A Simple Accelerogram Correction Method to Prevent Unrealistic Displacement Shift

L. Borsoi, A. Ricard
Framatome, Département EE/C, Tour Fiat, F-92084 Paris-la-Défense Cedex 16, France

ABSTRACT

The double integration of discretized accelerograms gives generally unrealistic velocity and displacement shifts - i.e. the displacement continually increases with time -. This can be for different reasons : deterioration of recorded signals due to data acquisition system, truncation of the initial part of the signal, mathematical method for synthesized seismic signals,...
This paper presents a very simple correction method based on Lagrange multipliers to prevent these unrealistic shifts. The initial accelerogram is changed into a corrected accelerogram, which is very close to the first one, but satisfies some physical constraints such as the final velocity and/or the final displacement and/or the displacement average equal to zero. The mathematical formulation is reduced to a minimization of a quadratic form (the distance between the 2 accelerograms) with linear constraints.
After explanation of the method, the paper shows and discusses some applications such as the correction of synthesized seismic accelerograms.

1. INTRODUCTION

According a mathematical viewpoint, the integration of discretized accelerograms (synthesized or recorded) does not set any special problem using classical assumptions such as the linearity of acceleration between two data points.
Unfortunately, velocity and displacement shifts - i.e. the displacement continually increases with time - are generally found in the double integration. This is for different reasons : deterioration of recorded signals due to data acquisition system, approximate knowledge of mechanical filter characteristics for seismograph, truncation of the initial part of the signal, digitization process, mathematical formulation for synthesized seismic signals, etc. This phenomenom, although obviously unrealistic, has often no practical influence and thus does not receive any special attention. For example, in a lot of seismic analyses, time history base accelerations are only used for computing equivalent inertia loads which are applied on the structure. The fact that the structure anchors may shift in the space, according an unrealistic way, does not matter, since this has no consequences on the focused results : relative displacements and forces developped in the structure.
Netherveless, for some applications, velocity and displacement shifts must absolutely be

prevented. It is the case of seismic analyses of multi-excited structures which are calculated from absolute displacements. It is clear that two anchors, whose motions differently shift - for example they are computed from uncorrelated accelerograms - can artificially and drastically load the structure.

In other respects, accelerogram data for shaking tables have to give displacements which fluctuate around zero due to the limitation of hydraulic cylinders paths.

It should be also noted that it is a crucial point for seismologists to obtain realistic displacements from recorded seismic accelerograms, especially to define the low frequency part of design ground response spectra.

A lot of methods exist to prevent velocity and displacement shifts. The less sophisticated is only to modify the initial velocity and displacement. But this solution is not always satisfactory because, on the one hand the found initial conditions may be completely unrealistic - without any relation with the actual ones which are due to the truncated initial part of the signal , on the other hand it can be impossible to consider non zero initial conditions.

The greatest part of correction methods are base line corrections. The general shape of the shift (the base line) may be determined (through a regression technic, for example) and remove from the initial signal. This kind of correction can also be performed in frequency domain : the low frequency responses, which are assumed to create shifts, are cancelled.

This paper presents an other simple approach based on Lagrange multipliers.

The initial accelerogram is changed into a corrected accelerogram which is very close to it but satisfies some "physical" constraints such as the zero final velocity and/or displacement. These constraints may be expressed as linear fonctions of the discretized accelerograms. Consequently, the mathematical formulation of the problem is simplely reduced to a classical minimization of a quadratic form (the Euclidean norme between the uncorrected and corrected accelerograms) with linear constraints.

The major advantages of the method are its high simplicity to obtain the Lagrange multipliers and its general applicability.

After explanations about this method (§2), corrections of synthesized seismic accelerograms are shown and discussed in the paper (§3).

2. MATHEMATICAL BACKGROUND

Let (a_i^*) i=1,N the N values of the initial discretized accelerogram defined with the constant time step h ; the value a_i^* corresponds to the time t=ixh ; the initial condition at time t=0. is supposed to be zero $(a_o^* = 0.)$.

Using the classical assumption of acceleration linearity between two data points,

the velocity (v_i^*) i=1,N (initial condition $v_o^* = 0.$) and

the displacement (d_i^*) i=1,N (initial condition $d_o^* = 0.$)

can easily be determined.

As mentionned in the introduction, shifts are generally found in the integration. Therefore the final velocity v_n, the final displacement d_n, like the displacement average e_n $(e = \sum_{i=1,N} d_i)$, are not zero and may have large values.

If t_n corresponds to the end of the excitation, it seems "physical" that the final velocity v_n should be zero or near zero : at the end of the earthquake the ground does not move any longer and/or the velocity of structures oscillates around zero with small values.

