Boundary Element and Speckle Photography Method for Solving Elastoplastic Problems

L. Hadjikov, V. Kavardjikov, V. Valeva
Bulgarian Academy of Sciences, Institute of Mechanics and Biomechanics, POB 373, 1090 Sofia, Bulgaria

Abstract

The stress-strain state of metal specimens in the vicinity of a stress concentrator (circular hole) is investigated in case of a quasi-static loading. The displacements are evaluated numerucally by the Boundary Element Method (BEM) and they are estimated experimentally by speckle photography. The experimentally and theoretically obtained results are compared and considered. A unified method for a simultaneous employment of both techniques is suggested. The experimental and theoretical techniques complement each other which results in an enhanced capability of the method proposed.

1. Introduction

The effect of residual or viscous non-elastic deformations on the stress and strain states in the vicinity of various stress concentrators (holes, notches, cracks, etc.) is intensively investigated in the recent years. In this connection the Finite Element Method (FEM) [1] and the BEM [2] are widely applied for theoretical predictions. An experimental method for separation of the plastic zones in chromorheologic materials, based on the local changes of the colour of specimen is used in [3]. The holographic and speckle interferometry methods [4,5] are subject of increased attention too.

The above-mentioned numerical and experimental methods could be used together. In some cases this improves the accuracy of the experimental technique [6] while in others the experimental information facilitates the solution of the boundary-value problem [7].

The aim of the present paper is to investigate the stress and strain states in the vicinity of stress concentrators in metal specimens loaded with a force slowly increasing in time. It is achieved by applying both BEM and MSP (Method of Speckle Photography).

2. Experimental method of speckle photography

A number of specimens have been investigated but since their presentation should increase improperly the size of the present short note, only the results concerning circular holes specimens are discussed here (see Fig.1). The specimens are made of steel leaves St-3 type with thickness $\delta=2\pm0.05$mm. The mechanical characteristics of this kind of steel at room temperature 20^0C are quoted in Table I. All experiments are conducted on a standard testing machine ZD-10/90 with constant velocity of stretching 0.5mm/min. The relation stretching force-time is schematically presented in Fig.2a(9).

The displacements registration and the defining of the forms of the nonelastic zones on the specimen surface is accomplished by speckle photography. The principle scheme of an automatic system for sequential specklegrams registration and of its operation in time is given on Fig.2a,b and its detailed description could be find in [8]. The moments of time at which the respective specklegrams are registered, are automatically marked on the loading curve (Fig.2a(9)). The information coded on the specklegrams is decoded in two different ways. The first one is the traditional way, based on the well known formulae [9] :

$$U_1=\frac{\lambda L}{M p_{x_1}} \qquad\qquad U_2=\frac{\lambda L}{M p_{x_2}} \tag{1}$$

defining the displacements U_1 and U_2 in x_1 and x_2 directions, respectively at an arbitrary point on the specklegram. Here L is the distance between the specklegram and the screen, λ - wave-lenght of the laser radiation, M - scale factor, p_{x_1} and p_{x_2} - distances between two neighbouring bright interferometric bands, projected on x_1 and x_2 directions, respectively.

The second method of specklegram decoding is proposed by the authors (described in details in [10]). By this method a two-dimensional picture of the inelastic (plastic) zones on the studied surface could be obtained.

3. BEM for solving two-dimensional elastoplastic problems

3.1 Setting of the problem

A two-dimensional stressed inelastic medium occupying the region Ω (see Fig.3) in cartesian coordinate system $x_1 0 x_2$ is considered. The displacements $U=\bar{U}$ are defined at the portion Γ_1 of the boundary Γ , while at the remainder of Γ the stresses $\tau=\bar{\tau}$ are known. Subject of this problem is the study of the stressed and strained state of the region Ω at slow increase of the stresses $\bar{\tau}$ along the boundary.

