A Parametric Study of a Layered Medium Based on the Vibration Tests

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Abstract

The purpose of this paper is to clear the general nature of a complex stiffness in a dynamic soil-foundation interaction analysis based on experimental results. To this end, we try to collect complex stiffnesses depending on frequencies resulted from vibration tests of foundation blocks on an actual ground. Before studying and comparing the complex stiffness, we obtain the dimensionless expressions for all these stiffnesses. As most of these vibration tests have been done on a soil layer, the experimental data should be discussed considering with the effect of a soil layer-foundation interaction. Furthermore, we obtain the theoretical solution of layered media over a half-space based on three-dimensional elastic wave propagating theory, and the relation between the experimental data and the theoretical solution are examined.

1. Introduction

The purpose of this paper is to investigate the general nature of a complex stiffness depending on frequencies resulted from the experimental vibration tests of foundation blocks, which have been collected. In the last decade, the theoretical studies with soil-foundation interaction analyses have been remarkably developed, therefore, it is possible to solve various applicable problems such as arbitrary shape foundations, embedded foundations and the other embedded structures. As theoretical methods are classified to the continuum derived from elastic propagation theory that contains semi-analytical methods and discrete manners which are called F.E.M., F.D.K., lumped mass system and others, herein, we don't describe these in detail. Comparing with such many papers, there have been few papers with the experiments to discuss on the availability of analytical methods. In recent years, the experimental attempts of soil-foundation interactions have been increasing with the theoretical developments to experimentally understand the dynamic behaviors in soil-foundation interactions. It is effective to excite at the top of a rigid foundation on a ground, and for this reason, most experimental tests have been performed in such a manner.

Each experimental test has been performed for the particular objective to clear the dynamic properties of the interactions on the special ground. From this reason, there are various differences in the experimental conditions such as the soil-profiles, the shape of foundations and the exciting systems, which influence test results. After sorting and analyzing many collected complex stiffnesses obtained from the experiments of foundation blocks and synthetically understanding test data, the general dynamic behaviors with
stiffness and damping coefficients in soil-foundation interactions are presented. It will be considered that our following discussions will useful to predict the actual behaviors of soil-foundation interactions, and to establish the dynamic criteria to the earthquake design of the structures having interaction problems. The test data have been limited to such cases of the foundation blocks rested on the ground as the frequency dependency of the complex stiffness.

Since most vibration tests of foundation blocks have been performed on a stratum, we must systematically analyze these data to be clear the effect contributed by a stratum, that can be represented by some parameters. To discuss all the test data in a similar manner, it is necessary to express nondimensional complex stiffnesses. For this reason, such resulted data have generally been presented by dimensional expressions, and we must presently determine in the interaction analysis what the reasonable value of the shear wave velocity to normalize test data in, then the following wave velocity in employed in this paper. At the first, we obtain the stiffness values at the resonance frequency based on the experimental data, and calculate the shear wave velocity from those values using the static theoretical equation of a rigid circular foundation in a half-space interaction analysis. This velocity of the so-called equivalent shear wave velocity has been obtained through the simulation method with try and error, which has been the common manner in the vibration tests of foundation blocks.

Though the above method to obtain such velocity is simple one of the ideas, it can be expected to have the availability for analyzing test data without complexity. The objective data are the swaying and rocking complex stiffnesses on a horizontal exciting and the vertical complex stiffnesses on the vertical exciting. In order to compare the test data with the theoretical solution based on the three-dimensional wave propagating theory, the appropriate parameters should be determined. To this end, we define such two parameters that relate to the effect of soil-foundation interactions on a stratum. These two parameters are geometry \((Z_l/\sqrt{\mathcal{A}})\) \(Z_l\): the first layer depth on a ground, \(\sqrt{\mathcal{A}}\): the length of a foundation block,) and material properties being impedance ratio \((\sigma_1V_s1/\sigma_2V_s2)\), \(\sigma_1\): the first layer density, \(\sigma_2\); the second layer density, \(V_s1\): the measured shear wave velocity of the first layer, \(V_s2\) the measured shear wave velocity of the second layer). Through these parameters, the understanding about the relation between the test data and the theoretical solutions, and the effect of a stratum are shown in the following.