By the same way, it can be assumed that the final displacement d_n should be zero. This condition, less obvious than the previous one, supposes that residual displacements do not occur during earthquake. More, if the final displacement is zero, it seems logical to assume that the displacement average is also zero. In other words, it appears "realistic" that both integrated velocity and displacement fluctuate symetrically around zero (the zero final displacement induces the zero velocity average).

Therefore the problem is to construct a modified accelerogram (a_i) i=1,N which is "as close as possible" to a_i^*, but satisfies the previously described "physical" constraints :

$$v_n = 0. \tag{1-a}$$

$$d_n = 0. \tag{1-b}$$

$$e_n = 0. \tag{1-c}$$

The proximity condition between a_i^* and a_i may be mathematically expressed by minimizing the Euclidean distance D between the two accelerograms :

$$D = \sum_{i=1}^{N} (a_i - a_i^*)^2 \tag{2}$$

On the other hand, it can be easily shown (see appendix) that the constraints (1) are linear combinations of a_i accelerations :

$$v_n = \sum_{i=1}^{N} \alpha_i \, a_i \tag{3-a}$$

$$d_n = \sum \beta_i \, a_i \tag{3-b}$$

$$e_n = \sum \gamma_i \, a_i \tag{3-c}$$

The problem is thus reduced to a classical problem of minimization of a quadratic form with linear constraints :

$$\min_{a_i} D = \sum_{i=1}^{N} (a_i - a_i^*)^2 \tag{4}$$

with

$$\sum \alpha_i a_i = 0. \tag{5-a}$$

$$\sum \beta_i a_i = 0. \tag{5-b}$$

$$\sum \gamma_i a_i = 0. \tag{5-c}$$

This is equivalent to minimize the fonctionnal F :

$$\min . \ F = \frac{1}{2} D + \lambda_1 \sum \alpha_i a_i + \lambda_2 \sum \beta_i a_i + \lambda_3 \sum \gamma_i a_i \tag{6}$$

where λ_1, λ_2, λ_3 are Lagrange multipliers.

To solve (6) it can be written :

$$\frac{\partial F}{\partial a_i} = 0. \qquad i=1,N \tag{7}$$

$$\frac{\partial F}{\partial \lambda_i} = 0. \qquad i=1,2,3 \tag{8}$$

The relation (7) gives :

$$a_i = a_i^* - \lambda_1 \alpha_i - \lambda_2 \beta_i - \lambda_3 \gamma_i \qquad (9)$$

whereas the relation (8) gives back the imposed constraints (5).
By introducing (9) in the relations (5), the following linear equation system is obtained :

$$\begin{aligned}
\Sigma \alpha_i a_i^* - \lambda_1 \; \Sigma \alpha_i^2 - \lambda_2 \Sigma \alpha_i \beta_i - \lambda_3 \Sigma \alpha_i \gamma_i &= 0. \\
\Sigma \beta_i a_i^* - \lambda_1 \; \Sigma \alpha_i \beta_i - \lambda_2 \Sigma \beta_i^2 - \lambda_3 \Sigma \beta_i \gamma_i &= 0. \qquad (10) \\
\Sigma \gamma_i a_i^* - \lambda_1 \; \Sigma \alpha_i \gamma_i - \lambda_2 \Sigma \beta_i \gamma_i - \lambda_3 \Sigma \gamma_i^2 &= 0.
\end{aligned}$$

Once determined the Lagrange multipliers by solving this system, the modified accelerogram (a_i) is generated from the relation (9).
The mentionned constraints (1) are obviously not the only possible ones.
They may be extended or reduced (for example, the final displacement may be fixed to a known value). The number of constraints provides the number of Lagrange multipliers and thus the size of the system (10) which must be solved.

3. APPLICATIONS

Two correction examples are shown in this paragraph. Both are related to synthesized seismic accelerograms consistent with NRC response spectrum (RG 1.60).
The first accelerogram, whose duration is 20. seconds and the definition time step 10. ms, is deduced from an actual earthquake recorded at Long Beach. The recorded signal has been modified in order to correspond to the NRC spectrum. Thus the resulting accelerogram (fig.1), although synthesized, is not purely artificial due to its origin.
On the figure 1 are plotted this accelerogram and the corrected accelerogram which respects the three constraints previously described (§2).
No distinction between the 2 accelerograms can be made on this figure : the correction which is applied on each acceleration data point is very slight.
Consequently it is clear that the 2 accelerograms will give the same response spectra or power spectral densities.
The figures 2 and 3 show respectively the velocities and the displacements obtained after integration of the uncorrected and corrected accelerograms.
The difference between the velocities are relatively weak and constant after 10. seconds. As required, the final velocity is well fixed to zero. The discrepancies are more marked for the displacements. The shift is cancelled and the final displacement like the displacement average are zero. Considering the linear shape of the displacement shift, it is clear that a non zero initial velocity condition would also permit to prevent the shift. It must be noted that the applied correction is quite different (see fig. 2).
The second accelerogram which is corrected is purely artificial. It has been generated by superposing a great number of sinus in a trapezoidal enveloppe (with an exponential decay). Its duration is 15. seconds and its time step about 15. ms. As previously, figures 4,5 and 6 present respectively the acceleration, the velocity and the displacement of the uncorrected

