A New Technique for Generating Spectrum Compatible Accelerogram

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B Abstract
A new technique for generating spectrum compatible earthquake accelerogram is presented. Simpliefied linearised schemes are used to determine the weights of the modulated sinewaves used to represent the ground acceleration in conformity with the instants of time of attaining the maximum responses of the SDOFs. Some typical numerical results are presented in the paper.

1. Introduction

For the seismic design of a nuclear power plant warranting a high degree of safety the basic input should be known with a reasonable degree of accuracy. The response spectrum method is well known to lead to overconservative designs. Further, theoretical analyses and actual observations have evinced the need for the details of the input motion for predicting dynamic response to and damage potential of an earthquake ground motion. Even if the response spectrum method were chosen for design and analysis, the time history of the ground motion has to be obtained for obtaining the floor response spectra since usually, only ground response spectrum is available as an input.

Several methods of generating artificial time-history of earthquake ground motion have been proposed [1-5].

The earlier practice of 'suitable modifying' a recorded accelerogram to obtain the desired time-history has been abandoned and one of the currently used procedure is to obtain a specified design-response-spectrum (SDRS) compatible accelerogram (SCA). The SCA, apart from matching the target response spectrum should satisfy the following constraints arising out of the seismotechtonic features of the site, viz. (i) peak ground acceleration, (ii) total duration, (iii) time dependent enveloping functions defining the rise time to strong motion, duration of significant shaking and decay of the strong motion and (iv) rate of zero crossings.

Basically, the methods for generating SCA can be divided in two classes. In one, a reference accelerogram is chosen and the Fourier Amplitude Spectrum (FAS) is suitably altered [1-2] such that the time-history generated response spectrum (THRS) closely matches the SDRS. Several variations are possible to alter the FAS, viz. (i) scaling the entire FAS up or down or (ii) scaling the FAS locally where the matching is not good. It may be noted that in linear scaling of the FAS both the real and imaginary parts of the fourier spectrum at any given frequency are multiplied by the same constant and thus, the phase characteristics of the chosen accelerogram are preserved.

In a second variation of this method based on a reference accelerogram one uses a filter in case the THRS is very much above the SDRS and add a damped sinusoid in case the THRS falls below the SDRS. However, this process of local scaling alters the phase relationship of the reference accelerogram.

In the second class of methods [3-5] no reference accelerogram is used as such but the ground acceleration is assumed to be represented by a superposition of suitably modulated sinusoid with an assumed phase relationship among the various frequency components. The gross characteristics of the earthquake ground motion are represented either in terms of an envelope time function [3,4] or the phase differences distribution [5] based on ground motion parameters.

Then generation of a SCA involves the computation of the N weights (G_j) of various modes in the assumed form of the ground acceleration when the SDRS is sampled at N points.

The present method described in this paper differ from others in basically two aspects.

(i) The spectral values are explicitly used at each stage of computation and, (ii) the instants of time (T_i) to attain the peak response are actually evaluated in conformity with the G_j's rather than being chosen as random numbers.

The total formulation of the 2N non-linear equations involving G_j (j=1,2,n) and T_i (i=1,2,...n) is given. However, only the linear iterative methods of solution are presented in this paper.

Some sample case studies are presented to show the efficacy of this method where a well known method [4] has failed to converge after several iterations. Next, a realistic SDRS is taken as the input and the corresponding time-history is shown.

2. Theory

2.1 Analysis of a SDOF System

The equation of motion of a single-degree-of-freedom (SDOF) system subjected to a base acceleration excitation ($\ddot{x}_g$) is

$$\ddot{x} + 2\zeta\omega\dot{x} + \omega^2 x = -\ddot{x}_g \tag{1}$$

where $\omega^2 = k/m$ and $\zeta = c/(2\omega m)$

Assuming zero initial conditions, the final solutions is given by the following Duhamel's integral

$$x = -\frac{1}{\bar{\omega}} \int_0^t e^{-\zeta\omega(t-\tau)} \sin[\bar{\omega}(t-\tau)]\, \ddot{x}_g(\tau)\, d\tau \tag{2}$$

$$\bar{\omega} = \omega(1-\zeta^2)^{1/2}$$

If the base excitation is expressed as a modulated sine wave i.e.

$$\ddot{x}_g = (e^{-\alpha t} - e^{-\beta t}) \sin(\Omega t - \phi) \tag{3}$$

then the displacement response (x) can be written as

$$x = I_1 \cos\phi + I_2 \sin\phi \tag{4}$$

where

$$\hat{I}_k = (e^{-\alpha t} A_k + e^{-\beta t} B_k)\cos(\Omega t) + (e^{-\alpha t} C_k + e^{-\beta t} D_k)\sin(\Omega t)$$
$$+ e^{-\zeta \omega t}\left[E_k \cos(\bar{\omega} t) + F_k \sin(\bar{\omega} t)\right];\ k = 1,2 \tag{5}$$

