Strain-Rate Dependent Material Modeling for Extreme Loading Cases

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Abstract
Impact and impulsive loadings in general lead to high stress and strain rates within the structural element materials involved. Increasing the strain rates in general leads to higher material strength and strains. To take account of these effects in structural design a comprehensive material model based on the COWPER-SYMONDS-BODNER-formulation for a homogeneous viscous rate dependent material has been implemented into the multipurpose computer programme FLOWERS. A comparison of experimental results obtained from tests with two-span continuous beams under sudden removal of a support with computer calculations showed good agreement.

1. Introduction
The design of reinforced concrete structures which are likely to be subjected to impact or impulsive loadings calls for special attention. Impact loads in general are due e.g. to collisions of airplanes, ships, cars or missiles with structures whereas impulsive loadings are initiated by gas explosions, blasting or by a sudden removal of one or several supports in a structural system. Under these loading cases the structures affected are normally not required to respond elastically. In addition the rise time of the load up to its maximum value is very short leading to high stress or strain rates within the structural materials involved. As has been shown in numerous research work during the last few years the mechanical properties of the materials like yield stress, tensile strength, uniform elongation and ultimate strain are rate dependent. Increasing the strain rate in tests with concrete and reinforcing steel specimens in general leads to higher material strength and strain.

Tests with reinforced concrete structural elements at increased loading rates show similar effects: an increase of the ultimate bearing capacity and especially of the ultimate deformation. To achieve a rational and economic design of structures or structural elements for these extreme loading cases the altered mechanical behaviour should be taken into account. As experimental investigations are restricted to some specific and well defined loading
cases and test specimens, complex structures in practice have to be investigated by theoretical analysis. Developments in the finite element method have extended the possibilities for refined analytical studies involving the influences of various parameters such as rate effects upon material properties.

An overview on all these topics and problems in connection with impact and impulsive loadings, with the strain rate dependent material behaviour, the finite element modelling, etc. is given e.g. in AMMANN [1] or DARGEL [2].

2. Material Modelling and Rate Effects

Reinforced concrete is a composite material of concrete and reinforcing steel. An altered behaviour of reinforced concrete structural elements as it is observed in nearly all the tests with an extremely short rise time of the impact or impulsive loading (see e.g. AMMANN [3], [4], BAM [5]) therefore may be based on the altered behaviour of either the reinforcing steel or of the concrete or of both components. Special attention has to be payed for the problem of bond between the two components (see VOS [6]).

For non-prestressed reinforcing steel the strength and strain are increased at elevated strain rates. During impact and impulsive loadings strain rates up to about \( \dot{\varepsilon} = 5 - 10 \text{ s}^{-1} \) may occur. This value is nearly two magnitudes higher than it is during earthquake loadings. Figure 1 shows the influence of an elevated strain rate on the ultimate strength of hot-rolled reinforcing steel. The scatter of the test results is rather large. Nevertheless, linear regression lines in linear/logarithmic plots are commonly used (see e.g. BAM [5]). This regression line takes the form of

\[
\frac{f_{\text{dyn}}}{f_{\text{stat}}} = 1 + c \cdot \log \frac{\dot{\varepsilon}}{\dot{\varepsilon}_{\text{stat}}}
\]

with

\[
\begin{align*}
\frac{f_{\text{dyn}}}{f_{\text{stat}}} &= \text{dynamic strength} \\
\dot{\varepsilon} &= \text{strain rate}
\end{align*}
\]

Similar relations have been found for the strain values. Corresponding values for the constant \( c \) can be found in AMMANN [7] or [5].

A comparison of test-results in AMMANN [7] shows that for a range of strain rates from \( \dot{\varepsilon} = 10^{-5} \) up to \( 5 \text{ s}^{-1} \) the strength values are increased for about 10 to 15% whereas the values for the ultimate elongation or the ultimate strain differs much more. An increase of up to 200% for the ultimate strain of cold-worked reinforcing steel is possible (see e.g. LIMBERGER in [5]).
For the implementation of this strain rate dependent material behaviour into a computer programme the well known COWPER-SYMonds-BODNER-formulation as it is e.g. described in PERZyna [8] has been used. It is a homogeneous viscous rate dependent material law which in one-dimensional form can be written as

\[ \frac{f_{\text{dyn}}}{f_0} = 1 + \left( \frac{\dot{\varepsilon}}{\dot{\varepsilon}_0} \right)^n \]  

(2)

where \( f_0 \) is approximately the quasi-static strength and \( \alpha \) and \( n \) are material dependent constants. For \( \dot{\varepsilon} = \dot{\varepsilon}_0 \) the dynamic material strength is doubled. It is a very similar formulation to eq. (1). An analog mathematical formulation has been adopted for concrete. This interpretation is justified taking investigations of ZECH, WITTmann [9]) into account.

