta_i, U_i\): the first participation factor due to the results of the eigenvalue analysis using Matrix Condensation Method

\(W_i\): mass weight of the i-th layer

With the result of this static analysis, if the total axial energy is equal to the equivalent incremental rotation energy, the equation refer to the strain energy is written as follows;

\[ \sum_j (N_{ij} \Delta u_{ij}) = \sum_j N_{ij} (\Delta \theta_{el} \kappa_{ij}) \]

\[ \ldots \ldots (4) \]

where \(N_{ij}\): axial force of the i-th element in the i-th layer

\(\Delta u_{ij}\): incremental axial displacement of the j-th element in the i-th layer \((\Delta u_{ij} = U_{ij} - V_{i-1,j})\)

\(V_{ij}\): axial displacement of the j-th element in the i-th layer

\(\Delta \theta_{el}\): equivalent incremental rotation angle in the i-th and (i-1)th layer (shown in Fig. 2)

\(\theta_{el}\): equivalent rotation angle the i-th and (i-1)th layer \((\Delta \theta_{el} = \theta_{el} - \theta_{el,i-1})\)

\(\kappa_{ij}\): distance between the j-th element and neutral axis in the i-th layer

Equivalent incremental rotation angle due to Mohr theorem is expressed as follows;
\[ \Delta \theta_{ei} = \int_{-l_i}^{l_i} \frac{M_i}{E_i} \, dx = \frac{M_i h_i}{E_i l_i} \quad \ldots \ldots (5) \]

where
- \( M_i \): bending moment of the layer in the \( i \)-th layer
- \( h_i \): floor height
- \( E_i l_i \): flexural stiffness of the \( i \)-th layer

Let \( M_i = \sum_j (N_{ij} e_{ij}) \) in eq. (5), the equivalent flexural stiffness is written as follows:

\[ E_i l_{ei} = \frac{\sum_j (N_{ij} e_{ij})}{\Delta \theta_{ei}} \cdot h_i \quad \ldots \ldots (6) \]

On the other hand, the shear deformation in the \( i \)-th layer, reducing the relative flexural displacement \( (\delta_{ei}) \) from the relative horizontal displacement \( (\delta_{ei}) \), is expressed as follows:

\[ \delta_{si} = \delta_{ei} - \delta_{Mi} = \delta_{ei} - \frac{\theta_{ei} + \theta_{ei-1}}{2} \cdot h_i \quad \ldots \ldots (7) \]

Then, the equivalent shear stiffness is expressed as follows:

\[ G_A e_i = \frac{Q_i h_i}{\delta_{si}} \quad \ldots \ldots (8) \]

where
- \( Q_i \): shear force of the layer in the \( i \)-th layer

4. EXAMPLE 1 Inner concrete

As a calculation example of this system, the analytical eigenvalue results obtained by the three methods using the models, which are the three dimensional finite element full model and the condensation model due to Matrix Condensation Method, and the lumped mass model due to Strain Energy Method, are compared for the model which imagines the two loop inner concrete building of a PWR type nuclear power plant.

In the three dimensional finite element model, a shell element is used for the primary shelter wall and its inner floor, and a solid element is used for the secondary shelter wall.

Analytical model is shown in Fig. 3. In the eigenvalue calculation due to Matrix Condensation Method, 22 locations are selected as the condensation point, and the eigenvalue analysis is conducted to each node in consideration of two degrees of freedom in \( \delta_y \) and \( \delta_x \) directions. The lumped mass model using the equivalent stiffness due to Strain Energy Method is shown in Fig. 4 and the resultant stiffness is shown in Table 1.

Analytical results: The first mode due to the three dimensional finite element model is shown in Fig. 5. The comparison between the eigenvalues due to each method is shown in Table 2. The participation modes are compared in Fig. 6.

5. EXAMPLE 2 Auxiliary building

As the other example, the eigenvalues obtained by the three methods are compared for the model which imagined a nuclear power plant auxiliary building. The auxiliary building is the reinforced concrete structure having four floors, and is mainly composed of the elements of an earthquake-proof wall. The half model of the building is used as the three dimensional finite element model in consideration of its symmetry. The analytical
model is shown in Fig. 7 in which the master points are located at the four corners of each floor, and the 16 points are selected.

Analytical results: The comparison of eigenvalues by each method is shown in Table 3 and also that of the participation function mode is shown in Fig. 8. The shear sectional area and the moment inertia of section are geometrically calculated from the structural drawings using a usual method, and the results and the stiffnesses obtained by Strain Energy Method are compared in Table 4.