2. Outline of the collected test results and the theoretical solutions

The number of collected reports, which have clearly drawn the complex stiffnesses depending on frequencies resulted from such vibration tests, is 11. For one report, we have calculated the complex stiffness based on its response curve and phase lag. The item in total test data is summarized in Table (1). We have limited to such experimental test cases as the given conditions in the reports are the foundations having no embedments, and following items to be specified: the shape and size of foundations, the soil-profile, the resonance curve and phase lag of the test results, and the exciting force amplitude.

As shown in Table (1), the test data in the horizontal excitation case have been mostly resulted from the experiments of a stratum, while in the vertical excitation case, the ratio of a stratum to a half-space is about 0.5. The classifications are shown in Fig. 1 according to each parameters of the given experimental conditions, but one of them (case (e)) is different from the others because \(V_s\) in case (e) being the resulted value of the ex-
experiments. The parameters to be used in the theoretical solutions with soil-foundation interaction should be determined by taking into account each of test conditions shown in Fig. 1. Though an actual soil-profile is often a multi-layered structure, considering diagram in Fig. 1, the layered model for the theoretical analysis to enable to represent collected test data is two layered media, which is a uniform stratum over a half-space. In addition to such stratum solutions, the half-space solution also is obtained because of a significant fundamental model. Theoretical solutions derived from wave-propagating theory, herein, neglect the internal damping. After obtaining complex stiffnesses in the solution, it should be normalized in the similar manner to the case of the test data in order to obtain the dimensionless expression, and the comparison of the test data with the solution is studied. The parametric analyses for the solutions are summarized in Table (II). However, for that comparing, we should take care of such different points as multi-layered media, the variations of the impedance ratio, Poisson's ratio and the gravity in the experiment ground. We mainly investigate the effect of the stratum of the test data based on the above two parameters for the geometry and material properties. Comparing the dynamic properties of a stratum with those of a half-space, it is well known that the response of the stratum foundation interactions has the characteristic nature which is contributed by two kinds of surface wave to be Rayleigh and Love waves.

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Such special behaviors yields in the imaginary part of a complex stiffness more evidently than the real part of it. From this reason, the tendency of the frequency dependencies of damping coefficients is considerably worth noting. In other words, the effect of a stratum strongly acts on damping coefficients, because the nature of energy dissipations is essentially different from a half-space. This special feature is caused by the reflection waves on the boundary conditions. As mentioned before, those three cases, swaying, rocking and vertical of test data have been collected, and our investigations to these data are described concentrating on the swaying case.

The equivalent shear wave velocity which already has been explained is derived from the following equations.

\[ KH = \frac{2\pi Ga}{2-\nu} \quad KR = \frac{\pi Ga^3}{2(1-\nu)} \quad KV = \frac{\pi Ga}{1-\nu} \quad \text{eq. (1)} \]

KH, KR, KV: Swaying, rocking and vertical stiffnesses, \( \nu \): Poisson's Ratio, \( G \): Shear Modulus, \( a \): The equivalent radius of a circular foundation

Considering both points which are the systematical manner and the large size of foundations, herein, the equation (1) is employed. Equation (1) is assumed to be uniform of the stress distribution on the contact surface. Obtained \( \bar{V}_s \) by using the equation (1) based on test results is reflecting on the various given conditions in the experiments. Therefore, \( \bar{V}_s \) is regarded as a kind of an average shear wave velocity in the stratum. Each value of \( \bar{V}_s \) for the test results is drawn in Table (II) and Fig.1. Though the evaluation methods with \( \bar{V}_s \) is very significant, there have been little discussions. In addition to \( \bar{V}_s \), the measured shear wave velocity of the first layer, \( V_{sl} \) is employed as the supporting value to study test data by the dimensionless expressions.

3. Studies on the experimental data and the theoretical solutions

3.1 The stiffness and damping coefficients with experimental data

After the collecting complex stiffnesses to be normalized by using \( \bar{V}_s \), a stiffness which means a real part of displacement functions to be a reciprocal of a stiffness is shown in Fig. 2-1 - Fig. 2-3. The expression of displacement functions is more appropriate than that of a stiffness to understand the test data synthetically, for the distribution of these is a narrow region. In the physical meaning, the real part of a displacement function corresponds to a stiffness, thus, they are defined as a stiffness in this paper because of simplification. With regard to the expression by using \( V_{sl} \), the stiffness of the swaying is drawn in Fig. 2-1'. The damping coefficients of the experiments, which are obtained from both imaginary and real parts of a complex stiffness are drawn in Fig. 3-1 - Fig. 3-3. The similar expressions by using \( V_{sl} \) are drawn in Fig. 3-1'. As each of theoretical solutions is drawn in Fig. 2-1 - Fig. 3-3 accompanying with the test data, the investigations of the experimental cases will be described at first.