and corrected accelerograms (3 constraints : d_n=0., v_n=0., e_n=0.). The corrections appear a little on the beginning of the accelerogram, but can not be seen after. The suppression of the displacement shift is quite obvious but in this example, on the contrary of the first one, the artificial aspect of corrections becomes evident. The modifications on the first second of accelerogram are not very physical, like, on the other hand, the decay of the velocity or displacement to zero at the end of the excitation. Nethertheless it must be present to mind that the initial accelerogram is also completely artificial.

4. COMMENTS

Excluding synthesized seismic accelerogram generation, the problem which is treated can be resumed like that : let μ_i the actual and unknown accelerogram ; even discretized, its integration does not create any shifts if the time step is appropriate ; (let δ_i the obtained displacement). For reasons mentionned in the introduction this accelerogram is approximated by the accelerogram a_i^* which is very close to μ_i but which presents some non physical characteristics (such as the non zero final velocity). The correction method proposed in this paper transforms the accelerogram a_i^* into an other one a_i which is also very close to μ_i and which respects some physical constraints which were previously missing.

But it must be understood that the correction process is only satisfactory according a mathematical viewpoint. In other words there is no special reasons that the corrected accelerogram a_i perfectly coincides with the true accelerogram μ_i. Consequently the corrected displacement d_i although more realistic than the uncorrected displacement d_i^*, may be still relatively far from the actual displacement δ_i.

It should be noticed that the problem to create the accurate accelerogram from the approximate one (without any special knowledge about the deterioration process) is very ambitious and, besides, belongs to the ill posed problem category.

5. CONCLUSIONS

This paper presents a very simple accelerogram correction method which prevents unrealistic velocity and displacement shifts. This method is based on Lagrange multipliers which force the solution to respect some contraints which appear physical. The number of constraints does not matter and the method can easily be extended or reduced.

In some applications, the obtained results are satisfactory and sufficient : the uncorrected and corrected accelerograms are very close and give the same response spectrum or spectral power density, the integration of the corrected accelerogram gives well the imposed constraints like the zero final velocity and/or the zero final displacement and/or the zero displacement average.

The physical meaning of corrections and consequently of integrated results has to be studied and compared with other approaches. This can be done through experiments which involve acceleration and displacement transducers.

The purpose of this paper was only to focus attention on this kind of correction.

APPENDIX

Determination of α_i, β_i, γ_i coefficients

A lot of formulations can be derived to determine α_i, β_i and γ_i coefficients (eq. 3). The following one, which is the less sophisticated, is based on linearity assumptions of acceleration, velocity and displacement. In fact these assumptions are obviously incompatible : if the acceleration is linear, the velocity is parabolic and the displacement cubic. But it can be shown that, if the time step h is small enough, these linearity simplifications do not significantly change the applied corrections (with respect to accurate parabolic and cubic formulations).

According the linearity assumptions, it can be written :

$$v_{i+1} = v_i + \frac{h}{2}(a_i + a_{i+1}) \quad (11\text{-a})$$

$$d_{i+1} = d_i + \frac{h}{2}(v_i + v_{i+1}) \quad (11\text{-b})$$

$$e_{i+1} = e_i + \frac{h}{2}(e_i + e_{i+1}) \quad (11\text{-c})$$

For simplicity the initial conditions are supposed to be zero : $a_o=0.$, $v_o=0.$, $d_o=0.$, $e_o=0.$ Using the equations 11, it is easy to derive the α_i, β_i and γ_i coefficients.

- final velocity v_n

$$v_n = \sum_{i=1}^{N} \alpha_i a_i \quad \text{with} \quad \alpha_i=h \text{ for } i=1, N-1; \quad \alpha_i=h/2 \text{ for } i=N$$

- final displacement d_n

$$d_n = \sum_{i=1}^{N} \beta_i a_i \quad \text{with} \quad \beta_i = h^2 (N-i) \text{ for } i=1, N-1; \quad \beta_i = \frac{h^2}{4} \text{ for } i=N$$

- displacement average e_n

$$e_n = \sum_{i=1}^{N} \gamma_i a_i \quad \text{with} \quad \gamma_i = \beta_i \times \frac{h}{2} + \sum_{j=i}^{N} \beta_j h$$

(The equality $e_n = \sum_{i=1}^{N} \beta_i v_i$ is used).