3. Implementation into Computer Programme

For the implementation of the strain rate dependent material model into a computer programme the system FLOWERS developed by ANDERHEGGEN et al [10] at ETH Zurich has been selected. FLOWERS is a multipurpose programme system for static and dynamic, linear and non-linear analysis. One of the main advantage of the system is that the non-linear, dynamic analysis can be handled with an interactive, on-line procedure. By this way some specific output results (like displacements, stresses and strains at selected points) or convergence tolerance criteria (like displacement increments or cut-of-balance loads) can be continuously visualized on the terminal screen and if necessary, the calculation can be interrupted and restarted again from a restart point set earlier, but now e.g. with a reduced time (or load) increment, with a changed analysis procedure (from static to dynamic analysis or from implicit to explicit or vice versa) or with a modified strategy for the implicit analysis procedure (automatic selection of an adequate time-step, criteria for updating the stiffness-matrix, convergence criteria, etc., see also BAZZI [11]).

A standardized library of element models and material models with well defined user interfaces enables the user to introduce his own elements or material models into the programme system. To be able to analyse the tests with the simple and two-span continuous beams described in AMMANN [3] and MUEHLEMATTER [12] the above mentioned strain rate dependent, one-dimensional material model for reinforcing steel and for concrete was implemented into FLOWERS in combination with a layered element for two- and three-dimensional framed reinforced concrete structures.

4. Comparison of Test Results with Computer Calculations

For the comparison of experimental results with computer calculations the tests with a total of 4 two-span continuous beams, described in MUEHLEMATTER [12] and the tests with 23 simple beams described in AMMANN [3] have been chosen. The paper is restricted to the most complex case of a two-span
continuous beam, fixed in longitudinal direction, with sudden removal of the mid-support. Figure 2 shows an overview on the test set-up with the final deflection of the beam after the test. The span-width, cross-section and amount of reinforcing steel along the beam, the chosen element mesh and the position of two deflection measuring points are introduced in Fig. 3. For symmetry reasons only one half of the beam is modelled. During the tests the deflections at selected points, the support reactions and some strains on the reinforcing steel and on concrete have been measured continuously and stored on magnetic tape.

One span of the beam is modelled with eight layered elements. Ten layers of concrete and two separate layers for the reinforcing steel are used in an element. For the initial configuration of the two-span continuous beam a static analysis for the dead load has been performed. The sudden removal of the mid-support is modelled by a gap-element acting on compression only and which endures at the time of completion of static load incrementation a large initial positive strain so that a gap is formed. A new "static" system in the form of a simple, hinged-hinged beam is now acting. Due to this sudden change in the static system dynamic solicitations are induced. For this subsequent dynamic analysis an explicit time integration procedure is used with a time increment of 0.0002 seconds.

Figure 4 shows a comparison of the measured and calculated displacements at the two selected points MP 20 and MP 23 introduced in Fig.3. Up to t = 0.25s practically no discrepancies may be observed. Experimental observations show that during this first stage of solicitation the strain rates in the longitudinal reinforcing steel are very low (\( \dot{\varepsilon} = 0.05 - 0.1 \text{ s}^{-1} \), see MUEHLEMATTER [12]). For the case where no strain rate dependent material modelling is used (\( \dot{\varepsilon} = 0 \)) a difference of about 10% occurs for the maximum deflection. Taking account of the strain rate dependent material behaviour (\( \dot{\varepsilon} \neq 0 \)) leads to smaller discrepancies with the test results as can be seen in Fig. 4. Especially the maximum deflection is well represented. Experimental observations (see MUEHLEMATTER [12]) reveal that after t = 0.30 s strain rates up to \( \dot{\varepsilon} = 1.5 \text{ s}^{-1} \) can occur. This gives a reasonable explanation for the smaller discrepancies between test results and the \( \dot{\varepsilon} \neq 0 \)-results. On the other hand it has also to be said that the differences between the two calculations with and without strain rate dependent behaviour are not dramatic. This was also recognised in calculations with the simple beams under impulsive loading. Although final maximum deflections of more than 10% of the span-width occurred in the tests the recalculations with and without taking strain rate effects into account differed not more than a few percent. For this type of loading configuration the influence of strain hardening is about of the same order of magnitude (see also AMMANN [1]).
References


Fig. 1: Influence of an increased strain rate on the ultimate strength of hot-rolled reinforcing steel (from [7])

Fig. 2: Final deflection of a two-span continuous beam after sudden removal of the mid-support (test DML.1 from MUEHLEMATTER [12])

Fig. 3: Configuration of element-mesh

Fig. 4: Comparison of test results (test DML.1 from MUEHLEMATTER [12]) with computer results with and without taking strain rate effects into account