(Stiffness) The tendency of the stiffness is different on each of swaying, rocking and vertical cases. In the swaying case, the general tendency of the dimensionless frequency \( (a_0 = \omega \bar{A}/\bar{V}_s) \) on the stiffness is evident, the positions of two data, however, are far from the others. The rocking stiffnesses of the test data except one datum have similar tendency to the swaying case. The number of the vertical test data in a half-space and a stratum is about equal while the greater part of the test data in the swaying and rocking cases are strata. Therefore, the tendency of the vertical case is mixture of a half-space and a stratum.
(Damping) The damping coefficients of the swaying test data are distributed within the narrow region, and show the qualitative tendency that increases with increasing the dimensionless frequencies. As for the rocking data, the general nature is considered to be the similar tendency to the swaying data, but there are some differences in each rocking data. The tendency of the quality of the vertical test data is not clear because of the different datum composition from the swaying and rocking data.

The tendency of damping coefficient in a half-space is essentially different from that of a stratum.

3.2 The relation between the experimental results and the theoretical solutions

As the solutions drawn in Fig.2-1 - Fig.3-1, its understanding is briefly related on occasion. Firstly, the different expression between \( \bar{V}_s \) and \( V_{sl} \) is considered. The value of \( V_{sl} \) is not contributed by any variation of the conditions to be included in the experiments. The test data of the dimensionless expressions by using \( V_{sl} \) in Fig.2-1' and Fig.3-1' are distributed more widely than that of \( \bar{V}_s \) in Fig.2-1 and Fig.3-1, for the nondimensional method of \( V_{sl} \) are reflected clearly by the differences of variations of the experimental conditions. On the other hand, such systematic method using \( \bar{V}_s \) has the effect to cancel the above differences. Through the contribution of using \( \bar{V}_s \), the test data are distributed within a narrow range and have the clear property of fundamental behaviors in the interaction problems. The dimensionless expressions for the solution are obtained in the similar manner to the test data. The value of \( V_{sl} \) has been defined, that is, 100m/sec. When the parameters become small, it will be more reasonable to use \( V_{s2} \) than \( V_{sl} \). Considering such condition, the dimensionless expressions by \( V_{s2} \) are also illustrated in Fig.2-1' and Fig.3-1'.

In the dimensionless expressions by using \( \bar{V}_s \), the case of the small parameter value such as 0.1 (21/\( \bar{A} \)) is not shown as the first resonance frequency isn't found within the common frequency range. Though the resonance peaks in this solution are caused only by a stratum, the dimensionless expression by \( \bar{V}_s \) can play an appropriate role as an instruction in the understanding of the corresponding test data. As for a half-space ground, these is no resonance peak, so that we obtain only expressions by \( V_{sl} \). The investigations of the solutions in Fig.2-1 - Fig.3-3 are as follows. The effect of the stratum which has the tendency depending on frequencies, is more clear in the low frequency range than the high frequency range, and on the small values of the geometry parameter, such a tendency is remarkable. The difference between a half-space and a stratum is large in the damping coefficients. We study how the effect of a stratum is in the test data.

(Stiffness) The tendencies of the test data in the swaying and rocking cases (Fig.2-1, Fig. 2-2) agree with that of the solution. As the parameter 0.5 of the solution evidently shows the effect of the stratum, some of the test data closely represent such natures, with regard to two data to be far from the others in the swaying case, if the solution is derived to the range of higher frequencies, these data seem to show the similar tendency to the solution. The vertical test data qualitatively agree with the solutions, however, the half data of these don't quantitatively agree with the solutions. The tendency of the other half data which show the effect of the stratum in the solutions is appreciable. As seen in Fig.2-1', and Fig. 3-1', about the half of test data by using \( V_{sl} \) or \( V_{s2} \) are distributed in the region indicated by two kinds of solutions, and the parameter of which are 0.5 (1) - 2.0(1). The form of (1) means the expression by using \( V_{sl} \). The
other test data are distributed in the both sides of those solutions, and the upper and lower limits of the existence of those data are respectively 0.1 (1) and 0.1 (2). The dimensionless expressions of the test data using the measured velocities are distributed in the wide range because of the variations of the experimental conditions. Upon these points, it may be concluded that the test data by using \( U \) correspond to the solutions in the similar expressions. We are unable to grasp the evident relation between the vertical test data and the solutions in the Fig. 2-3 and Fig. 3-3, which is caused by the datum composition.

