Experimentally Observed Phenomena vis-à-vis Current Constitutive Theories

M. Korzen

Technische Hochschule Darmstadt, Institut für Mechanik III, Hochschulstrasse 1,
D-6100 Darmstadt, Germany

Abstract

An attempt is made to compile under systematic aspects the large variety of available experimental information concerning the material AISI 304 as well as DIN 1.4948. The experiments shall be classified according to the loading histories in four different groups.

Many constitutive equations have been proposed in the literature aiming to predict the phenomena observed in the above experiments. We furthermore present a classification of such theories according to a systematic pattern including a representation of some basic features.

We conclude with some remarks on the equilibrium stress-strain curve.

1. Constitutive equations - Experiments

Constitutive equations are the key ingredient of structural analysis. They describe the specific properties of a material. For the identification of these properties experiments are required. Manifestly, there exists a great number of possible ways of carrying out these experiments. Therefore the question arises, as to which of these experiments are necessary to determine the specific properties of the material at hand as well as how they should be systematically compiled following a systematic pattern.

The material particularly considered is the high-temperature resisting steel AISI 304 or X6 CrNi 1811 (DIN 1.4948). We limit our attention to isothermic uniaxial stress cases with macroscopically homogeneous states of stress and small strain.

2. Loading histories and experimental phenomena

In principle, the aforementioned pattern has been stimulated by ideas of ONAT /1/ and KREMPFL /2/. These authors look upon the material as well as on the model as an operator (Fig. 1). These operators map a given time-dependent input into a correspondent time-dependent output. The response to some particular input is called an experimental phenomenon.

The form of the input function over time and the experimental possibility of stress or strain-control are the basis of the compilation. As a matter of fact, all experimental inputs which have been found in the literature are characterized by piecewise constant loading rates. This implies a natural order of the compilation. All experimental informations can be devided into four different groups (Fig. 2).
3. Compilation of experiments: Monotonic loadings without holdtime

**STRAIN-CONTROL**

Fundamental shape of stress-strain curve at room temperature (KREML /4, Fig. 2/):
- Linear region with strong slope (Region I).
- Non-linear transition region (Region II).
- Linear region with weak slope (Region III).

**Influence of strain-rate on shape of stress-strain curve:**
- No influence on fundamental shape ([KREML /4, Fig. 2/, CONWAY et al. /5, Tables 7.1-7.7/]).
- Positive strain-rate sensitivity at room temperature ([KREML /4, Fig. 2/]).
- Non-monotonic sensitivity at elevated temperatures ([CONWAY et al. /5, Tables 7.1-7.7/]).

**Influence of positive jumps of strain-rate on shape of stress-strain curve in region III at room temperature ([KREML /4, Fig. 2/]):**
- Strong slope directly after jump.

**Influence of temperature on shape of stress-strain curve:**
- No influence on fundamental shape ([GREENSTREET /6, Fig. 1/]).
- Non-monotonic temperature sensitivity ([SWINDEMAN /7, Appendix A/]).

**STRESS-CONTROL**

Fundamental shape at room temperature ([KUJAWSKI et al. /8, Fig. 1/]):
- Three regions (as in strain-control).
- More gradual and larger transition region (region II).

**Influence of jumps of stress-rate on shape of stress-strain curve at room temperature:**
- Negative jumps: No sensitivity ([KUJAWSKI et al. /8, Fig. 1/]).
- Positive jumps: Gradual increase of slope directly after jump ([KUJAWSKI et al. /8, Fig. 1/]).

4. Compilation of experiments: Monotonic loadings with holdtime

**STRAIN-CONTROL** (Fig. 3)

Fundamental shape of basic relaxation curve at room temperature:
- Existence of relaxation (No detailed information about shape available) ([BOOIJ et al. /9, Fig. 15/]).
- No dependence of the relaxed state on the rate of loading ([BOOIJ et al. /9, p. 8/]).

Fundamental shape of basic relaxation curve at 550 °C ([INTERATOM /10, Fig. II.6.1.2-18,19/]):
- Strong decay of stress directly after loading stop.
- Monotonically decreasing slope tending to zero.
- Finite nonzero value of stress for large hold-times.