3.2 Governing equations

In the present paper the constitutive relation of Maxwell-Gourevich-Rabinovich (see [11]) is employed. In the two-dimensional case it reads:

$$\frac{\partial \varepsilon_{11}^{\circ}}{\partial t}=\frac{f_{11}^{\circ}}{\eta^{\circ}}\,, \qquad \frac{\partial \varepsilon_{22}^{\circ}}{\partial t}=\frac{f_{22}^{\circ}}{\eta^{\circ}}\,, \qquad \frac{\partial \varepsilon_{12}^{\circ}}{\partial t}=\frac{2 f_{12}^{\circ}}{\eta^{\circ}}$$

$$f_{11}^{o} = \sigma_{11} - \frac{1}{2}\sigma_{22}, \quad f_{22}^{o} = \sigma_{22} - \frac{1}{2}\sigma_{11}, \quad 2f_{12}^{o} = 3\sigma_{12},$$
$$f_{zz}^{o} = \frac{1}{2}\left[\left(f_{11}^{o} + f_{22}^{o}\right) \pm \sqrt{\left(f_{11}^{o} - f_{22}^{o}\right)^2 + 4 f_{12}^{o2}}\right] \tag{2}$$
$$\frac{1}{\eta^{o}} = \frac{1}{\eta_{o}^{o}} \exp\left(\frac{1}{m^{o}} / f_{zz}^{o} /_{max}\right)$$

The meaning and the values of the mechanical parameters of eqs. (2) are presented in Table I. By ε^{o} are denoted the residual deformations. Making use the Betti's theorem of reciprocity of mechanical work and eqs. (2), the two-dimensional problem of plane stress state is translated into the following system of boundary integral equations:

$$[C_{ji}]\{u_j\} = \int_{\Gamma}\left([U_i^{*(j)}]\{\tau_i\} - [T_i^{*(j)}]\{u_i\}\right)d\Gamma + \\ + 2G\int_{\Gamma}\left([U_i^{*(j)}]\{\varepsilon_{ik}^{o} n_k\}\right)d\Gamma - 2G\int_{\Omega}\left([U_i^{*(j)}]\{\varepsilon_{ik,k}^{o}\}\right)d\Omega \tag{3}$$

where U^{*} and T^{*} are the fundamental solutions of the elastic problem and n_k (k=1,2) - directory cosines of the outer normal vector to the boundary Γ. The formulae for coefficients C_{ji} are presented in [2]. In a linear approximation of the unknown functions, eqs. (3) are reduced to the following algebraic system:

$$[A]\{u\} = [B]\{\tau\} + \{\Phi^{o}\} \tag{4}$$

and accounting the specific boundary conditions we have:

$$[\bar{A}]\{x\} = \{\Phi^{o}\} \tag{5}$$

The algorithm of solution consists of the following steps. Initially, eqs. (5) together with eqs. (2) are solved for a given time stage and the unknown values of the stresses and displacements at the boundary are obtained. The displacements in the whole region Ω are calculated by means of eqs. (3) in which $[C_{ji}]$ is set equal to the unit matrix. The formulae for calculating the stresses in Ω are given in [2]. Thus a complete time stage is calculated and the same steps are repeated in the next time stage. Elastic solution i.e. $\{\Phi^{o}\} = \{0\}$ is used as an initial condition of the inelastic problem.

The speckle photography and the boundary element methods could be used together according to the block-scheme shown on Fig.4.

4. Results and discussion

The specimen presented on Fig.1 is tested by means of the above described methods. On Fig.5 and Table II are shown displacements at certain characteristic points at mean stress σ_m =62.5 MPa. The theoretical predictions and the experimental results show good coincidence. The plastic zone around a circular hole at σ_m =251 MPa as observed by the method described in [10] is shown on Fig.6a. On Fig.6b a similar result obtained by 3 on the basis of a different technique and of a different material of the specimen is shown. The qualitative agreement is obvious. On Fig.7 the quantitative and qualitative comparison between computed by BEM and measured through MSP for a portion of the plastic zone occurring at σ_m =251 MPa is presented.