(Damping) The test data except two data in the swaying case of Fig. 3-1 are distributed in such a region as the solutions indicate and the good agreement between the test data and solutions can be seen. Apparently, the test data represent characteristic behavior resulted from the solutions of a stratum. As for the rocking case, the relation of both is similar to the swaying case, but the value in the greater part of test data is larger than the solutions. In general, it is more difficult to obtain such agreements for the rocking case than the swaying case because the damping coefficients of the stratum are considerably small in the lower range of the frequency. For the reason of this quantitative disagreement, it will be caused by the contributions of viscous damping. Taking into account this condition, the difference can be acceptable. Through these discussions, the effect of the stratum is clear. In the following sections, furthermore, we study in detail such effects of damping coefficients under the condition which limits test data to only a stratum case.

1.3 Discussions on the effects of a stratum concerning with damping coefficients

The test data limited to a stratum case are drawn in Fig. 4-1 - Fig. 4-3 together with the corresponding theoretical solutions. Since the swaying test data are precisely distributed within the region to be indicated by the solutions, the agreement of both is considerably well. For the vertical case as shown in Fig. 4-3, that relation is similar to the swaying case. As already pointed out, the tendency of all the vertical test data have not been clear, now limited test data correspond to the theory. As for the rocking data, the limiting test data to the stratum case as shown in Fig. 4-2, the advanced observations about such relation can't be obtained. Therefore, the availability of limited data isn't appreciable and from this reason, it is necessary to consider again another view point such as internal damping. Through the studies, since we have been able to verify the good agreement between the swaying test data and the corresponding solutions, we will introduce the regression equation based on the swaying test data. Two kinds of the regression curve will be calculated, one of which is a cubic equation to directly represent the general properties of the test data and the other of which is a broken line consisted of two lines, that is, bi-linear curve, to represent the special nature of the stratum test data.

The curve of the obtained cubic equation is illustrated in Fig. 5 together with corresponding test data of the stratum case. The cubic equation of the regression is as follows.

\[
h = 0.615a^3 - 3.66a^2 + 21.51a - 9.672 \quad \text{h: damping coefficient, eq. (2)}
\]

Considering the curve and the tendency of the frequency dependency in the solutions, the regressive broken line based on the test data is illustrated in Fig. 6. The cross point of two lines is calculated through try and error manner to make error smallest, and each available range of two lines is as follows.
As shown in the Fig. 6, the regression line closely agrees with the tendency of the strain eq. (3)

\[ a_0 = \sqrt{A} \omega / V_s \]

\[ a_0 = \sqrt{A} \omega / V_s \]

The number in each figure means geometric parameter in the theoretical solution.
solutions within the range $a < 1.5$ increases under the rising gradient different from a half-space solution with increasing the frequencies and within the range $a \geq 1.5$ under the similar gradient to a half-space solution.

Considering such a point as the variation of the geometrical parameter of the test data being wider than the defined solutions herein, the resulted regression curve is appropriate and the nature of the stratum represented by the test data conforms to the solutions. In the past, as there has not been any attempt to present discussion methods for many test data, our manner in this paper is considered to be reasonable.

4. Conclusion

The many complex stiffnesses resulted by the tests of foundation blocks on the ground, which have been collected, have been studied. These complex stiffnesses of the test data remarkably show the effect of a stratum and agree with the theoretical solutions of the stratum cases. It has been clear that such effects act considerably on the dynamic damping properties. The quantitative difference between the test data and the solutions in the rocking case has been remained to be discussed in future. This presented method, herein, which is one of the ideas to understand many such test data, has been available for the variations of the given conditions on the experiments.

5. References