Simulated Earthquake Motion Using Algebraic Function Phase
Most Fitting Simultaneously Against Response Spectra
with Different Damping

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ABSTRACT
One of the earthquake motion simulations computer generated is termed the wave superposition method. Its elemental wave amplitudes are determined by target convergent calculation on some smoothed response spectra. Phases are usually given with uniform random numbers. We seldom experienced on response spectrum over the level of the target response spectrum with lower damping. We therefore established a generation method to fit the response spectra with simultaneously with 2 kinds of damping.

1. INTRODUCTION
We define a Target Acceleration Response Spectrum with 5% damping (SAT(5%)) , and generated Simulated Earthquake Motion (SEM) accompanying Acceleration Response Spectrum with 5% damping (SA(5%)) with practically the same characteristics of SAT(5%). The SEM generating procedure is a superposition method of several sinusoidal waves. Individual elementally sinusoidal wave amplitudes are determined by convergent calculations aimed at SAT(5%). Here, the Target Acceleration Response Spectrum with 1% damping (SAT(1%)) which is multiplied SAT(5%) by the factor depending on frequency is compared with the SEM Acceleration Response Spectrum with 1% damping (SA(1%)). Generally, SA(1%) does not always coincide with SAT(1%). SA(1%) is also a higher level than SAT(1%). This paper describes a new approach to coincide SAT(1%) and SA(1%) and also SAT(5%) and SA(5%) in any natural period domain. The procedure is a compensation of elementally sinusoidal wave phases. SA(5%) agrees with SAT(5%) in convergence calculations. After that the relative inclination of SA(1%) against SAT(1%) and raising or lowering of the response level in frequency domain are possible to control by supplying certain phase characteristics. Its phase characteristics are presented a mixed phase of algebraic function and random numbers a set of $\pi$ through $\pi$. The above SA(1%) adjustments are carried out by the control of the coefficients in an algebraic function in each phase.

2. OUTLINE OF THE SEM GENERATING METHOD
SEM acceleration time history $f(t)$ is generated with superposition by eq.(1).
\begin{equation}
    f(t) = \sum_{i=1}^{N} A_i \cos(\omega_i t + \phi_i) \tag{1}
\end{equation}

Where, $N$ is a number using a Fourier integral or inverse Fourier integral, $\omega_i$ is Fourier frequency, $I(t)$ is a envelop function simulating the earthquake motion in SEM. Amplitude $A_i$ is determined by several convergence calculations compatible to SAT(5%). Phase $\phi_i$ is usually proposed in random numbers. Here the mixed algebraic and random number phase is denoted by eq. (2).

\begin{equation}
    \phi_i = \alpha f_i + \delta e^{-k f_i} - d + \omega e^{-z f_i} (-\pi, \pi) \tag{2}
\end{equation}

\begin{align*}
    f_i & : \omega_i / (2\pi) \\
    (-\pi, \pi) & : \text{uniform random numbers from } -\pi \text{ through } \pi
\end{align*}

According to eq. (2), when coefficients $\omega, z$ are 0, $\phi_i$ is only algebraic function, when $\alpha, d$ are 0, $\phi_i$ is only uniform random number. Compatibility compensation between SAT(1%) and SA(1%) is performed in algebraic function part of $\phi_i$. Coefficient $\alpha$ is concerned only with parallel shift, not on the response spectrum level, and we do not use it as an examination parameter.

3. **SA(1%) TRENDS BY NUMERICAL CALCULATION**

3.1 **SA(1%) LEVEL AGAINST CHANGE OF COEFFICIENT $d$**

Dotted lines in Fig. 1 are SAT(1%) and SAT(5%). Real lines SA(1%) and SA(5%). SA(5%) is shown as a convergence calculation as target SAT(5%). As a result, the curve on SA(5%) of SEM overlaps on SAT(5%). When SA(1%) is compared with SAT(1%) in Fig. 1, both fit in the wide period ranges, especially in less than 0.1 sec. period. In this case, the coefficients in eq. (2) are $d=170, \kappa=0.26, \omega=0.26, z=0$, and SAT(5%) sets up in earthquake 7.0 magnitude with 20Km epicentral distance. We examined the suitability of SAT(1%) and SA(1%) which $d$ and $\kappa$ in Fig.1 are standardized in the following discussion. The coefficient $d$ in Fig. 2 set up by replacing 170 by 500, but coefficients $\alpha, \kappa, \omega, z$ are the same as in fig.1. From Fig.2, we recognize that SA(1%) is larger than SAT(1%) in all periods, while SA(5%) and SAT(5%) are the same values. Fig.3 are the curves of decreasing the $d$ value to 50. In this case, SA(1%) shows a lower level than SAT(1%) in all periods. Subsequently the SA(1%) level increases in keeping with $d$, and decreases with $d$.