The combined usage of both methods has the advantages. By MSP the displacements along the boundary could be measured and then in its turn the size of the matrix in eqs. (5) to be reduced. Thus, the numerical efficiency of the BEM rises significantly. On the other hand the ability of speckle photography method to determine experimentally the exact shape of the inelastic zone, since this information can prove crutial for constructing zone - fitted discretization for the numerical method, gives the opportunity for a considerable reduction of the computational time.

References

/ 1 / ZIENKIEWICZ, O.C., The Finite Element Method in the Technique, Moscow, 1975 (in Russian).

/ 2 / Progress of Boundary Element Methods, Vol. 1, Edited by C.A.Brebbia, London: Pentech Press.

/ 3 / RAUTU, S., LEIBZON, E., An Experimental Determination of the Fields of Strains and Stresses in the Elastoplastic Range, Experimental Mechanics, March, 1983.

/ 4 / Mechanics of Fracture 7, Experimental Evaluation of Stress Concentration and Intensity Factors, Ed. by G.C. Sih, London, 1981.

/ 5 / JONES, R., WYKES, C., Holographic and Speckle Interferometry, Cambridge: Cambridge Univ. Press, 1983.

/ 6 / YASUSHI MITSUI, SHUN-YA YOSHIDA, Boundary Element Method Applied to Photoelastic Analysis, Journal of Engineering Mechanics, Vol. 109, No. 2, April, 1983.

/ 7 / BESKOV, A.N., Combined Investigation of Three-Dimensional Bodies by Potential and Holography Methods, Ph.P. Thesis, Kiev, 1981 (in Russian).

/ 8 / KAVARDJIKOV, V., TONTCHEV, T., KOULEV, P., HADJIKOV, L., Automated System for Registration of Small Displacements on the Basis of Double-Exposure Speckle Photography Method, Tekhnitcheska Misl, Sofia, 1983, in press (in Bulgarian).

/ 9 / KOUDRIN, A.B., POLOUKHIN, P.I., TCHITCHENOV, N.A., Holography and Deformation of Metals, Moscow: Metallurgia, 1982 (in Russian).

/ 10 / HADJIKOV, L., KAVARDJIKOV, V., Experimental Separation of a Plastic Zone by Speckle Photography Method Applied to Metal Specimens with Circular Holes, Fiziko-Khimitcheska Mekhanika, Sofia, 1984, in press (in Bulgarian).

/ 11 / GOUREVICH, G.I., Deformability of Media and Propagation of Seismic Waves, Moscow, 1979 (in Russian).

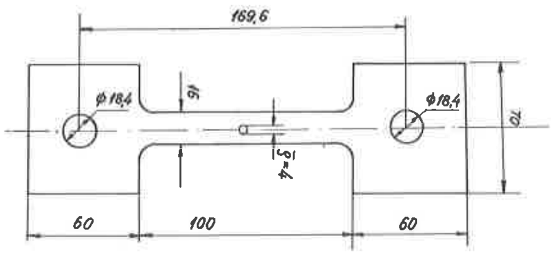

Fig.1 Form and dimensions of the specimen

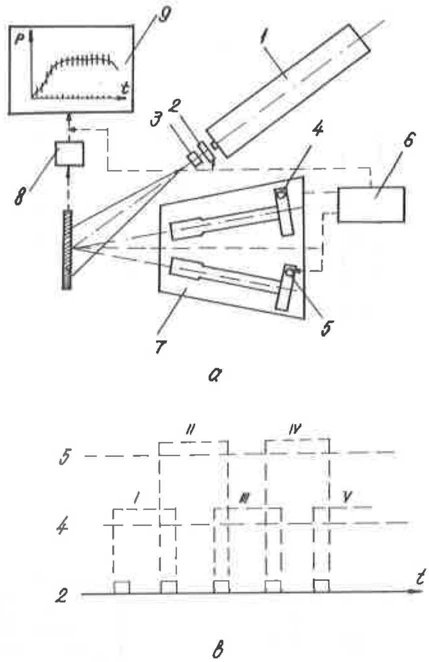

Fig.2 a) A principle scheme of an automatic system for specklegrams registration: 1-laser, 2-optical obturator, 3-beam expander, 4,5-cameras, 6-control block, 7-support, 8-gauge, 9-recorder