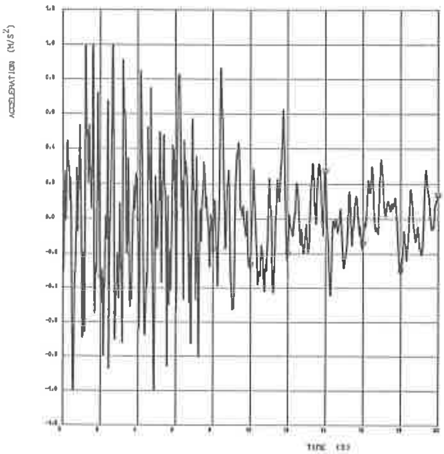

Fig. 1 : Acceleration (Accelerogram 1)

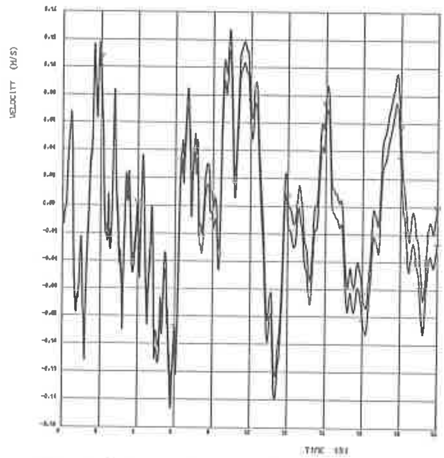

Fig. 2 : Velocity (Accelerogram 1)

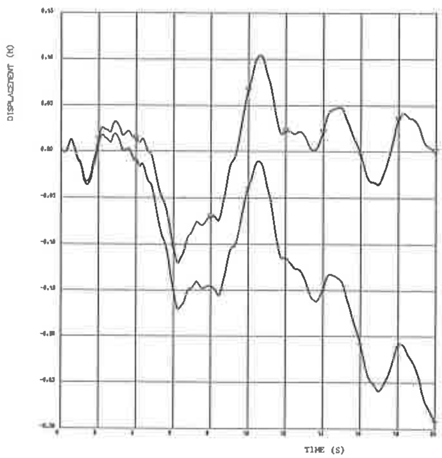

Fig. 3 : Displacement (Accelerogram 1)

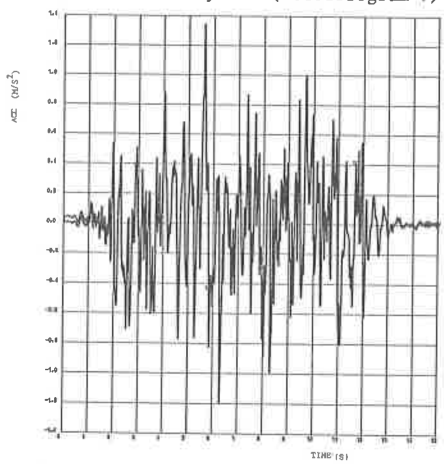

Fig. 4 : Acceleration (Accelerogram 2)

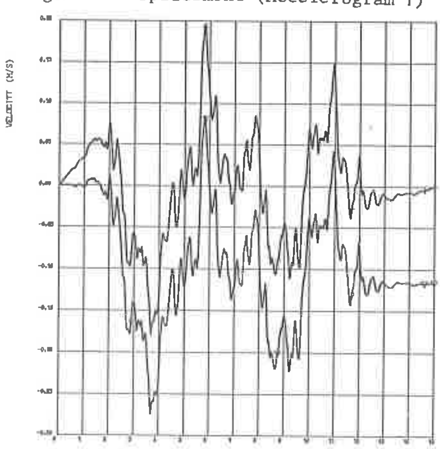

Fig. 5 : Velocity (Accelerogram 2)

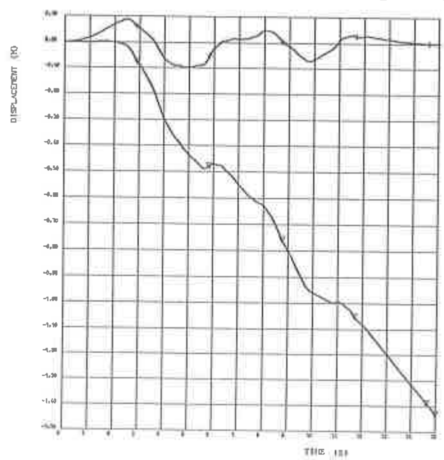

Fig. 6 : Displacement (Accelerogram 2)