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Prediction of Elastic-Plastic Response of Structural Elements Subjected to Cyclic Loading

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Abstract

A simplified elastic plastic analysis is developed to predict stress strain and force deformation response of structural metalic elements subjected to irregular cyclic loadings. In this analysis a simple elastic plastic method for predicting the skeleton force deformation curve is developed. In this method, elastic and fully plastic solutions are first obtained for unknown quantities, such as deflection or local strains . Elastic and fully plastic contributions are then combined to obtain an elastic plastic solution. The skeleton curve is doubled to establish the shape of the hysteresis loop. The complete force deformation response can therefore be simulated through reversal by reversal in accordance with hysteresis looping and material memory. Several examples of structural elements with various cross sections made from various materials and subjected to irregular cyclic loadings, are analysed. A close agreement is obtained between experimental results found in the literature and present predictions.

1. Introduction

As an important step in the fatigue and dynamic analyses of structural elements, the local stress-strain history must be predicted from the load history applied to these elements. This necessitates using the irregular history stress-strain behaviour described in previous work [1] in conjuction with an elastic plastic analysis of these structural elements. Sophisticated elastic-plastic analysis techniques, such as finite elements could in theory be applied on a reversal by reversal basis during an applied irregular load history. However, this would be prohibitively expensive and time consuming for histories containing more than a few load reversals. Rigorous analysis is needed, which makes special provisions for handling cyclic and dynamic loads, so as to reduce this analysis task to reasonable proportions. A method for doing this is developed in this paper. This method can be used to simulate the deformation response of structural elements subjected to cyclic proportional loading and it will be described in this paper with applications to various examples of structural elements.

2. Procedure of Predicting Cyclic Deformation Response

Given a force history or deformation history, which is to be imposed on a structural element, the following procedure is implemented in the present work to predict the behaviour of the dependent force or deformation variable :

(1) A description of the cyclic force-deformation curve (skeleton curve) is first obtained from an experimental test, such as, an incremental step test or via a single elastic plastic analysis. The single analysis needed is one for monotonic loading along the cyclic stress-strain curve of the material. A simple elastic-plastic method for predicting the skeleton force deformation curve is proposed in section (3) of this paper.

(2) The skeleton curve shape is then doubled, using Masing's hypothesis, to establish the shape of the force-deformation hysteresis loop.

(3) With the knowledge of the imposed reversal by reversal history of the independent variable (either force or deformation), the complete force deformation behaviour is simulated through reversal by reversal application of the skeleton and doubled force-deformation curve in accordance with hysteresis looping and material memory behaviour [2].

This procedure was verified in reference [3], where it was shown that a beam in bending obeys moment - curvature relationships similar to the mathematical stress - strain rules for memory and hysteresis loop shape of uniaxial specimen. As a result of this study and similar work on notched specimens, previous investigators [2-3] have proposed the generalization that the above procedure would apply to any force deformation system of variables of more complex structural components.

Computer models are developed in the present work to implement the above procedure to simulate the cyclic response of structural elements subjected to cyclic loadings.

3. Simple Method for Predicting the Force Deformation Skeleton Curve

The cyclic force-deformation curve can be obtained using an elastic plastic analysis. This analysis is performed, just as for monotonic loading, except that the cyclic stress-strain curve is employed. A stable cyclic stress-strain curve given in the form of eq. (1) given below, is usually used and cyclic stress-strain curves corresponding to various specific numbers of cycles can also be used to simulate cyclic hardening and softening of the material.

$$\epsilon_a = \epsilon_{ea} + \epsilon_{pa} = \frac{\sigma_a}{E} + \left(\frac{\sigma_a}{A}\right)^{1/s} \qquad (1)$$

However, using the stress strain relation of the form given in eq.(1) in the elastic-plastic analysis of structural elements will result in complicated solutions requiring iterating technique even when the geometry of the structural element is simple. Therefore, a simplified technique is developed here to obtain rather simple solutions.

In this technique, linear elastic and fully plastic solutions are first obtained for unknown quantities, such as, deflection or local maximum strains. Elastic and fully plastic contributions are then combined to obtain an approximate elastic plastic solution. The advantage of the present proposed approach lies on the fact that in common structural configurations, linear elastic and fully plastic solutions can be easily obtained in a simple form. To check the accurancy of this prediction technique, several examples of simple structural elements are analysed, as given below. In these examples, both elastic and fully plastic solutions are obtained based on bending theory. However, for complicated geometries, a rather sophisticated techniques, such as, the finite elements is certainly needed. Based on this technique, an approximate elastic plastic moment-curvature and load-deflection, relations will be obtained for statically determinate beams of various cross sections, which typically exist in most of metalic structures.