6. CONCLUSIONS

When comparing the analytical eigenvalue results due to Full Matrix Method, Matrix Condensation Method and Strain Energy Method, there exist fairly good coincidence in respect of both the first eigen mode and eigenvalue. As the respect to the second or more order mode, smaller degrees of freedom of models shows higher frequency, but very close results can be obtained each other.

When using the eigenvalue due to Matrix Condensation Method, we can get higher order basic frequency mode not including local unwanted slave mode. The eigenvalue analysis due to the lumped mass model by Strain Energy Method shows the good coincidence in comparison with another two methods, therefore, this system is an effective method in evaluation of the stiffness of large scale structures.

REFERENCES


![Diagram of Evaluation System of a Structure](image)
Fig. 3 Analytical model (Inner concrete)

Table 1 Resultant stiffness of lumped mass model

<table>
<thead>
<tr>
<th>Node NO.</th>
<th>Weight Wi (t)</th>
<th>Member NO.</th>
<th>Shear Stiffness Ae (m²)</th>
<th>Flexural Stiffness Iei (m⁴)</th>
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</thead>
<tbody>
<tr>
<td>1</td>
<td>170.86</td>
<td>1</td>
<td>3.00</td>
<td>115.9</td>
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<tr>
<td>2</td>
<td>627.9</td>
<td>2</td>
<td>10.81</td>
<td>424.4</td>
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<td>3</td>
<td>396.9</td>
<td>3</td>
<td>11.49</td>
<td>210.8</td>
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<tr>
<td>4</td>
<td>345.68</td>
<td>4</td>
<td>79.66</td>
<td>5322.0</td>
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<tr>
<td>5</td>
<td>397.53</td>
<td>5</td>
<td>89.46</td>
<td>794.60</td>
</tr>
<tr>
<td>6</td>
<td>2606.4</td>
<td>6</td>
<td>124.3</td>
<td>10341.0</td>
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<tr>
<td>7</td>
<td>4056.7</td>
<td>7</td>
<td>182.3</td>
<td>10817.0</td>
</tr>
</tbody>
</table>

Fig. 4 Lumped mass model

Fig. 5 1-th mode (Full Matrix Method)

Fig. 6 Participation modes by each method

- Full Matrix Method
- (Strain Energy Method)
- (Matrix Condensation Method)
Fig. 7 Analytical model (Auxiliary building)

Fig. 8 Participation modes by each method

Table 2 Eigenvalues by each method (Inner concrete)

<table>
<thead>
<tr>
<th>Mode NO.</th>
<th>Matrix Condensation Method</th>
<th>Strain Energy Method</th>
<th>Full Matrix Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>period (sec)</td>
<td>frequency (Hz)</td>
<td>period (sec)</td>
</tr>
<tr>
<td>1</td>
<td>0.8971</td>
<td>1.115</td>
<td>0.8972</td>
</tr>
<tr>
<td>2</td>
<td>0.04863</td>
<td>0.956</td>
<td>0.04809</td>
</tr>
<tr>
<td>3</td>
<td>0.03929</td>
<td>2.345</td>
<td>0.03491</td>
</tr>
</tbody>
</table>

Table 3 Eigenvalues by each method (Auxiliary building)

<table>
<thead>
<tr>
<th>Mode NO.</th>
<th>Matrix Condensation Method</th>
<th>Strain Energy Method</th>
<th>Full Matrix Method</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>period (sec)</td>
<td>frequency (Hz)</td>
<td>period (sec)</td>
</tr>
<tr>
<td>1</td>
<td>0.1392</td>
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<td>2</td>
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<tr>
<td>3</td>
<td>0.0365</td>
<td>27.40</td>
<td>0.0341</td>
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Table 4 Comparison of the resultant stiffness

<table>
<thead>
<tr>
<th>Member NO.</th>
<th>Strain Energy Method</th>
<th>Geometrically calculated</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Shear Stiffness (Aei (m³))</td>
<td>Flexural Stiffness (Iei (m⁴))</td>
</tr>
<tr>
<td>1</td>
<td>350.5</td>
<td>3,953 × 10⁴</td>
</tr>
<tr>
<td>2</td>
<td>476.4</td>
<td>1,869 × 10⁵</td>
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<tr>
<td>3</td>
<td>487.7</td>
<td>2,585 × 10⁵</td>
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<tr>
<td>4</td>
<td>594.4</td>
<td>2,716 × 10⁵</td>
</tr>
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