**Influence of strain level on shape of basic relaxation curve at 550 °C ([INTERATOM /10, Fig. II.6.1.2-18,19/]):**
- No influence on fundamental shape.
- Non-linear increase of initial stress with increasing strain level.
- Constant difference between initial stress and stress after 1000 hours hold-time.

**STRESS-CONTROL** (Fig. 4)

Fundamental shape of basic creep curve at room temperature ([KUJAWSKI et al. /8, Fig. 2,3/):
- Increasing strain during hold-time.
- Decay of strain rate during hold-time.
Influence of stress-rate on shape of basic creep curve at room temperature (KUJAWSKI et al. /8, Fig. 2.3/):
- No influence on fundamental shape.
- Decreasing initial strain with increasing stress-rate. (Positive stress-rate sensitivity)
- Increasing strain-rate at constant hold-time with increasing stress-rate.
- Increasing decay of strain-rate at constant hold-time with increasing stress-rate.

Influence of stress level on shape of basic creep curve at room temperature (KREML/4, Fig. 3/):
- No influence on fundamental shape.
- Existence of creep in region I of stress-strain curve.
- Increasing strain-rate at constant hold-time with increasing stress level.

Influence of stress-rate on shape of basic creep curve at 550 °C (INTERATOM /11, Fig. 11.6.1.3.-29/):
- Influence on fundamental shape with high stress-rates.
- Non-linear increase of initial strain with increasing stress-rate. (Negative sensitivity)

Influence of stress level on shape of basic creep curve at 649 °C (Dead load machine):
- No influence on fundamental shape (GREEN STREET /6, Fig. 9/).
- Increase of strain-rate at constant hold-time with increasing stress level (GREEN STREET /6, Fig. 9/).

Influence of temperature on shape of basic creep curve (Dead load machine) (INTERATOM /11, Fig. 11.6.1.3-7, 8, 9, 10/):
- No influence on fundamental shape.
- Non-linear increase of strain-rate at constant hold-time with increasing temperature.
- Large strains at constant hold-time with increasing temperature.

5. Compilation of experiments - Non-monotonic loadings without hold-time

STRAIN-CONTROL

Fundamental shape of basic cyclic stress-strain curve at room temperature (BODIJ /9, Fig. 16/):
- Hysteresis loops with increasing stress range with increasing number of cycles.
- Bounded stress range after finite number of cycles.

Influence of strain range on shape of basic cyclic stress-strain curve at elevated temperature (CORUM et al. /12, Fig. 2.5/):
- No influence on fundamental shape.
- Increase of bounded stress range as increasing strain range.
- Decrease of number of cycles until reaching bounded stress range.
- If there is a jump in strain range, the bounded stress range is not uniquely determined by the former.

Influence of strain rate on shape of basic cyclic stress-strain curve at elevated temperature (CONNAY /13, p. 8-9/):
- No influence on fundamental shape.
- Increase of saturated stress with increasing strain rate.

Influence of temperature on shape of basic cyclic stress-strain curve at elevated temperature (CORUM et al. /12, Fig. 2.11/):
- No influence on fundamental shape.
- Non-monotonic influence on stress level at the reversal points of the hysteresis loops.
STRESS-CONTROL

- Boundedness of strain range depends on stress range and medium stress (TURNER /14, Fig.3/).

6. Compilation of experiments: Non-monotonic loadings with hold-time

- Experiments of this group are very complex. Therefore examples are presented only.
- At room temperature creep and relaxation is more dominant in regions with smaller slope of stress-strain hysteresis loop (KUJAWSKI et al. /8, Fig. 14,15/).
- In the case of cyclic symmetrical loading with intermittent relaxation periods there is a lower bounded stress range with hold-time than without hold-time at elevated temperature (CONWAY /13, p. 8/).

7. Constitutive theories: Theories of rate-independent type

CLASSIFICATION
- Differential type theories with classical yield condition formalism (MRÓZ /16/).
- Theories with updating procedures (LIU et al. /16/, ELLEUCH et al. /17/).
- Theories with arc-length parametrization (VALANIS /18/).

BASIC FEATURES
- Cyclic hardening or softening behavior possible.
- Rate independent hysteresis loops.
- No creep and relaxation processes are possible. (All states are relaxed.)