3.2 **SA(1%) INCLINATION BY CHANGING COEFFICIENT $\kappa$**

We examined the characteristic trends of SA(1%) by adjustment of only coefficient $\kappa$ as a coefficient standardization in Fig.1. Fig.4 shows the curves of SA(5%) and SA(1%) when changing from $\kappa=0.26$ to 0.5. According to this coefficient, the SA(1%) short period domain is lower than SAT(1%), but the long period domains in SA(1%) are higher than SAT(1%), namely, replacing coefficient $k$ carries out inclination compensation for SA(1%) against SAT(1%). Fig.5 are the reversed smaller $\kappa$ curves. This trend shows the reverse characteristics into Fig.4.
namely, the short period domain in SA(1%) is larger than SAT(1%), whereas the long periods domain in SA(1%) is less than SAT(1%). Coefficient $k$ is multiplied by frequency $(f_1)$ in eq.(2). $\Phi_1$ frequency control is performed by coefficient $\kappa$. Exponential term in $\Phi_1$ rapidly decreases, proportionally with $f_1$ and $\kappa$, and phase characteristics become nearly pulse. The SA(1%) shape can be estimated from this phase trend. From these numerical results, replacing only $d$ corresponds to the raising or lowering of the SA(1%) level whereas only $\kappa$ changing corresponds to the inclination of SA(1%) against SAT(1%). Time history, giving the coefficients such that $\alpha=2\pi, d=170, k=0.26, w=0, z=0$ is shown in Fig.6. The shape of this time history differs from the envelop form, compared with natural earthquake records, for the sake of giving to be alike envelop for natural earthquake, $w$ and $z$ are adopted non-zero values in $\Phi_1$.

3.3 SA(1%) DUE TO CHANGE OF COEFFICIENT $w$

We now established a new standard SA(1%) where in $d=170, k=0.26, w=500, z=1.0$ for the following discussion. SA(1%) and SA(5%) in Fig.7 are on above conditions. SA(1%) in Fig.7 have a large fractuation in the long period region, but the short period region of less than 1.5 sec. is a smooth curve, which is the response spectrum due to be algebraic function phase. Time history, using this phase characteristics, is shown in Fig.12, and denotes the envelop like a time history, using random phase. Fig.8 shows that coefficient $w$ in Fig.7 replaced from 500 to 100. From this, fractuation region of SA(1%) shift from short periods toward long. Fig.9 show that the coefficient $w$ inverses, replacing from 500 to 3000. The trend of the curve in Fig.9 is the inverse of the characteristics of Fig.8.

3.4 CHANGE OF SA(1%) DUE TO CHANGE OF $z$

Coefficient $z$ replaced from 1 to 2. Fractuation region in Fig.10 moves toward long periods, compared to Fig.7 and is a rapid jump that the smoothed region in the short periods transfers from fractuation region in the long periods. When $z$ decreases so that 0.5 in Fig.11. SA(1%) is gradually shift from the random phase type response (fractuation region) to the algebraic phase type response (smoothed region).

4 CONCLUSION

We recognized that the height and inclination of the response spectrum with lower damping is controlled by giving the SEM Fourier phase. We proposed the algebraic function phase in eq.(2). Its function was shown mainly by the exponential function of coefficient $d$ which is multiplied on an exponential term to control the entire SA(1%) level and the power of exponential term $k$ governs the SA(1%) inclination. Therefore, SEM produced by this method can fit any SA(1%) shape, using the suitable $d,k$. Giving an envelope like a natural earthquake record, the random number elements are added in the long period regions. When using any coefficients $d,k,w,z$, we give the SEM the fitness to SA(1%) and can simulate a likeness of natural earthquake record in time history.
Fig. 1 Response spectra standardized only exponential function $p_{phase}$.

Fig. 2 Coefficient $d$ ($d=500$) larger than in Fig. 1.

Fig. 3 Coefficient $d$ ($d=50$) smaller than in Fig. 1.
Fig. 4 Coefficient $\zeta$ ($\zeta>0.5$) larger than in Fig. 1.

Fig. 5 Coefficient $\zeta$ ($\zeta<0.1$) smaller than in Fig. 1.

Fig. 6 Acceleration and Velocity time history having only exponential function phase.
Fig. 7 Response spectra generated by mixed phase of algebraic and random number. (d=170, $\zeta=0.26, w=500, z=1.0$)

Fig. 8 SA(1%) and SA(5%) due to $w=100$

Fig. 9 SA(1%) and SA(5%) increasing $w$ from 500 to 3000.
Fig. 10 SA(1%) and SA(5%) increasing $z$ to 2.0.

Fig. 11 SA(1%) and SA(5%) decreasing $z$ to 0.5.

Fig. 12 Acceleration and Velocity time history having mixed phase.