3.1 Cyclic Moment Curvature Relation

According to the present approach, elastic and fully plastic solutions will be obtained separately. Therefore, the total strain distribution is separated into elastic and plastic strain distributions, as given by eq. (1). It is also assumed that plane sections remain plane and perpendicular to the longitudinal fibres of the beam and linear strain distributions can therefore be assumed. Applying equillibrium conditions for both elastic and plastic conditions and combining elastic and plastic components of strains an approximate elastic plastic solution is obtained as follows:

$$\epsilon_a = \epsilon_{ea} + \epsilon_{pa} = \frac{M_a}{K_1} + \left[\frac{M_a}{K_2}\right]^{1/S} \qquad (2)$$

To check the accuracy of eq. (2), several examples of structural elements loaded in pure and cantilever cyclic bending made from various materials were analysed. Constants employed in these equations are given in table (I). Comparison between predictions based on the above analysis and experimental results are shown in figure (1).

A close agreement is obtained between the present approach and the experimental results [4] shown in the figure. In addition, the present approach is in agreement with the exact solution given in reference [4]. Also, limit elastic and plastic solutions are given in figure (1) for comparison with other solutions. As expected, the elastic solution provides an accurate value at low applied moment values, and fully plastic solution predicts exact values of strains at very high values of applied bending moments, as shown in the figure.

3.2 Load Deflection Relations

The method discussed above can be followed also to predict an elastic plastic load-deflection relation of cyclically loaded beams. Elastic and fully plastic solutions are obtained first, then an approximate solution for the deflection can be obtained.

The method of virtual work has been used here to predict both elastic and plastic deflection values. Therefore, eq. (2) should be rearranged to obtain elastic and fully plastic formules for the slope at any distance. An elastic plastic solution for the deflection can therefore be approximated as follows:

$$\Delta = \Delta_e + \Delta_p = \frac{P_o}{F_1} + \left[\frac{P_o}{F_2}\right]^{1/S} \qquad (3)$$

Solutions given by eq. (3) are given in figures (2) and (3) for the case of cantilever beam with rectangular and rolled cross sections. A close agreement between analytical results based on eq. (3) and experimental results [5-6] of the above examples are indicated in figures (2) and (3). Values of the constants employed in the solution are given in tables (I) and (II).

4. Simulation of Cyclic Force Deformation Response

Following the procedure outlined in section (2) and employing the skeleton curves developed above, force deformation response of structural elements subjected to irregular cyclic loadings have been simulated, using a computer program developed in the present study [7]. Several examples of structural elements subjected to irregular cyclic loading were analysed using this computer program. These examples include rectangular beams loaded in pure and cantilever cyclic bending. The materials and dimensions of these structural elements are outlined in table (I) and (II). A close agreement is obtained between experimental [6] and predicted results, as shown in figure (4). Effects of hardening and softening on the cyclic response of structural elements can easily be simulated in the present computer program by simply employing the values of material constants A and S corresponding to a specific number of cycles as discussed in details in reference [7].

5. Conclusions

A simple elastic-plastic method for predicting the skeleton force-deformation curve is developed. In this method, elastic and fully plastic solutions are first obtained for unknown quantities, such as, deflection or local maximum strain. Elastic and fully plastic contributions are then combined to obtain an approximate elastic plastic solution.

The skeleton curve shape is then doubled, using Masing's hypothesis, to establish the shape of the force-deformation hysteresis loops. The complete force-deformation behaviour is simulated through reversal be reversal application of the skeleton and doubled force-deformation curve, in accordance with hysteresis looping and material behaviour described in the previous work.

Based on this approach, a computer program was developed. Several examples of structural elements with rectangular and rolled cross sections subjected to irregular cyclic loadings, were analysed using this program. A close agreement is obtained between experimental results found in the literature and present predictions.