b) Operation of the system in time

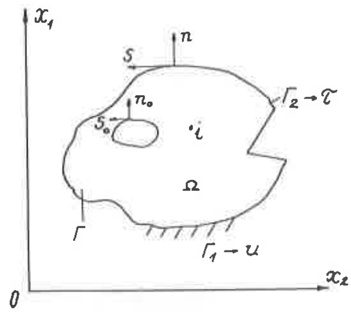

Fig.3

Table I Material parameters

Young's modulus E GPa	Poisson's ratio ν	Initial viscosity coefficient η_c^o GPa.min	Logarithmic modulus of velocity m^o N/mm	Volume coefficient μ^o
210	0,3	6,408.10^7	16,5	0,0

Table II

No	u_1 [μm]		u_2 [μm]	
	exper.	theory	exper.	theory
1	1.40	1.94	0.00	0.00
2	2.60	2.05	0.00	0.00
3	2.45	2.43	0.00	0.00
4	2.37	2.18	-0.18	-0.14
5	1.80	1.84	-0.46	-0.58
6	1.75	1.99	-0.86	-0.75
7	1.39	1.20	-1.04	-0.88
8	0.42	0.56	-1.47	-1.59
9	0.00	0.00	-1.22	-1.13
10	0.00	0.00	-1.00	-0.96
11	0.00	0.00	-0.82	-0.84
12	1.92	1.65	-0.39	-0.33

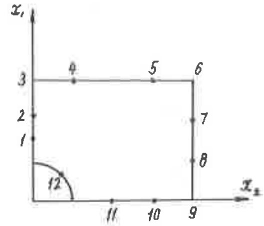

Fig.5

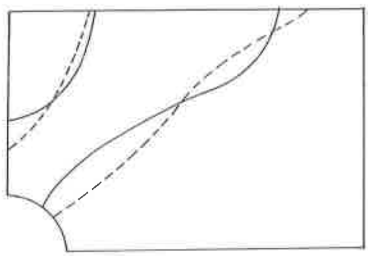

Fig.7 Comparison of the theoretical and experimental solutions: --- experiment, —— theory

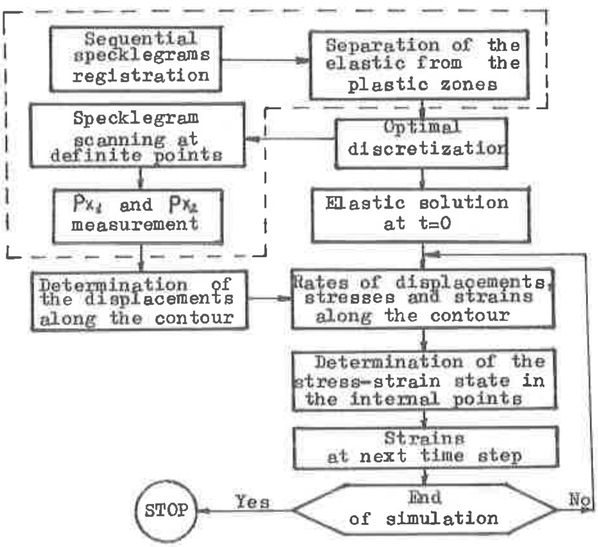

Fig.4 Block-scheme of the boundary element and speckle photography hybrid method

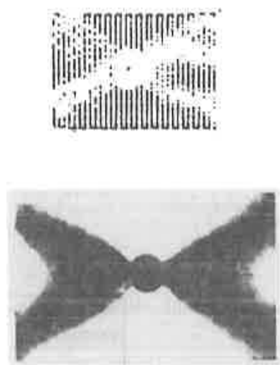

Fig.6 Experimentally obtained elastoplastic zones