2.2 Generation of SCA

The earthquake ground acceleration is chosen to be of the following form

$$\ddot{x}_g = (e^{-\alpha t} - e^{-\beta t}) \sum_{j=1}^{N} G_j \sin(\Omega_j t - \phi_j) \tag{6}$$

Then the response ($y_i(t)$) of the SDOF having an undamped natural frequency ω_i and damping factor ζ_i will be given by

$$y_i(t) = \sum_{j=1}^{N} G_j \left[P_{1ij}(t)\cos(\phi_j) + P_{2ij}(t)\sin(\phi_j)\right] \tag{7}$$

where $y_i(t)$ will be x_i, $\dot{x}_i$ and $\ddot{x}_i$ for displacement, velocity and acceleration response respectively. Correspondingly, P_{kij} will be I_{kij}, $\dot{I}_{kij}$ and $\ddot{I}_{kij}$ respectively. The maximum response of the ith system to the excitation given by equation (6) will be attained at a time (T_i) when

$$\dot{y}_i(T_i) = \sum_{j=1}^{N} G_j \left[\dot{P}_{1ij}(T_i)\cos(\phi_j) + \dot{P}_{2ij}(T_i)\sin(\phi_j)\right] = 0 \tag{8}$$

In order that the accelerogram, equation (6) is compatible with the SDRS the following equations must be satisfied.

$$S_i = \left| \sum_{j=1}^{N} \left[P_{1ij}(T_i)\cos(\phi_j) + P_{2ij}(T_i)\sin(\phi_j)\right] \right| \ \forall i = 1, N. \tag{9}$$

Equations (8) and (9) provide the 2N non-linear equations to solve for the 2N unknowns G_j and T_i ($i, j = 1, N$). However, a simplified linear scheme of solution is presented in this paper.

2.3 Description of the Numerical Methods

The instants of time (T_i) for maximum response are solved from the set of equations (8). In one scheme, this is achieved by setting the characteristic determinant equal to zero and these values of T_i are used to obtain the G_j's from

$$y_i(T_i) = \operatorname{sgn} S_i \tag{10}$$

where sgn is chosen as $\pm$ 1.

In another scheme, T_i's are first solved from equation (8) for an assumed set of values of G_j which is then updated by equation (10) using the calculated T_i's. The iterations are continued till a converged solution consistent with the constraints is obtained.

2.4 Numerical Analysis

The second method is similar to the Levy-Wilkinson (LW) method 4 with the difference that the spectral values are explicitly used in each iteration for evaluation of the weights G_j's. It is observed from various numerical studies that the (LW) method fails to converge particularly when the sampled frequencies (ω_i) on the SDRS are different from the frequencies (Ω_i) appearing in the expression for the ground acceleration. This is illustrated by a simple example. A test spectrum (SDRS) was generated for

$$\ddot{x}_g = \sin\frac{2\pi t}{10} + \sin\frac{4\pi t}{10}$$

and was sampled at 10 points for ζ = 0.02. It was intended to generate $\ddot{x}_g = G_1 \sin\frac{2\pi t}{10} + G_2 \sin\frac{4\pi t}{10}$ i.e. to calculate G_1 and G_2 such that it is compatible with the above mentioned SDRS. Results of a 10 point matching by LW method are shown in Table-I. For brevity, results upto 5 iterations are only shown; actually they are worse in subsequent iterations. The present scheme however, converged to the expected solution (i.e. $G_1 = G_2 = 1$) in two iterations starting from the same assumed values of G_1 as used in the LW method.

Next, a typical realistic SDRS (Fig.1) is chosen for generating a SCA. A comparison of the SDRS and THRS is shown in Table-II and the computed acceleration history is shown in Fig.2.

3. Discussion

The formulation of the nonlinear problem is exact and the method by itself is non-iterative. The simplified linear scheme of solution presented in this paper is an improvement over some of the existing methods since the equality of the spectral values of SDRS and THRS is explicit and time for attainment of the maximum response is actually calculated rather than being chosen as a random number.

References

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TABLE - I

Results of SCA Generation by LW Method

$\omega_i = 0.1\,i$; $\Omega_i = \frac{2\pi i}{10}$; $(i = 1, 10)$; $\zeta = 0.02$