8. Constitutive theories: Theories of viscoelastic type

- Engineering creep theory (RABOTNOV /19, § 49/).
- Linear and non-linear theories of viscoelasticity in integral formulation (FINDLEY et al/20/).
- Linear and non-linear theories of viscoelasticity in differential formulation including internal variable formulation (ONAT et al. /21/).

BASIC FEATURES
- Cyclic softening.
- Elastic behavior at static and very fast loadings.
- Unique equilibrium stress-strain curve. (No hysteresis)
- Rate dependent hysteresis loops.
- Boundedness of relaxation processes in any case.
- Boundedness of creep processes depends on the shape of equilibrium stress-strain curve.

9. Constitutive theories: Combined theories

CLASSIFICATION
- Combination of viscoelastic constitutive equations with differential type equations of classical plasticity (group I) (PERZYNA /22/).
- Theories of viscoelastic type using the arc-length of difference between total strain and strain at fast loadings as a variable (group II) (WALKER /23/).

BASIC FEATURES OF THEORIES OF GROUP I:
- Existence of nontrivial equilibrium hysteresis cycles is only possible if the constitutive equation is of so-called 'fluid-type'.
- Hardening is not in any case present at static loadings.
- Hardening depends in most cases on rate of loading.
- Creep and relaxation processes are possible.

BASIC FEATURES OF THEORIES OF GROUP II

- Existence of hardening from zero state.
- Non-linearity of equilibrium stress-strain curve from zero state.
- Equilibrium stress-strain curve is of endochronic type formulated over the history of inelastic strain.
- Creep and relaxation processes are possible.

10. Discussion

   One basic method of analyzing complex constitutive equations consists in the application of
   these for static and very fast loadings, as well as loadings histories with hold-times.
   It should be remarked however, that both static and very fast loadings cannot be technically
   carried out. Nevertheless, loading processes with hold-time following a well defined pre-
   loading history enables us to roughly determine the shape of the equilibrium stress-strain
   curve. The relaxed as well as the retarded states can be approximately determined. The
   characteristic time-scales can hence be estimated for a material subject to a specific
   temperature and loading history. With this in mind, the shape of the relaxation and creep
   curves in connection with the form of the equilibrium stress-strain curve can, in the opinion
   of the author, provide us a key to the description of inelastic material behavior.

11. References

/1/ ONAT, E.T.: Description of mechanical behavior of inelastic solids. Proceedings of the

/2/ KREMPFL, E.: On the interaction of rate and history dependence in structural metals.

/3/ MORROW, J.: Cyclic plastic strain energy and fatigue of metals. ASTM Special Technical

/4/ KREMPFL, E.: An experimental study of room temperature rate sensitivity, creep and
     relaxation of AISI type 304 stainless steel. Journal of the Mechanics and Physics of

/5/ CONWAY, J.B.; STENTZ, R.H.; BERLING, J.T.: Fatigue, tensile and relaxation behavior of
     stainless steels. University of Cincinnati, Cincinnati, Ohio, USA, 1975.

/6/ GREENSTREET, W.L.: Structural analysis technology for high-temperature design.

/7/ SWINDEMAN, R.W.: Representation of high-temperature tensile behavior of reannealed
     type 304 stainless steel by the VOCE equation. Journal of Engineering Materials and

/8/ KUJAWSKI, D.; KALLIANPUR, V.; KREMPFL, E.: An experimental study of uniaxial creep,
     cyclic and relaxation of AISI type 304 stainless steel at room temperature. Journal of

/9/ BODIN, J.; VAN DER WERFF, K.: A Description of plasticity experiments on austenitic
     stainless steel tubes. Delft University of Technology, Delft, Netherlands,
     Report WTHD-No. 77, August 1975.
/10/ INTERATOM: IA-Bericht 55.06265.6, August 1983.
/11/ INTERATOM: IA-Bericht 55.05483.6, August 1982.
**Fig. 1** Constitutive equations and material as operators
(figure on the right hand side after MORROW /3, Fig. 3/)

**Fig. 2** Pattern of loading histories

**Fig. 3** Basic relaxation

**Fig. 4** Basic creep