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TABLE (I)

MATERIAL AND GEOMETRY CONSTANTS FOR FORCE-DEFORMATION RELATIONS FOR RECTANGULAR CANTILEVER

$$\epsilon_o = \frac{M_o}{K_1} + \left(\frac{M_o}{K_2}\right)^{1/S} \qquad \text{Equ. (2)}$$

$$K_1 = \frac{2}{3} E b c^2 \quad , \quad K_2 = \frac{2Abc^2}{S+2}$$

$$\delta = \frac{P_o}{F_1} + \left(\frac{P_o}{F_2}\right)^{1/S} \qquad \text{Equ. (3)}$$

$$F_1 = \frac{2Ebc^3}{L^3} \quad , \quad F_2 = \frac{2Abc^{S+2}(1/S+2)^S}{(S+2)\,L^{1+2S}}$$

MATERIAL	E MPa	A MPa	S	b cm	c cm	GEOMETRY
R Q C 100 STEEL	199955	903.2	009	*	*	PURE BENDING
1020 MILD STEEL	206860	944.6	0.22	0.635	0.635	** CANTILEVER OF SPAN 14.6 cm

* Value of bc^2 is assumed one.

** In case of cantilever $M_o = P_o L$, where P_o is the applied load, and L is the span of the cantilever.

TABLE (II)

MATERIAL AND GEOMETRY CONSTANTS FOR LOAD-DEFLECTION RELATIONS FOR ROLLED CANTILEVER

$$\Delta = \frac{P_a}{F_1} + \left(\frac{P_a}{F_2}\right)^{1/S} \qquad \text{Equ. (3)}$$

$$F_1 = \frac{3EI}{L^3} \quad , \quad F_2 = 2A(1/S+2)^S \left[(b_1-b_2)\,c_1^{S+2} + b_2\,c_2^{S+2}\right] / (S+2)(L)^{1+2S}$$

MATERIAL	E MPa	A MPa	S	W18X50 ROLLED SECTION b_1 cm	b_2 cm	c_1 cm	c_2 cm	I cm^4	L cm
A 36 STEEL	206850	1227.3	0.22	0.91	19.05	21.41	22.86	33423.4	236.2

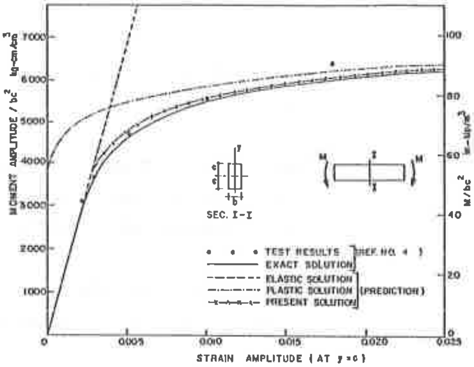

Figure 1 Actual and Predicted Moment Strain Relation for Cyclic Loading of Rectangular Beams.

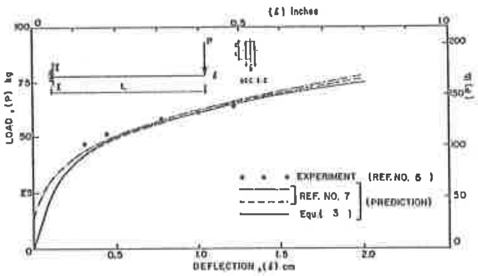

Figure 2 Actual and Predicted Load-Deflection Relation for Cantilever Beam.

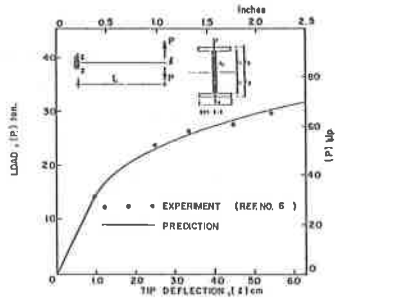

Figure 3 Actual and Predicted Load Deflection for Rolled Cantilever Beam.

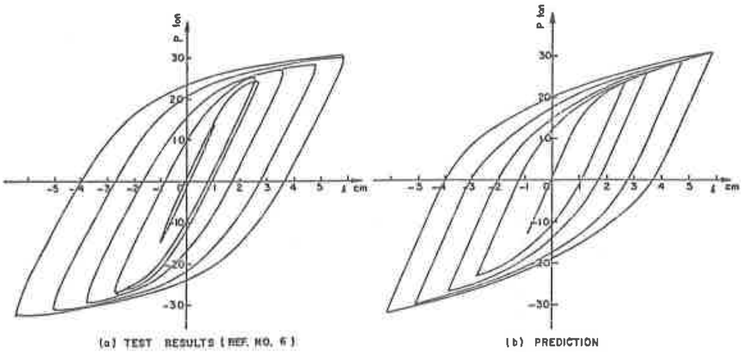

Figure 4 Actual and Predicted Cyclic Load-Deflection Response for Rolled Cantilever Beam.