Sl. No.	Specified values of velocity spectrum	Computed Values of $/V_{max}/$ after 1 iteration	after 3 iterations	after 5 iterations
1	0.3148E + 01	.2604E + 02	0.8345E + 01	0.1520E + 02
2	0.3198E + 01	.2644E + 02	0.8493E + 01	0.1565E + 02
3	0.3431E + 01	.2743E + 02	0.7585E + 01	0.1265E + 02
4	0.5103E + 01	.3248E + 02	0.8211E + 01	0.1463E + 02
5	0.7884E + 01	.41448E+ 02	0.8479E + 01	0.1489E + 02
6	0.1730E + 02	.5296E + 02	0.9114E + 01	0.1602E + 02
7	0.1177E + 02	.4308E + 02	0.8842E + 01	0.1572E + 02
8	0.6614E + 01	.2633E + 02	0.8054E + 01	0.1466E + 02
9	0.5580E + 01	.2468E + 02	0.8228E + 01	0.1454E + 02
10	0.5574E + 01	.2860E + 02	0.8652E + 01	0.1502E + 02

TABLE - II

Comparison of SDRS and THRS 10 Point Matching of Acceleration Response Spectrum

Sl. No.	Frequency, f_j (Hz)	Spectral Acceleration (g) SDRS	THRS
1	33.33	0.2400	0.2442
2	22.72	0.7785	0.7794
3	15.35	1.0080	1.0070
4	10.37	1.0080	1.0110
5	7.00	1.0080	1.0070
6	4.73	1.0080	1.0050
7	3.20	1.0080	0.9954
8	2.16	0.7945	0.7715
9	1.46	0.5708	0.5360
10	0.99	0.4101	0.3669

$\Omega_j = \omega_j = 2\pi f_j$

$\alpha = 0.2$; $\beta = 0.5$; $\zeta = 0.02$

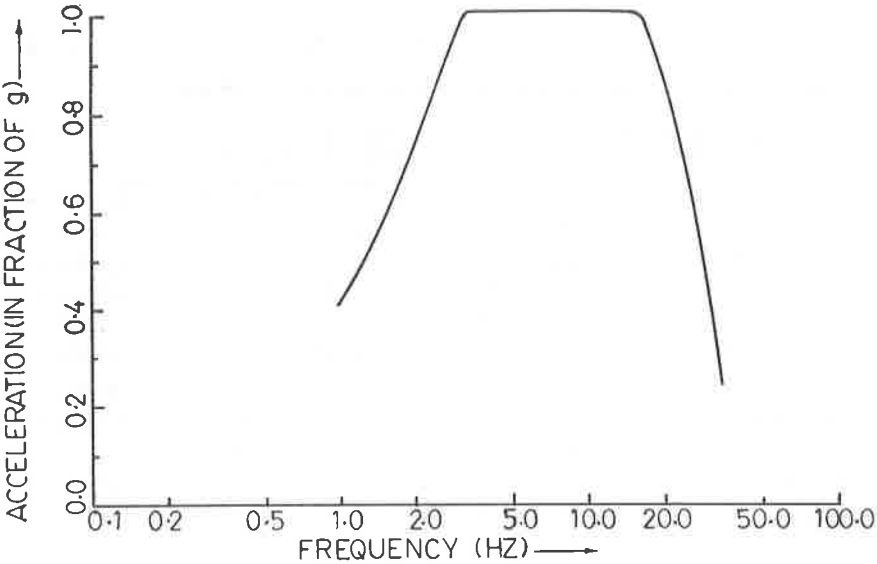

FIG.1: SPECIFIED DESIGN RESPONSE SPECTRUM (SDRS)

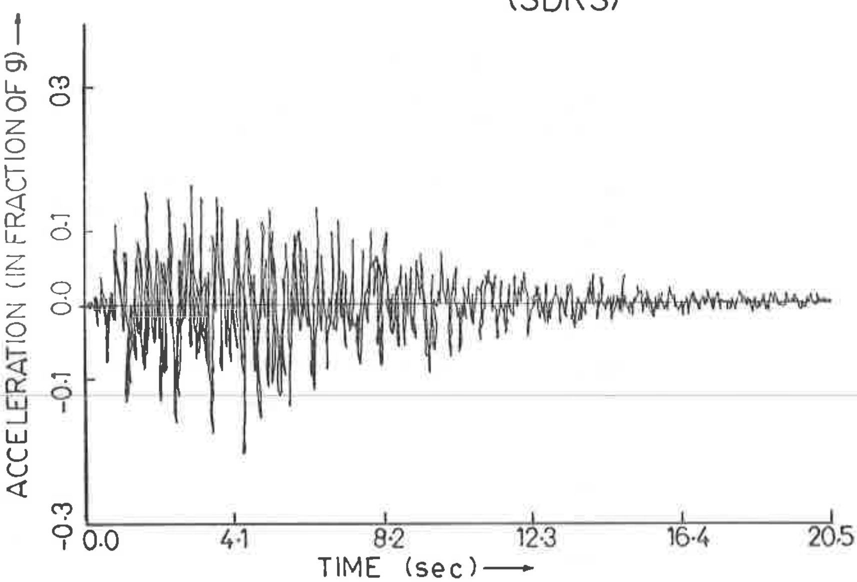

FIG. 2 : GENERATED ACCELEROGRAM